Dynamic Property Optimization of Suspension MBD Model based on Sensitivity Analysis *

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Abstract
In recent years, the needs to achieve the eigenvalue optimization considering the NVH performance increase in the initial design of suspension systems. This paper presents an application of MBD (Multi-Body Dynamics) model to dynamic property optimization. A vehicle suspension is modeled by MBD and the vibration properties are analyzed based on the linearization of the system equation. The model can solve the dynamic properties as modal parameters such as natural frequencies, modal dampings and mode shapes. The targets of this model are to calculate the eigenvalue sensitivity with respect to each design parameter (mass, stiffness, damping and geometry of the link) and to optimize the eigenvalue by combining the structural modifications of these high sensitivity elements. By the sensitivity analysis, we can make sure which elements contribute to the dynamic characteristics of the system. The feasibility of the dynamic property optimization is examined by applying the presented approach to the suspension system model. The structural modification is carried out based on the sensitivity analysis in order to attain the required natural frequency change. As a result it is shown that the natural frequency is optimized based on the presented sensitivity analysis.

Key words: Sensitivity Analysis, Multi-Body Dynamics, Modal Analysis, Vibration, Response Surface, Computer Aided Engineering, Suspension

1. Introduction

Recently, many studies have been done on Multi-Body Dynamics (MBD). Those studies focus on the aspects such as contact problem, multi-flexible-body analysis in conjunction with FEM (1).

The vibration analysis is also conducted based on the linearization of the non-linear system equation. The applications to NVH (noise, vibration, harshness) problems are also reported in the field of automobile industry. For example, J. Yu (2) shows the analysis of vehicle chassis transmissibility for steering shimmy and judder by using the MBD model including strut suspension, sub-frame and steering system. However, the studies on inverse problem are rarely found, which include system identification and structural optimization.

As for the automobile suspension, various performances such as drivability, ride comfort and NVH need to be satisfied. Especially, the demand for NVH performance increases in order to secure the comfort.
Therefore, the dynamic property optimization by using sensitivity is applied to the link model like automobile suspension MBD model. The sensitivity analysis method is used, which deals not only with the change in mass, stiffness and damping but also with the change of geometry (geometrical position) in the mechanical linkage. It is necessary to examine the effectiveness of the first-order approximation in the prediction of dynamic property change with respect to the design variable (mass, stiffness, damping and geometry).

In this paper, the validity of the sensitivity analysis is investigated: the dynamic property changes predicted by the sensitivity analysis are compared with the result of real structural change that is expressed by the response surface. Moreover, the sensitivity analysis is applied to a strut suspension system. The natural frequency optimization is carried out for the system.

The frequency of interest is lower than 100 Hz under the assumption that each body of the model is a rigid body, and the stiffness and damping elements are included in rubber bush, coil spring and shock-absorber.

2. Vibration Analysis by MBD and Sensitivity Analysis

For the dynamic analysis of the suspension system, the analysis generally needs to include the large displacement. However, for the analysis of vibration phenomena such as harshness and road noise, the linear analysis that assumes small displacement is effective. This is because the eigenvalue analysis and FRF (Frequency Response Function) evaluation can be used for the linearized system.

2.1 Vibration Analysis by MBD (3), (4)

The linearized equation is derived on the assumption that the displacement is small from the neutral position. The small displacement in generalized coordinate of \( q \) is expressed by \( \delta q \), and the Jacobian of constraint conditional equation is \( \Phi_q \). The dynamic equation with the constraint is

\[
M \ddot{\delta q} + C \dot{\delta q} + K \delta q + \Phi_q^T \lambda_{\text{const}} = F_{\text{ext}}
\]  

where \( M, C, \) and \( K \) are mass, damping and stiffness matrix in the generalized coordinate, respectively. The \( \lambda_{\text{const}} \) is a Lagrange multiplier and \( F_{\text{ext}} \) is the external force. The matrix size of \( \Phi_q \) is \( m \times n \) (\( m \) is number of constraint DOFs (Degrees of Freedom), and \( n \) is number of DOFs in the generalized coordinate.). The QR decomposition of \( \Phi_q^T \) matrix gives

\[
\Phi_q^T = \begin{bmatrix} Q_u & Q_v \end{bmatrix} \begin{bmatrix} R_u^T \ 0 \end{bmatrix} = Q_u R_u
\]

where \( Q_u (n \times m), \ Q_v (n \times (n - m)) \) are orthogonal matrices and \( R_u (m \times m) \) is a right upper triangular matrix. Consequently,

\[
\begin{bmatrix} Q_u & Q_v \end{bmatrix}^T \times [Q_u \ Q_v] = I_n
\]

(\( I_n \) is a \( n \times n \) identity matrix) The following equation exists between matrices \( Q_v \) and \( \Phi_q^T \).

\[
Q_v^T \Phi_q^T = Q_v^T (Q_u R_u) = 0
\]

This equation shows that the column space of \( \Phi_q^T = Q_u R_u \) and that of \( Q_v \) are orthogonal to each other. Generalized coordinate \( q \) can be orthogonally decomposed to \( u \) which is parallel to the constraint force and the vector \( v \) which is orthogonal to it. Therefore, \( u \) is a dependent coordinate, and \( v \) is an independent coordinate. The generalized coordinate \( q \) is expressed as follows.

\[
q = Q_v, v
\]
Degrees of freedom of Eq. (1) can be reduced to DOFs concerning independent coordinate \( \mathbf{v} \). This reduction leads to
\[
\ddot{\mathbf{M}}\ddot{\mathbf{v}} + \dot{\mathbf{C}}\dot{\mathbf{v}} + \mathbf{K}\mathbf{v} = \mathbf{F}_{\text{ext}}
\]  
(6)
where \( \mathbf{M} = \mathbf{Q}_v^T\mathbf{M}\mathbf{Q}_v \), \( \dot{\mathbf{C}} = \mathbf{Q}_v^T\mathbf{C}\mathbf{Q}_v \), \( \mathbf{K} = \mathbf{Q}_v^T\mathbf{K}\mathbf{Q}_v \), \( \mathbf{F}_{\text{ext}} = \mathbf{Q}_v^T\mathbf{F}_{\text{ext}} \). \( \mathbf{M}, \dot{\mathbf{C}} \) and \( \mathbf{K} \) are mass, damping and stiffness matrices with the size of \( n \times n \) respectively. Eq. (6) is a linearized vibration equation, which can be solved by eigenvalue analysis. Thus, conventional modal analysis can be applied to the linearized model. Furthermore, the linearization gives the profits such as the stable and easy calculation of the response analysis with high accuracy. The assumption of small displacements is thought to be valid for the vibration analysis.

2.2 Sensitivity Analysis

The sensitivity analysis is a variation analysis: the variation of objective function with respect to the design variable change is evaluated by means of differentiation. In this chapter the sensitivity analysis of the linearized MBD model is introduced. The objective function can be eigenvalue, FRF response, etc. and the design variable can be the geometry of model as well as mass, stiffness and damping.

2.2.1 Sensitivity of First Order

Firstly, the change of stiffness matrix is considered. Suppose that a design variable \( \gamma \) have a slight change \( \varepsilon \), which causes the initial stiffness matrix \( \mathbf{K}_0 \) to be changed into \( \mathbf{K} \). In general, the relation between the design variable (e.g., dimension of a body) and the stiffness matrix is a nonlinear function, this relation can be expressed as follows by using the first order approximation of the Taylor's expansion.
\[
\dot{\mathbf{K}} = \mathbf{K}_0 + \frac{\partial \mathbf{K}}{\partial \gamma} \varepsilon
\]  
(7)
where \( \frac{\partial \mathbf{K}}{\partial \gamma} \) is the first order sensitivity of stiffness matrix to design variable \( \gamma \). Because this sensitivity is in the independent coordinate system, the sensitivity in the original coordinate system is also derived. The sensitivity in the original coordinate and that in the independent coordinate have a relationship as follows.
\[
\left( \frac{\partial \mathbf{K}}{\partial \gamma} \right) = \left( \frac{\partial \mathbf{Q}_v^T\mathbf{K}\mathbf{Q}_v}{\partial \gamma} \right) = \mathbf{Q}_v^T \left( \frac{\partial \mathbf{K}}{\partial \gamma} \right) \mathbf{Q}_v + \mathbf{Q}_v^T \left( \frac{\partial \mathbf{Q}_v}{\partial \gamma} \right) \mathbf{Q}_v + \mathbf{Q}_v^T \left( \frac{\partial \mathbf{Q}_v}{\partial \gamma} \right) \mathbf{Q}_v
\]  
(8)
where we can get \( \frac{\partial \mathbf{Q}_v}{\partial \gamma} \) by using derivation of Eq. (4). The sensitivity of mass matrix and the damping matrix can be derived in a similar way. Because the vehicle suspension system contains the damping element such as bush, shock absorber and tire, the assumption of the proportional damping is not effective. Hence, the suspension model in this research is treated as general viscous damping system, which allows having complex modes. Thus, the system of the general viscous damping has \( \dot{\mathbf{D}} \) and \( \dot{\mathbf{E}} \) matrix. The sensitivities of those matrices are
\[
\left( \frac{\partial \dot{\mathbf{D}}}{\partial \gamma} \right) = \begin{bmatrix}
\frac{\partial \mathbf{C}}{\partial \gamma} & \frac{\partial \mathbf{M}}{\partial \gamma} \\
\frac{\partial \mathbf{M}}{\partial \gamma} & 0 \\
\end{bmatrix}
\]  
(9)
\[
\left( \frac{\partial \dot{\mathbf{E}}}{\partial \gamma} \right) = \begin{bmatrix}
\frac{\partial \mathbf{K}}{\partial \gamma} & 0 \\
0 & -\frac{\partial \mathbf{M}}{\partial \gamma} \\
\end{bmatrix}
\]  
(10)
The sensitivity of $r$-th eigenvalue is obtained by using the $\hat{D}$ and $\hat{E}$ matrices sensitivity. The eigenvalue change is expressed by

$$\lambda_r = \lambda_{0r} + \left( \frac{\partial \lambda_r}{\partial \gamma} \right) \varepsilon$$

(11)

The sensitivity of the eigenvalue is obtained by

$$\left( \frac{\partial \lambda_r}{\partial \gamma} \right) = -\psi^T r \lambda_{0r} \left( \frac{\partial D}{\partial \gamma} \right) + \left( \frac{\partial E}{\partial \gamma} \right) \psi_r$$

(12)

$\psi_r$ shows $r$-th eigenvector in the initial design. Not only the sensitivities with respect to mass, stiffness and damping but also the sensitivity with respect to the geometry of the link joint is derived. The geometry change includes the change of the mass, moment of inertia and the position of the center of gravity of body and the change of the stiffness and the damping of the system, etc. The sensitivity for the geometry change is derived with all these effects considered. Therefore, many kinds of sensitivities such as $\frac{\partial M}{\partial \gamma}, \frac{\partial K}{\partial \gamma}, \frac{\partial C}{\partial \gamma}, \frac{\partial Q}{\partial \gamma}$ are related to the geometry sensitivity.

2.2.2 Sensitivity by Difference Approximation

In the above section, the sensitivity analysis by analytical approach is presented. In addition, the sensitivity analysis by the difference approximation can also be used. The approach is based on the re-analysis with slightly changing the design variable. It is considered that the variation of objective function relative to the change of design variable is an approximation to the analytical sensitivity. Computational cost for the difference analysis is not serious comparing with the analytical sensitivity analysis.

In this study, the sensitivity analyses by two approaches are utilized. Effectiveness of the sensitivity analysis of the MBD model is examined in the following chapters.

3. Application to the Two-Link Model

The property prediction by the sensitivity analysis is compared with the response surface. The effectiveness of the sensitivity analysis is examined from the viewpoint of prediction error. Generally, the response surface is an interpolation analysis with the surface function assumed (6). In this study, the analysis is iterated with small intervals so as to get the response precisely. Therefore, the response analysis is referred as the exact solution in this study.

3.1 Modeling

In this chapter, the two-link suspension (Fig. 1) is modeled by the approach described in the chapter 2. The effective range of the sensitivity analysis for the prediction of natural frequency change is investigated by the above-mentioned approach. In other words the nonlinearity of the eigenvalue with respect to the design variable change is examined in case of a simple structure.

The three dimensional model in Fig. 1 has 7 DOFs (=12-5), consisting of two bodies (2 x 6 DOFs) with the constraint of revolution joint (5 DOFs). These bodies are connected to the rigid wall through the bush elements. It is assumed that the bush has stiffness and damping elements in 6 DOFs.

By using this model, we evaluate the sensitivity of eigenvalue with respect to the following design parameters: the position of the mount bush (point A), the revolution joint (point B), or the lower arm bush (point C); and it is assumed that the point moves on the horizontal plane $(XY)$. 
3.2 Effective Range of Sensitivity Analysis

The dynamic characteristics of the MBD model of Fig. 1 are summarized. The damped natural frequencies and the mode shapes for the fundamental three modes (1st - 3rd) are summarized in Table 1 and Fig. 2 (a)-(c). Mode shape movements are described with respect to translation and rotation elements in Table 1.

The modification of the position of the revolution joint (point B) is investigated. The change of the damped natural frequency according to positional change of point B is predicted by the proposed sensitivity analysis.

Table 1. Modal Parameter of the Two Link Model

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>1st Mode</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damped Natural Frequency (Hz)</td>
<td>22.4</td>
<td>52.4</td>
<td>68.9</td>
</tr>
<tr>
<td>Translation:</td>
<td>Strut Y</td>
<td>Arm X</td>
<td>Arm Z</td>
</tr>
<tr>
<td>Rotation:</td>
<td>Arm Z</td>
<td>Strut Y</td>
<td>Arm Z</td>
</tr>
</tbody>
</table>

Let us assume the modification of the joint-B location of the two-link model. The sensitivity prediction of the eigenvalue change is compared with the re-analysis (response surface). Figures 3 and 4 show the results of the 1st mode and the 3rd mode, respectively. These figures show the change of the natural frequency according to the shift of joint location within the range of ±100 mm in $XY$ plane, and the numbered contours are amount of the damped natural frequency change (Hz). The right-hand figures show the
results of the prediction by sensitivity analysis (SA) and the left-hand show the results of response surface (re-analysis) (RS). The result of response surface (re-analysis) almost shows a real frequency change because the response calculation is iterated with small intervals.

![Figs. 3 Compare SA with RS at the 1st Mode](image1)

![Figs. 4 Compare SA with RS at the 3rd Mode](image2)

It is understood that the 1st mode has relatively low sensitivity with respect to the joint geometry, and on the other hand the 3rd mode has high sensitivity. In both cases, the sensitivity analysis is able to well predict the real frequency changes. It is noted that in higher modes having local modes often show stronger nonlinearity.

### 3.3 Prediction Error of Sensitivity Analysis

The prediction error of the sensitivity analysis is evaluated by taking the difference between the results of the left-hand and the right-hand with the following normalization.

\[
\text{Error Ratio} = \left| \frac{f_{\text{Actual}} - f_{\text{Predicted}}}{f_{\text{Actual}}} \right| \times 100 \%
\]  

(13)

If the error ratio described as above is less than 3 %, the prediction by the sensitivity analysis is thought to be applicable.

![Fig. 5 Error of Sensitivity (1st Mode)](image3)

![Fig. 6 Error of Sensitivity (3rd Mode)](image4)
Even for the result of the 3rd mode the effective range is still as much as ±40 mm, which corresponds to the 15 % to the original length of the lower arm.

4. Application to a Strut Suspension Model

4.1 Modeling

In this chapter, a strut suspension (Fig. 7) is considered for the structural modification in order to investigate the effectiveness of sensitivity analysis. In addition, the modification parts are chosen from the candidates by comparing the sensitivity values. Design parameters are mass, stiffness, damping and geometry, and the objective function is the damped natural frequency. The frequency of interest is less than 100 Hz, where the harshness and the riding comfort evaluations are involved. Each body (arm, wheel, knuckle, etc) is assumed to be rigid and is connected through the bush or the spherical joint based on the literature (4). For the strut suspension model, the sensitivity is evaluated by using the difference analysis as described in section 2.2.

![Strut Suspension Model](Fig. 7)

4.2 Effective Range of Sensitivity Analysis

The model has 27 DOFs in total. The fundamental three natural modes are summarized in Table 2.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
<th>4th Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damped Natural Frequency (Hz)</td>
<td>22.0</td>
<td>51.8</td>
<td>86.5</td>
</tr>
<tr>
<td>Mode Shape</td>
<td>Translation:</td>
<td>Wheel Z</td>
<td>Wheel X</td>
</tr>
<tr>
<td>Rotation:</td>
<td>L/Arm X</td>
<td>L/Arm Z</td>
<td>L/Arm Z</td>
</tr>
</tbody>
</table>

Let us assume that the position of the point ③ in Fig. 7 is a design variable, and the geometry of the point is shifted on the $XY$ plane. The sensitivity analysis prediction is compared with the real change of damped natural frequency. Figures 8 is the damped natural frequency change predicted by the sensitivity analysis (right), and by the response surface (left) with respect to the 3rd mode, which has relatively high sensitivity of the location-③ change.
Prediction error is shown in Fig. 9, which is calculated by Eq. (13). The effective area of error less than 3 % is roughly within ±30 mm from the original location. It is little narrower than the simple two-link model.

For the design investigation in NVH performance, the large geometry change is not acceptable because many design requirements arise for the suspension system, such as strength, durability, and drivability etc. Considering the above conditions, it is concluded that the sensitivity analysis can be applied to the design investigation of the suspension system.

4.3 Distribution of Sensitivity

In this section, the sensitivities of the modification candidate locations are evaluated and the optimum design change is chosen by comparing the sensitivity magnitudes. The candidate design variables are mass, stiffness, damping and geometry of bodies (lower arm, knuckle, etc.).

For example, Table 3 shows the geometry sensitivities of the 3rd natural frequency with respect to the geometry change of each point. From Table 3, it is understood that the damped natural frequency is greatly influenced by the $X$-coordinate of front lower arm bush and $Y$-coordinate of lower arm joint.

Moreover, Table 4 shows the stiffness sensitivity of the 2nd natural frequency. As for the stiffness, it is turned out that the stiffness of F/R lower arm bush around $X$-axis most influence with respect to the natural frequency. Thus, we understand which elements influence the damped natural frequency by the sensitivity analysis. For the application of the sensitivity, the design change which influences a certain natural frequency without changing other natural frequencies can be investigated. In addition, the elements of high sensitivities in Table 3, 4 are used as the modified parameters in the following section.
Table 3. Distribution of Geometry Sensitivity of the 3rd Mode
(Gray numbers show high sensitivity parameters)

<table>
<thead>
<tr>
<th>Direction</th>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Lower Arm Bush</td>
<td>-1.152</td>
<td>0.221</td>
<td>0.029</td>
</tr>
<tr>
<td>Rear Lower Arm Bush</td>
<td>0.537</td>
<td>0.680</td>
<td>-0.023</td>
</tr>
<tr>
<td>Lower Arm Joint</td>
<td>0.650</td>
<td>-0.944</td>
<td>0.083</td>
</tr>
<tr>
<td>Tie-Rod End Near Upright</td>
<td>-0.649</td>
<td>0.066</td>
<td>0.015</td>
</tr>
<tr>
<td>Tie-Rod End Near Rack-and-Pinion</td>
<td>0.624</td>
<td>-0.062</td>
<td>-0.029</td>
</tr>
<tr>
<td>Upper Mount Bush</td>
<td>0.001</td>
<td>-0.005</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Units: $(Hz/cm)$

Table 4. Distribution of Stiffness Sensitivity of the 2nd Mode
(Gray numbers show high sensitivity parameters)

<table>
<thead>
<tr>
<th>Direction</th>
<th>$\theta_X$</th>
<th>$\theta_Y$</th>
<th>$\theta_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F Lower Arm Bush</td>
<td>0.090</td>
<td>0.063</td>
<td>0</td>
</tr>
<tr>
<td>R Lower Arm Bush</td>
<td>0.153</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Shock Absorber</td>
<td>0.002</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>Coil Spring</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Upper Mount Bush</td>
<td>0.002</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>Tire</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Units: $(Hz/(N-mm/rad))$

4.4 Optimization of Strut Model

The dynamic characteristics optimization of the strut model is considered. The dynamic property is optimized by using the sensitivity distribution concerning each design variable. The flow of the optimization process is shown in Fig. 10. The objects of the optimization are set to raise the 2nd and 3rd natural frequencies by 20 %, respectively.

The sensitivities with respect to various design variables (M, K, C and geometry) are evaluated. The candidate variables were screened out into the following four design variables, which have relatively high sensitivities with respect to the objective frequencies. The extent of structural modification is limited to the small extent, e.g., where the physical interference does not occur, and the modifications of four variables are decided as follows.

1. 15 % lightening of the knuckle assembly (Wheel, Brake, Knuckle)
2. addition of the stiffness value by 17 kNm/rad around $X$ axis of rear lower arm bush
3. the position of front lower arm bush moved toward $-X$ by 30 mm
4. the position of lower arm joint moved toward $-Y$ by 30 mm
The property change can be obtained by changing these four structural changes. Figure 11 shows the dynamic stiffness (Input: the ground displacement in $Z$ direction, Output: the transmitted force at upper mount in $Z$ direction), which relates to the 2nd natural mode. Figure 12 shows the dynamic stiffness (Input: the ground displacement in $X$ direction, Output: the transmitted force at upper mount in $X$ direction) relating to the 3rd natural mode. Figure 12 shows the resonant peak of the 4th natural mode (86.5 Hz) as well as that of the 3rd natural mode (51.8 Hz).

The predicted changes of the damped natural frequencies are summarized in Table 5, compared with those actual changes. The predicted frequencies have less than 5 % error (by Eq. (13)) compared with the actual changes. The natural frequency changes of the 2nd and the 3rd modes are attained at the same time, where both frequency changes are more than 20 % relative to the original frequency. Furthermore, the peak level of the 2nd mode in the dynamic stiffness in Fig. 11 decreased by 17 %, and the modification is carried out without changing the natural frequency of the 4th mode as shown in Fig. 12.
Table 5. Result of Optimization

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>2nd Mode</th>
<th>3rd Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>1.8</td>
<td>4.1</td>
</tr>
<tr>
<td>②</td>
<td>2.6</td>
<td>-</td>
</tr>
<tr>
<td>③ Predicted Value (Hz)</td>
<td>-</td>
<td>3.5</td>
</tr>
<tr>
<td>④ Predicted Value (Hz)</td>
<td>-</td>
<td>2.8</td>
</tr>
<tr>
<td>Total Predicted Value (Hz)</td>
<td>4.4</td>
<td>10.4</td>
</tr>
<tr>
<td>Actual Value (Hz)</td>
<td>5.3</td>
<td>11.9</td>
</tr>
<tr>
<td>Error Ratio (%)</td>
<td>4.1</td>
<td>2.9</td>
</tr>
</tbody>
</table>

5. Conclusion

In this research, the effectiveness of the sensitivity analysis and the optimization of dynamic property method based on the sensitivity were discussed for the link mechanics that assumed the vehicle suspension. The conclusions are as follows.

1. The validity of the sensitivity was confirmed by comparing the sensitivity analysis concerning the geometry change with the response surface for the link systems such as the two-link model and the vehicle strut suspension model.

2. By comparing the sensitivities for various design variables, the design change to attain the required frequency change is investigated for the strut suspension model. It is showed that the candidate design variables are screened, and the dynamic property optimization with respect to natural frequencies can be carried out based on the sensitivity analysis.

3. The results of this paper suggest that the sensitivity analysis of the suspension MBD model is effective for the design optimization considering the NVH performance.

References

(5) A. Nagamatsu, Modal Analysis, Baifukan, 1990
(6) H. Yamada, Design of Experiments—Methodology—, JUSE Press, 2004