An Application of Active Magnetic Bearing to Gyroscopic and Inertial Sensors*

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Abstract
A new gyroscopic sensor using active magnetic bearing (AMB) is studied both theoretically and experimentally. The proposed sensor works as a two-axis gyroscopic sensor and also as a three-axis servo-type accelerometer, which utilizes the control function of AMB. Angular velocities and accelerations are estimated from the control signals for canceling gyroscopic torque and inertial forces acting on the shaft of the AMB. In measuring two-axis angular velocity simultaneously, the control signals canceling inertial torque must be considered in addition to the gyroscopic torque for precise measurement. This paper presents the principles of two-axis angular velocity and acceleration measurement by AMB. In order to investigate the feasibility of the proposed sensor, several basic studies are carried out with a conventional-size AMB.

Key words: Gyroscopic Sensor, Active Magnetic Bearing, Servo-Type Accelerometer, Angular Velocity Measurement, Multi-Axis Sensor

1. Introduction
This paper proposes a new gyroscopic sensor. In this sensor, active magnetic bearing (AMB) is applied to gyroscopic sensor. Gyroscopic sensors are used to measure angle or angular velocity. They have many applications such as navigation, homing and attitude control. In the fields of automobiles, robots and unmanned micro-airplanes, compact and high-accuracy gyroscopic sensors are required to achieve sophisticated motion and attitude control. Gyroscopic sensors are classified into various types according to the principle of operation. Vibrating gyros are mostly used in compact and low-cost applications because they are suitable for minimization and mass production. Recently MEMS technology has been used for them (1), (2). However, their detection sensitivity is lower than the other types, and they are therefore not suitable for high-performance applications. On the other hand, rotating gyros have high detection sensitivity than the other types. We focus attention on rotating gyros and apply AMB to them to realize a compact, high-accuracy, multi-axis, and low-cost sensor. It is to be mentioned that the miniaturization of sensors using AMB is possible because several micro magnetic bearings have already been developed (3), (4).

In the previous researches (5), (6), magnetic bearings have been applied to rotating gyros and floated pendulous accelerometers in order to realize high-performance sensors and the availability has been proved. In these works, AMB has only been used to support a proof mass or a rotor without friction in these sensors. In contrast, AMB is used to detect angular
velocity and acceleration in addition to non-friction support in the proposed sensor. AMB itself works as a two-axis gyroscopic sensor and also as a three-axis servo-type accelerometer, which utilizes control signals to cancel gyroscopic torque and inertial force acting on the AMB shaft. Single axis angular velocity measurement using a conventional-size AMB has been carried out as a first step (7). The results have indicated the feasibility of the proposed sensor. However, acceleration and two-axis angular velocity measurements have not been tried. This paper will present the principle of acceleration and two-axis angular velocity detections by AMB. In order to examine the principle, acceleration and two-axis angular velocity measurements are carried out.

2. Rotating Gyro

2.1 Gyroscopic action

Gyroscopic action is a motion that is based on the law of angular momentum. When a rotor spins about the $z$-axis at a constant angular velocity of $\omega_z$, a precession appears about the $y$-axis if torque acts about the $x$-axis in a right-hand coordinate system, as illustrated in Fig. 1. A precession about the $x$-axis also appears if torque acts about the $y$-axis. This torque is presented as the product of angular momentum $H$ and angular velocity of precession, $\omega_x$ or $\omega_y$. The equations of the torque are written as follows.

\[
\begin{align*}
T_x &= H\omega_x, \\
T_y &= -H\omega_x,
\end{align*}
\]

where

\[
H = I_z\omega_z,
\]

$T_x$: gyroscopic torque about the $x$-axis,

$T_y$: gyroscopic torque about the $y$-axis,

$I_z$: moment of inertia about the $z$-axis.

2.2 Conventional rotating gyro

A rate-integrating gyro is illustrated in Fig. 2 as an example of conventional rotating gyro. A flywheel is mounted on a gimbal, and the gimbal is supported by bearings. The flywheel spins at high rate. It has angular momentum, $H$. If the gyro itself rotates about the input axis, gyroscopic action rotates the flywheel with the gimbal about the output axis that is orthogonal to the spin axis and the input axis. A pickoff detects the relative angle of the gimbal to the base. In order to cancel the deviation, the pickoff output drives a servo amplifier that supplies current to the torque motor, commonly called torquer. Angular velocity can be measured based on the current because the current is proportional to the control torque canceling the gyroscopic torque. The detection sensitivity of a rotating gyro increases if the spin rate of the flywheel is high. Therefore, a rotating gyro is superior to other gyros in terms of detection sensitivity. However, friction in the bearings that support gimbal and flywheel generates drag torque. The friction also causes a drift in the output.

![Fig. 1 Gyroscopic action.](image-url)
Therefore sometime special bearings are used in the gyro or the flywheel is in liquid to solve the problems (8).

3. Magnetic Levitation Gyro

In this study, a five-degree-of-freedom active magnetic bearing illustrated by Fig. 3 is applied to gyroscopic sensor. We refer to the gyroscopic sensor as magnetic levitation gyro. Ten electromagnets and five displacement sensors are used in AMB. Two electromagnets (9, 10) control the axial translation of the AMB shaft. The other electromagnets (1 - 8) control the radial translations and rotations. These electromagnets and sensors control the five-axis motions independently. When the AMB stator moves, the relative displacement of the AMB shaft to the stator changes due to the inertia of the shaft. The control system works to reverse the displacement and return the shaft to its original position. The force required for the recovery can be estimated from the control signal. Therefore, AMB has a function of servo-type accelerometer. In addition, if the shaft spins, gyroscopic torque appears and the torque can be estimated from the control signal. The angular velocity can be determined according to Eq.(1). Therefore it is possible to measure the five-axis motions of the measuring object on which the AMB is mounted. In the proposed gyro there is no problem caused by friction since the shaft is supported by magnetic force. The gyro does not need any gimbal mechanism because AMB works as a two-axis gimbal. Therefore, high-accuracy, compact and low-cost sensor can be realized.

3.1 Principle of detecting acceleration

The principle of detecting acceleration is presented for three translations. Since there is no interaction between the three motions, the detection is performed independently. Inertial force acting on the shaft of AMB balances the control force produced by the electromagnets as mentioned above. The force balance equation in the $x$-direction is written in the following.

\[-ma_x = F_{ex},\]

Input axis

Spin axis

Gimbal Torquer

Pickoff

Bearing

Bearing

Output axis

Servo amplifier

Fig. 2  Schematic view of a rate integrating gyro.

\[1 - 10 : \text{Electromagnets}\]

Fig. 3  Schematic view of five-degree-of-freedom (5 DOF) active magnetic bearing.
where
\[ a_x : \text{acceleration in the } x\text{-axis direction}, \]
\[ F_{cx} : \text{control force in the } x\text{-axis direction}, \]
\[ m : \text{mass of the shaft}. \]

It is assumed that all the electromagnets are same. The control force is given by
\[ F_{cx} = K_i u_x - 4K_d x, \quad (4) \]
where \( x \) and \( u_x \) are the relative displacement of the shaft to the stator and the control input in the \( x\)-axis direction, respectively. \( K_i \) and \( K_d \) are the values determined from the characteristics of the electromagnet, the bias current and the gap between the electromagnet and the shaft at steady state. Substituting Eq.(4) into Eq.(3) gives
\[ -ma_x = K_i u_x - 4K_d x. \quad (5) \]
If the relative position of the shaft to the stator is maintained by active control, \( x \) is zero. Then Eq.(5) becomes
\[ a_x = -bu_x, \quad (6) \]
where
\[ b = K_i / m. \quad (7) \]

Equation(5) is an estimation equation for the \( x\)-axis acceleration. In a similar way, estimation equations for the \( y\)-and \( z\)-axis acceleration are written as follows.
\[ a_y = -bu_y, \quad (8) \]
\[ a_z = -bu_z. \quad (9) \]

3.2 Principle of detecting two-axis angular velocity

Angular velocities can be estimated from the control signal for canceling gyroscopic torque coming from angular velocity. In measuring two-axis angular velocity simultaneously, the control signals for canceling inertial torque coming from angular acceleration must be considered in addition to the gyroscopic torque for precise measurement. Estimation equations considering and neglecting the inertial torque are presented in this section.

Figure 4 shows a model of rotational motions in AMB. The angle \( \phi \) is the angular displacement of the shaft of AMB, \( \varphi \) is the angular displacement of the stator of AMB, and \( \Theta \) is the relative angle of the shaft to the stator. The subscript (\( x \) or \( y \)) indicates the motion about each axis. Thus,
\[ \begin{cases} \Theta_x = \phi_x - \varphi_x, \\ \Theta_y = \phi_y - \varphi_y. \end{cases} \quad (10) \]

When the stator rotates about the \( x \) and \( y\)-axis, the control torque \( T_{cx} \) and \( T_{cy} \) produced by the electromagnets, inertial torque and gyroscopic torque act on the shaft as illustrated in Fig. 4. Assuming that the shaft is axially symmetric, the torque balance equations are written as follows.

![Fig. 4 A model of rotational motions in a 5 DOF active magnetic bearing.](image)
\[
\begin{align*}
-I_x\ddot{\phi}_x - I_y\omega_y\dot{\phi}_y + T_{cx} &= 0, \\
-I_y\ddot{\phi}_y + I_x\omega_x\dot{\phi}_x + T_{cy} &= 0,
\end{align*}
\]

where

\[ I_x : \text{moment of inertia of the shaft about the } x \text{ and } y \text{-axis.} \]

Equation (11) becomes

\[
\begin{align*}
\dot{\phi}_x + a_k\dot{\phi}_y &= T_{cx} / I_x, \\
\dot{\phi}_y - a_k\dot{\phi}_x &= T_{cy} / I_x,
\end{align*}
\]

where

\[ a_k = I_x\omega_x / I_y. \]

The parameter \( a_k \) represents the effect of gyroscopic torque.

Adding \[-\dot{\phi}_y - a_k\dot{\phi}_x\] to both side of Eq.(12) gives the solution:

\[
\begin{align*}
\dot{\theta}_x + a_k\dot{\theta}_y &= -\dot{\phi}_x - a_k\dot{\phi}_y + T_{cx} / I_x, \\
\dot{\theta}_y - a_k\dot{\theta}_x &= -\dot{\phi}_y + a_k\dot{\phi}_x + T_{cy} / I_x.
\end{align*}
\]

It is assumed that all the electromagnets are located at a distance of \( l \) from the center of gravity of the shaft. The control torque is given by (9)

\[
\begin{align*}
T_{cx} &= K_x l u_{\theta x} + 4K_d l^2 \theta_x, \\
T_{cy} &= K_y l u_{\theta y} + 4K_d l^2 \theta_y,
\end{align*}
\]

where \( u_{\theta} \) is the control signal. Substituting Eq.(15) into Eq.(14) gives

\[
\begin{align*}
\dot{\theta}_x + a_k\dot{\theta}_y &= b_\theta u_{\theta x} - \dot{\phi}_x - a_k\dot{\phi}_y, \\
\dot{\theta}_y - a_k\dot{\theta}_x &= b_\theta u_{\theta y} - \dot{\phi}_y + a_k\dot{\phi}_x,
\end{align*}
\]

where

\[ b_\theta = K_x / l^2, \]

\[ K_y = 4K_d l^2 / l. \]

If the relative position of the shaft to the stator is maintained by active control, the left-hand side of Eq.(16) is zero. Then Eq.(16) becomes

\[
\begin{bmatrix}
u_{\theta x} \\
u_{\theta y}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\phi}_x \\
\dot{\phi}_y
\end{bmatrix}
\begin{bmatrix}
0 & a_k \\
a_k & 0
\end{bmatrix}
\begin{bmatrix}
\phi_x \\
\phi_y
\end{bmatrix} = 0.
\]

The Laplace transform of Eq.(19) is

\[
\begin{align*}
\mathcal{L}\left[u_{\theta x}(s)\right] &= \frac{1}{s^2 + a_k s + b_\theta}, \\
\mathcal{L}\left[u_{\theta y}(s)\right] &= \frac{1}{s^2 + a_k s + b_\theta}.
\end{align*}
\]

where \( \psi_x(s) \) and \( \psi_y(s) \) are the Laplace transforms of \( \dot{\phi}_x(t) \) and \( \dot{\phi}_y(t) \), respectively. From Eq.(21), we get

\[
\begin{bmatrix}
\dot{\psi}_x(s) \\
\dot{\psi}_y(s)
\end{bmatrix}
\begin{bmatrix}
\frac{b_\theta}{s^2 + a_k s + b_\theta} \\
\frac{1}{s^2 + a_k s + b_\theta}
\end{bmatrix}
\begin{bmatrix}
\psi_x(s) \\
\psi_y(s)
\end{bmatrix} = \begin{bmatrix}
u_{\theta x}(s) \\
u_{\theta y}(s)
\end{bmatrix},
\]

where \( \dot{\psi}_x \) and \( \dot{\psi}_y \) are estimate values of \( \dot{\psi}_x \) and \( \dot{\psi}_y \), respectively. Equation(21) shows that the two-axis angular velocities can be estimated from the control signals for two-axis rotations. However oscillating components appear in the estimated signals because the transfer function in the right hand side of Eq.(21) has neutral poles \( (s = \pm j\omega) \). They may be canceled if \( a_k \) in Eq.(20) is perfectly identified. Actually, however, the cancellation is imperfect because the spinning speed fluctuates in actual equipments. Assumed that \( \omega \ll a_k \), Eq.(21) is approximated as

\[
\begin{align*}
\dot{\psi}_x(s) &= \frac{b_\theta}{s^2 + a_k s + b_\theta} \begin{bmatrix}
u_{\theta x}(s) \\
u_{\theta y}(s)
\end{bmatrix}, \\
\dot{\psi}_y(s) &= \frac{1}{s^2 + a_k s + b_\theta} \begin{bmatrix}
u_{\theta x}(s) \\
u_{\theta y}(s)
\end{bmatrix}.
\end{align*}
\]
The inertial torque is considered in Eq.(22). If the inertial torque is ignored, the $\phi_1$ and $\phi_2$ are zero in Eq.(19). The solution is given by

$$
\begin{bmatrix}
\dot{\psi}_x(s) \\
\dot{\psi}_y(s)
\end{bmatrix} = \frac{b_s}{a_s} \begin{bmatrix}
0 & -a_k \\
0 & a_k
\end{bmatrix} \begin{bmatrix}
0 \\
u_{in}(s)
\end{bmatrix}.
$$

(23)

4. Acceleration Measurement

4.1 Experimental apparatus and method

In order to verify the principle of detecting acceleration as explained in the previous section, several experiments are carried out with a conventional-size AMB system. An experimental setup for acceleration measurement is shown in Fig. 5. The AMB is fixed on a turntable. The distance of the central axis (the $z$-axis) of the AMB from the rotational axis of the table is 90 mm. The central axis points to the vertical direction. The $x$- and $y$-axes correspond to the tangential and centripetal directions, respectively. A servo-type accelerometer for reference is mounted on the AMB. Its detection axis coincides with the $x$-axis of the AMB. Figure 5 shows the setup for acceleration measurement in the $x$-axis direction. In the $y$-axis measurement, the setup is changed for the $y$-axis to coincide with the tangential direction. The table is made to swing sinusoidally for generating acceleration to the AMB and the accelerometer. The mass of the shaft is 0.87 kg. The shaft does not spin about the $z$-axis in acceleration measurement. Equations(6) and (8) are used to estimate acceleration in the following measurements. The AMB is controlled independently in the five-axis (the $x$-, $y$- and $z$-axis translations and the $x$- and $y$-axis rotations) by I-PD controllers. The frequency responses of the closed-loop system in the $x$ and $y$-axis direction

![Fig. 5 A view of experimental setup for acceleration measurement.](image)

![Fig. 6 Frequency response of the closed-loop system in the $x$- and $y$-axes translation.](image)
are shown in Fig. 6, where the input is disturbance and the output is the displacement of the shaft in Fig. 6 (a) and the control force in Fig. 6 (b), respectively. The upper limit of control bandwidth was approximately 30 Hz. This frequency determines the upper limit of acceleration measurement bandwidth.

4.2 Experimental results of acceleration measurement

Figures 7 and 8 show the results of acceleration measurement when the turntable is driven to swing sinusoidally at frequencies of 1 Hz and 2 Hz, respectively. These results were obtained through a low-pass filter whose cutoff frequency was 40 Hz. There is approximately 30% error in amplitude between the acceleration measured by the AMB and the accelerometer for reference in both Fig. 7 and Fig. 8. The frequency responses in the measurements are shown in Fig. 9, where the input is the acceleration measured by the accelerometer and the output is the acceleration measured by the AMB. This transfer function should be same theoretically as the transfer function from the disturbance to control force as shown in Fig. 6 (b). In this measurement, they agree above 0.5 Hz while don’t agree below 0.5 Hz. In Fig. 9, errors appear because signal-to-noise ratio is small in the latter region. The gain is constant and the phase is nearly zero (within 4 degrees) in a range of 0.5 Hz to 10 Hz. The errors of amplitude can be eliminated by proper calibration so
that precise acceleration measurement using AMB is possible in a range of 0.5 Hz to 10 Hz.

5. Two-axis Angular Velocity Measurement

5.1 Experimental apparatus and method

An experimental setup for angular velocity measurement is shown in Fig. 10. The AMB and a high-accuracy fiber optical gyro for reference (FOG) are fixed on the turntable. The center of gravity of the shaft is located above the center of the table. The z-axis points to the horizontal direction. The input axis points to the vertical direction that is at 45 degrees to the x- and y-axes. Angular velocity is generated to the AMB and the FOG by swinging the table sinusoidally. Same angular velocity was generated about the x- and y-axes. The AMB shaft was driven by an induction motor at 155 rps about the z-axis. The shaft runs freely at 155 rps when the measurements are carried out. Equations (22) and (23) are used to estimate the angular velocity in the following measurements. The frequency responses of the closed-loop system about the x- and y-axes are shown in Fig. 11, where the input is disturbance and the output is the angle of the shaft in Fig. 11 (a) and the control torque in Fig. 11 (b), respectively. The upper limit of control bandwidth is approximately 30 Hz. This frequency determines the limit of angular velocity measurement bandwidth.

5.2 Experimental results of two-axis angular velocity measurement

The results of two-axis angular velocity measurement are shown in Figs. 12 and 13. These results are obtained through a notch filter for eliminating a component of unbalance vibration. Figures 12 and 13 show the angular velocities when the turntable is driven to swing sinusoidally at frequencies of 1 Hz and 2 Hz, respectively. The upper graphs are the estimations based on Eq.(23) neglecting inertial torque. The lower graphs are the
estimations based on Eq. (22) considering inertial torque. The amplitude errors are approximately 10% in both. The phase difference between the AMB and the FOG is smaller when inertial torque is considered than neglected. The frequency responses in the measurements are shown in Fig. 14. The estimation neglects the inertial torque in Fig. 14 (a) while considers it in Fig. 14 (b), where the input is the angular velocity measured by the FOG and the output is the angular velocity measured by the AMB. Angular velocity measurement is possible with proper calibration if the gain is constant and the phase is zero in these responses. The bandwidths are below 1 Hz in Fig. 14 (a) and 3 Hz in Fig. 14 (b), respectively.

In Fig. 14 (b), zeros are observed at $\omega_k \text{ rad/s}$ (10 Hz in these experiments) due to approximation as presented in section 3.2 and limit the bandwidth of the angular velocity.

![Fig. 12 Two-axis angular velocity measurement when the turntable is driven at 1 Hz.](image)

![Fig. 13 Two-axis angular velocity measurement when the turntable is driven at 2 Hz.](image)

![Fig. 14 Frequency responses of two-axis angular velocity measurement (the AMB / the FOG).](image)

(a) Using estimation equation (23)  
(b) Using estimation equation (22)
estimation. The upper limit of the bandwidth is 10 % of $a_k$ in Fig. 14 (a) while 30 % in Fig. 14 (b). Therefore, considering inertial torque extends the bandwidth of angular velocity measurement. The bandwidth can be extended further by increasing $a_k$. Increasing $a_k$ also improves detection sensitivity.

6. Conclusion

Gyroscopic sensor using AMB was proposed. The principles of detecting acceleration and two-axis angular velocity in the proposed sensor were derived based on the dynamics of AMB. Several experiments were carried out to examine the principles. Amplitude error was within 30 % and there was little phase difference in a range of 0.5 Hz to 10 Hz in acceleration measurement. Amplitude error was within 10 % and there was little phase difference as a frequency region lower than 30 % of $a_k$ in two-axis angular velocity measurement using estimation equation considered inertial torque. The results have demonstrated that AMB can measure acceleration and two-axis angular velocity; that the bandwidth of angular velocity measurement was extended by considering inertial torque; and that the bandwidth can be extended further by increasing $a_k$. In future, we will fabricate an AMB with large $a_k$ and minimize the AMB. More sophisticated methods of control and signal processing will be investigated for improving the accuracy of the proposed sensor.

References


