A Concept of Shock Absorption Mechanisms Using Buckling Phenomena and Hysteretic Spring Elements

Arata MASUDA**, Yohei MORI** and Satoshi SHIMABUKURO**

** Department of Mechanical and System Engineering, Kyoto Institute of Technology
Matsugasaki, Sakyo-ku, Kyoto, 606-8585, Japan
E-mail: masuda@kit.ac.jp

Abstract

In this paper, a preliminary study on a shape-recoverable shock absorption structure consisting of hysteretic spring elements combined with a buckling mechanism which could achieve both high energy absorption efficiency and high energy dissipation efficiency is presented. In order to illustrate the basic idea, a conceptual model consisting of a two-link mechanism and a hysteretic spring is investigated. The two-link mechanism with axial and lateral springs is frequently used as an elementary model for the buckling behavior of thin-walled structures. Without the lateral spring, it has a pair of stable equilibriums and one unstable equilibrium between them, so that it shows a snap-through buckling instability when loaded in the lateral direction. The load-deformation curve changes depending on the quantity of the lateral spring constant undergoing saddle-node bifurcation. After the bifurcation, the stable and unstable equilibriums except for the origin vanish, and only one stable equilibrium remains at the origin. This implies that, when combined with a strong spring, the load-deformation curve of the two-link mechanism has only a stable equilibrium at the origin, whereas it has another stable equilibrium when combined with a weak spring. Therefore, if combined with an appropriate hysteretic spring, the mechanism subjected to an impact will yield positive resistive force in the loading path, followed by stopping at the stable equilibrium with residual deformation when unloaded. After the impact, the residual deformation will be eliminated by applying a small amount of energy to trigger the bifurcation again. A proof-of-concept experimental model consisting of a two-link mechanism and a superelastic shape memory alloy (SMA) spring is developed for the demonstrative purpose. A prototype of superelastic SMA-based “woven” shells is also developed to show a possible application of the proposed concept.

Key words: Shock Absorption, Buckling, Bifurcation, Hysteresis, Shape Memory Alloy, Smart Structure

1. Introduction

Shell structures are lightweight, high-strength structures being utilized in many engineering fields such as automobiles, airplanes, space vehicles, ships, pressure vessels and tanks. Shell structures themselves perform as critical structural members that support and resist the structural loads, but also as “outer skin” of the structure that isolate the system from the external world and protect it from the environmental disturbances. Particularly for the transportation vehicles, their outer shells are exposed to the risk of the impact of objects. The impact of the objects may cause large deformation of the shell. In terms of reducing the impact load, large deformation is welcome because it will decrease the maximum reaction force, but may yield plastic deformation usually coming along with buckling to absorb the impact energy. Such large deformation with permanent change of the geometry may not only directly harm the instruments inside but also mar the geometric advantage of the shell structure and
turn it vulnerable. The consequent sudden loss of strength of the structure may lead to further damage and even catastrophic failure. Therefore, a shape-recoverable shock absorption structure is expected that has high performance of energy absorption without permanent change of geometry after the large deformation induced by the impact.

In terms of impact protection and crashworthiness, a number of energy-absorbing structures have been developed and utilized, particularly in the field of vehicle protection and personal safety. Most of them have been made of ductile materials like aluminum and steel, achieving high energy absorption and dissipation efficiencies by exploiting the nonlinear effects of plasticity and buckling deformation\(^{(1)}\). Such energy-absorbing structures, however, are designed for one-time use, not for multiple or repeated impact scenarios. An energy-absorbing mechanism with shape-recoverability is demanded to make the structure bearable even for successive impact load.

In these contexts, shape memory alloy (SMA) has been increasingly recognized as an attractive candidate for energy-absorbing material that could realize such shape-recoverable shock absorption structures because of its unique characteristics such as shape memory effect, superelasticity and remarkable energy-absorbing capacity. Lagoudas et al. examined the dynamic behavior of porous SMA\(^{(2)}\) and a SMA rod\(^{(3)}\) under an impact loading. Nemat-Nasser et al.\(^{(4)}-\(^{(6)}\) investigated the high strain-rate response of superelastic SMA rods and the rate-dependency of their phase transformation stress. They also reported an experimental study on the dynamic buckling and the shape-recovery of SMA thin cylindrical shells\(^{(7)}\). Suzuki et al.\(^{(8)}\) revealed that the anti-buckling effect of SMA column of the shape memory effect grade resulted in the larger capacity of energy absorption, preventing the generation of localized plastic hinges. Minakuchi et al.\(^{(9)}\) developed a honeycomb core made of SMA of the shape memory effect grade and a sandwich plate, and examined their shape-recoverability after impact.

As discussed in those precedent works, there are two principles for SMA to realize shape-recoverable shock absorption structures, i.e., shape memory effect and superelasticity. With the former effect, however, the whole deformed area in the SMA member has to be heated above the \(A_f\) temperature to recover to the original shape, that may require long time for full recover. In contrast, the SMA member with the latter effect can recover quickly and spontaneously. The use of superelasticity, however, may cause the risk of the deterioration of energy dissipation efficiency, which will lead to unexpected rebound of the colliding object.

In this paper, an approach to realize a shock absorption structure that can achieve both high energy absorption efficiency and high energy dissipation efficiency with quick recovering capability is presented by exploiting the geometrical nonlinearity of the buckling phenomena and shape-recoverable hysteretic material such as superelastic SMA. The shock absorption mechanism involved in the proposed structure is essentially composed of a snap-through buckling mechanism and a shape-recoverable hysteretic spring element. The proposed structure responds to the colliding object as a softening spring in the loading phase, then stops with residual deformation that will prevent the colliding object from rebounding. After the impact, it can return to its original shape using the elastic energy stored inside the structure, so that the shape-recovery can be very quick. In order to illustrate the basic idea, a conceptual model consisting of a two-link mechanism and a hysteretic spring is investigated both theoretically and numerically. Then, a proof-of-concept experimental model consisting of a two-link mechanism and a superelastic SMA spring is developed for the demonstrative purpose. A prototype of a superelastic SMA-based “woven” shell is also developed to show a possible application of the proposed concept.

2. Concept of the Proposed Structure

2.1. Characteristics of Shock Absorption Mechanisms

The fundamental function of a shock absorption mechanism is to give a reaction force against a colliding object to vanish its momentum and dissipate the kinetic energy to prevent
the rebound. The basic performance of the shock absorption mechanism is measured by two efficiency indices: energy absorption efficiency $\eta_a$ and energy dissipation efficiency $\eta_d$. The energy absorption efficiency is the ratio of absorbed energy depicted by $W_a$ in Fig. 1 to the maximum energy absorption capacity $P_{\text{max}} \cdot x_{\text{max}}$ where $P_{\text{max}}$ and $x_{\text{max}}$ are the maximum reaction force and the maximum stroke, respectively. Meanwhile, the energy dissipation efficiency is the ratio of dissipated energy depicted by $W_d$ in Fig. 1 to the absorbed energy $W_a$. Denoting the force-displacement relationship by $P_+(x)$ for the loading phase, and by $P_-(x)$ for the unloading phase, they are respectively defined as

$$\eta_a = \frac{W_a}{P_{\text{max}} x_{\text{max}}} = \frac{\int_{x_0}^{x_{\text{max}}} P_+(x) \, dx}{P_{\text{max}} x_{\text{max}}}$$

and

$$\eta_d = \frac{W_d}{W_a} = \frac{\int_{x_0}^{x_{\text{max}}} [P_+(x) - P_-(x)] \, dx}{\int_{x_0}^{x_{\text{max}}} P_+(x) \, dx}$$

Maximizing $\eta_a$ means minimizing the maximum impact load under the restricted stroke. Obviously, the possible maximum value of $\eta_a$ is one when the force-displacement curve forms a rectangle, i.e., $P_+(x) = P_{\text{max}}$ and $P_-(x) = 0$. On the other hand, maximizing $\eta_d$ means minimizing the energy returned to the colliding object. The possible maximum value of $\eta_d$ is one when the reaction force in the unloading phase $P_-(x)$ is identically zero.

The purpose of this study is to develop a self-recoverable shock absorption mechanism which could achieve both high energy absorption efficiency and high energy dissipation efficiency without leaving permanent change of geometry. To realize the shape-recoverability, the nonlinear bifurcation phenomena associated with snap-through buckling will be utilized as described in the following subsections.

2.2. Two-Link Model of Snap-Through Buckling and Its Bifurcation

In order to illustrate the basic idea of utilizing snap-through buckling and its bifurcation, a conceptual model introducing a two-link mechanism is investigated. The two-link mechanism $^{(10)-(12)}$ is frequently adopted as an elementary model to explain the snap-through buckling of thin-walled structures with small curvature as illustrated in Fig. 2 (a). It is composed of two massless rigid links rotatively connected at the center, two axial compressive springs representing the tangential (or horizontal, in Fig. 2 (a)) stiffness and a lateral spring representing the radial (or vertical, in Fig. 2 (a)) stiffness of the structure as schematically shown in Fig. 2 (b).

Let us firstly consider the case without the lateral spring. The reaction force against the displacement in the lateral direction is given by

$$P(x) = 2k_a \left( x - L \sin \theta_0 \right) \left\{ \frac{L \cos \theta_0}{\sqrt{L^2 - (x - L \sin \theta_0)^2}} - 1 \right\}$$

Fig. 1 Characteristic diagram of shock absorption mechanism.
where $x$ denotes the lateral displacement measured from the equilibrium position (the positive direction is specified by an arrow in Fig. 2 (b)), $\theta_0 > 0$ denotes the base angle of the links in the equilibrium position, $L$ is the length of the links and $k_a$ is the stiffness of the axial springs. Introducing a dimensionless parameter $\alpha = \tan \theta_0 > 0$, dimensionless displacement $x = x/X$ and dimensionless reaction force $P = F/F$, where the characteristic length $X$ and the characteristic force $F$ are chosen as $X = L \sin \theta_0$ and $F = 2k_a X \alpha^2$, respectively, the Eq. (3) will be rewritten as

$$P(x) = \frac{1}{\alpha^2} \left( \frac{1}{\sqrt{1 - \alpha^2 x(x - 2)}} - 1 \right)$$

This function is odd with respect to $x = 1$, having three equilibrium points at $x = 0, 1, 2$. The slopes at $x = 0$ and $x = 2$ are positive equal to 1, thus these equalibriums are stable, whereas the slope at $x = 1$ is found to be negative given by $-\frac{1}{2} \frac{1}{\sqrt{1 - \alpha^2}} = -\frac{1}{2} + O(\alpha^2)$, so that the equilibrium at this point, corresponding to the neutral position where the links align straight, is unstable. The function $P(x)$ is reasonably approximated by a cubic function $P_c(x)$ defined as

$$P_c(x) = \frac{1}{2} x(x - 1)(x - 2)$$

in the sense of

$$P(x) = P_c(x) + O(\alpha^2)$$

so that the cubic function $P_c(x)$ is hereafter used as the force-displacement function to describe the fundamental behavior of the mechanism.

Equation (5) implies that, in this configuration, the two-link mechanism behaves as a nonlinear spring that is stable with unit initial stiffness at the origin, and has extra equalibriums at $x = 1, 2$ which are unstable and stable respectively. When positively loaded in the lateral direction starting from the original equilibrium at $x = 0$, it initially behaves as a softening spring, but then shows snap-through buckling instability in the vicinity of the neutral position at $x = 1$, and jumps to the opposite side. After unloading, the mechanism settles in the stable position at $x = 2$ and never goes back to the original position.

Let us next consider to add the lateral spring with the dimensionless spring constant of $k$ between the center joint and the ground. The force-displacement function is then modified as

$$P(x) = \frac{1}{2} x(x - 1)(x - 2) + kx$$
where the overlines standing for the dimensionless values are omitted in order to simplify the notation. The equilibriums of the above system drastically change depending on the quantity of the lateral spring constant $k$ as one can see in Fig. 3, in which the force-displacement curves for $k = 0, 0.1, 1/8$ and $0.3$ are shown. For $k = 0$, the unstable and stable equilibriums are located at $x = 1$ and $x = 2$. When increasing the value of $k$, these two equilibriums move closer to each other and finally merge when $k = 1/8$. For the greater $k$, they vanish and only one stable equilibrium remains at the origin. This is a saddle-node bifurcation, and the mechanical behavior significantly changes across the bifurcation point: the mechanism indeed undergoes snap-through buckling when loaded, but when unloaded, it settles in the positive stable position before the bifurcation, whereas it returns to the original position after the bifurcation.

2.3. Basic Idea and Use of Shape Memory Alloy

As observed in the previous subsection, the mechanical properties of the two-link model with the lateral spring drastically change depending on the value of the lateral spring constant because of the saddle-node bifurcation. With a strong lateral spring, it behaves as a softening spring with only equilibrium at the origin, while with a week spring, it stops at the positive stable point upon unloading. In this section, we think of combining those behaviors into one by introducing a hysteretic spring.

Let us conceptually think of setting $k = k_2$ (hard) for loading phase to bring out high energy absorption efficiency, then switch to $k = k_1$ (soft) upon unloading, so as to stop the mechanism at the equilibrium in the opposite side to prevent the colliding object from rebounding. The synthesized force-displacement curve becomes a combination of the curves with $k = k_1$ and $k = k_2$ as plotted in Fig. 4 (a) where $k_1 = 0.1$ and $k_2 = 0.3$ are adopted for example. Such switching is automatically realized by using hysteretic material for the lateral spring which has a bilinear force-displacement curve shown in Fig. 4 (b).

In the positive equilibrium position after the impact, most of the absorbed energy still remains stored in the lateral and axial springs as the elastic potential energy. This means that the mechanical energy level of the mechanism in the positive equilibrium position is higher than that in the original equilibrium position. By inputting some activation energy to override the energy barrier between two equilibriums, the mechanism autonomously returns to the original position using this energy difference. For example, if the spring constant of the lateral spring can be switched again greater than $k_c = 1/8$ by inputting some energy in some way, then the bifurcation will undergo again, and the mechanism will autonomously return to its original shape using the energy stored in the springs. The whole cycle of the operation of the mechanism is shown in Fig. 5 with the demanded change of the spring constant of the lateral spring.

Thus, we demand some “smart” material satisfying the following requirements for the lateral spring to realize such a mechanism: (a) it should be recoverable from large deforma-
tion; (b) it should show significant hysteresis, and (c) it should be able to vary stiffness upon activation. Superelastic shape memory alloy (SMA) is definitely one of the most promising candidates for such material because its stress-strain characteristics present large recoverable strain over 6% with large hysteresis due to stress-induced martensitic phase transformation, and its equivalent stiffness increases upon heating due to the dependence of the transformation stress on temperature.

2.4. Numerical examples

In this section, the impact response of the two-link mechanism with the bilinear hysteretic spring presented in the previous subsection is numerically investigated. Immediately after the impact of an object, the dimensionless equation of motion of the mechanism with the colliding object is derived as

\[ \ddot{x}(t) + P(x(t)) = 0, \quad \text{I.C. } x(0) = 0, \quad \dot{x}(0) = v_0 \] (8)

where \( P(x) \) is the dimensionless reaction force given by

\[ P(x) = \frac{1}{2} x(x - 1)(x - 2) + k(x, \dot{x})x \] (9)

where the bilinear spring constant is represented as

\[ k(x, \dot{x}) = \frac{1}{2} (1 - \text{sgn}(x\dot{x}))k_1 + \frac{1}{2} (1 + \text{sgn}(x\dot{x}))k_2 \] (10)

where \( \text{sgn}(\cdot) \) stands for the signum function. Note that the characteristic time \( T \) is chosen as \( T = \sqrt{\frac{m}{2k\alpha}} \) in the above formulation where \( m \) is the equivalent mass attached to the center joint. To solve the equation, numerical integration is carried out assuming that the colliding object remains attached to the mechanism throughout the simulation.
The responses for various values of the initial velocity $v_0$ are plotted in Fig. 6 in which the force-displacement plots are shown in the left column while the phase plane diagrams (the relationship between the displacement and the velocity) in the right. For $v_0 = 0.8$, because the initial velocity is too slow, the mechanism cannot pass over the unstable equilibrium and returns back to the origin. In this case, the colliding object will undergo rebound. For $v_0 = 0.85$, it stops short of the stable point and remains there. For $v_0 = 1.05$, it stops beyond the stable equilibrium and comes back upon unloading, then rests at a point between the stable and unstable equilibriums. In these two cases, the mechanism performs as expected. For $v_0 = 1.1$ or larger, it once stops beyond the stable equilibrium and comes back directly to the origin because the recoverable energy is sufficiently large to pass over the unstable equilibrium. In this case, the colliding object will be rebounded.

As suggested in those results, the range of acceptable initial velocity, i.e., the collision speed, depends on the length of the part bounded by the unstable and stable equilibriums in which the reaction force takes negative value. Therefore, this length is one of the most important characteristics one has to consider in the design of shock absorption mechanisms based on the proposed concept.

3. Experiments

In order to demonstrate the feasibility and the potential of the proposed concept, a proof-of-concept model consisting of a two-link mechanism and a superelastic SMA lateral spring was developed. A superelastic SMA-based compression spring composed of a pair of superelastic NiTi wires set in arc shape as schematically shown in Fig. 7 (a), previously developed for the use of seismic isolation(13), was used as the lateral spring because its force-displacement characteristics exhibited shape recoverable softening behavior with significant hysteresis as shown in Fig. 7 (b). A two-link mechanism was fabricated using two rigid bars rotatively jointed with a center joint block which was vertically guided by a linear slider. The outer ends of the rigid bars were horizontally guided by another linear slider and supported by axial compressive springs. The SMA spring was connected between the center joint block and the base plate as shown in Fig. 8. The whole structure was mounted on a universal testing machine.

Compressive loading tests were conducted in two configurations. Firstly, the push rod fixed to the load cell of the testing machine was tied with the center joint block, and a displacement-controlled compression test with unloading phase was conducted. Then, the fastener between the center joint block and the push rod was released, and a displacement-controlled compression test with unloading phase was conducted. In the former configuration, the whole picture of the force-displacement curve for the loading-unloading cycle would be obtained. In the latter configuration, in contrast, the force-displacement curve would be bounded at the stable equilibrium in the unloading phase at which the reaction force would turn negative so that the push rod would leave from the center joint block.

Figure 9 shows the force-displacement diagrams of a two-link mechanism with $L = 60$
Fig. 6 Numerically simulated impact responses of the proposed mechanism.

mm, $k_a = 0.78$ N/mm and $\delta_a = 5$ mm. The blue line is the case the push rod was tied with the center joint block, and the green line is the case the push rod was released. Note that the dis-
placement shown in the figure means the displacement of the crosshead of the testing machine. Both the loading and unloading branches of the blue line clearly reflect the cubic nonlinearity of the link mechanism. This nonlinearity in the loading phase obviously contributes to the achievement of high energy absorption efficiency. The location of the equilibriums are, however, totally different between the loading and unloading branches as expected by the influence of the hysteresis of the SMA spring. In the unloading phase, the force-displacement curve (see blue line) had additional stable equilibrium at 43 mm, so that the mechanism stopped and remained at this point in the latter test (see green line). This property successfully makes the energy dissipation efficiency high. Although all the experiments were carried out under quasi-static loading condition, these results suggest that the proposed concept is promising and may perform as well even under impact loading.

After the latter test, the SMA spring was heated for a second using a hair drier. The link mechanism immediately returned to its original position. This result confirmed the validity of the idea of a “smart” shock absorption mechanism in which the residual deformation is automatically eliminated by reactivating the bifurcation by heating SMA. Note that the shape-recovery was very quick because it was driven by the potential energy stored in the springs,
not by the temperature-induced phase transformation of the whole deformed part as is used in the literature\(^9\).

4. Prototype of Woven SMA Shell

For the illustrative purpose, a prototype of a shell structure made of woven SMA was fabricated as a possible example of applications of the proposed concept. Superelastic NiTi thin wire with the diameter of 0.8 mm was woven by triaxial weaving to form a shallow spherical shell with the curvature of \(6.5 \times 10^{-3}/\text{mm}\) on the hexagonal frame as shown in Fig. 10. The whole structure was mounted on the testing machine, and a displacement-controlled compression test with unloading phase was conducted. In the test, the center of the shell was compressed by a cylindrical rod connected to the load cell.

Figure 11 shows the measured force-displacement curves with the strokes of 40 mm and 60 mm. One can observe the cubic nonlinearity in the loading phase as with the case of the two-link mechanism. They showed residual deformations in the unloading phase, which were also eliminated by heating just as we saw in the case of the two-link mechanism.

5. Conclusions

In this paper, a preliminary study on a shape-recoverable shock absorption structure consisting of hysteretic spring elements combined with a buckling mechanism has been presented. The proposed idea has been investigated both theoretically and numerically using a conceptual model consisting of a two-link mechanism and a superelastic SMA-based hysteretic spring. Then, a proof-of-concept experimental model has been developed for the demonstrative purpose. It has been concluded that the proposed shock absorption mechanism is promising which may achieve high energy absorption efficiency and high energy dissipation efficiency. Furthermore, the residual deformation would be eliminated after the impact by applying some heat to the SMA spring. Finally, a prototype of superelastic SMA-based “woven” shells has been developed to show a possible application of the proposed concept.

Acknowledgements

The authors gratefully acknowledge the support by the Ministry of Education, Culture, Sports, Science and Technology, Grant-in-Aid for Young Scientists (B), 17760183, 2005–2007.

References


Fig. 10  Woven SMA shallow spherical shell (triaxial weave).

(b) Photo of woven SMA shell.

Fig. 11  Force-displacement diagram of woven SMA shallow spherical shell.