Mass Measurement System Using Relay Feedback with Hysteresis*

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Abstract
Mass measurement using a relay feedback system was studied experimentally. The measurement system has an on-off relay with hysteresis and switches force acting on the object in relation to its velocity. Such nonlinear control induces a limit cycle in the feedback system. The mass of the object is determined from the period of this limit cycle. The apparatus manufactured for experimental study uses two voice coil motors (VCM’s), one of which is for driving the object and the other is for generating prescribed disturbances. The effects of system parameters and disturbances on measurement accuracy were examined experimentally.

Key words: Mass Measurement, Space Engineering, Nonlinear Control, Limit Cycle

1. Introduction

Various weightless or micro-gravity experiments have been carried out in the space shuttles. A number of new phenomena, which could not be observed on earth, have been disclosed in the experiments. The production of new materials and medicines in space is expected to start in the near future. Mass measurement under weightless conditions will be necessary to perform such production.

Conventional methods of mass measurement are classified into two types(1):
- Compare the weight force acting on an object of unknown mass with those of reference weights.
- Measure the change in length of a spring that is proportional to the weight force acting on an object.

Neither of them can perform measurement in space because they are based on gravitational acceleration on earth. Various methods of measuring mass under weightless conditions have been proposed:

(1) Measure the natural frequency of a spring-mass system(2).
(2) Apply the frequency-controlled method(3).
(3) Apply the concept of the dynamic measurement method(4).
(4) Measure the centrifugal force in rotating an object(5).
(5) Use a dynamic vibration absorber as a measurement device(6), (7).
(6) Apply the law of conservation of momentum(8), (9).
(7) Apply relay feedback(10), (11)

It is pointed out that the first method is sensitive to vibration transmitted from the surroundings; it takes longer time to measure in the presence of such vibration(4). The second method also measures the natural frequency of a spring-mass system(3). In the third method, mass is estimated based on the transient response of a spring-mass system(4). It needs sophisticated mass-estimation algorithms including the compensation of sensor dynamics. The fourth method has a critical problem that centrifugal forces generated during
measurement may cause vibration in the surroundings. The authors proposed the fifth method to overcome the problem of the fourth method\(^6\). It has been experimentally shown that they perform measurements with accuracy of percent order\(^7\). In the sixth method, mass is estimated from the ratio of the velocity of a reference mass to that of an object\(^8\), or the integral of impulse force in a collision\(^9\). A disadvantage of this method is the necessity of a setting mechanism of the initial conditions for accurate repetitive measurement.

In addition to measurement accuracy, the reduction of weight is also a critical requirement to space equipment so that measurement systems fit for lightweight should be considered. The purpose of the seventh method is to develop mass measurement systems whose mechanism is simpler than that of the fifth system. They are characterized by using relay feedback\(^10\), \(^11\). Relay feedback has been extensively investigated for more than a century\(^12\). It was successfully applied to auto-tuned PID controllers for process control\(^13\). The authors have proposed to apply relay feedback to mass measurement. There are two types. One of them uses an on-off relay with dead zone and feeds back the displacement of an object to be measured\(^10\). The other uses a relay with hysteresis and feeds back the velocity of the object\(^11\). The advantage of the latter over the former is the simplicity of period measurement. While the on-state and off-state periods must be individually measured in the former, a simple period or frequency measurement is sufficient in the latter. It is favorable in constructing measurement system. This paper, therefore, focuses on the latter.

An apparatus is fabricated for experimental study on the mass measurement. It has two voice coil motors (VCM's); one is for moving an object and the other is for generating disturbances. In actual measurement, a spring-like element must be added for avoiding the drift of the moving part. Since the moving part is suspended by a linear bearing for guiding the motion, some damping and friction are inevitable. The effects of parameters such as stiffness and damping on measurement accuracy are studied experimentally by simulating spring and damper with the voice coil motor for disturbance.

### 2. Measurement Principles

Figure 1 shows a schematic drawing of the developed mass measurement instrument. It is made up of four elements:

- actuator for moving an object to be measured,
- sensor for detecting the velocity of the object,
- controller for producing switching signals,
- amplifier for driving the actuator.

The operation of the measurement system is shown in Fig.2. The force \(F(t)\) produced by the

![Diagram](image-url)
The actuator is switched in relation to the velocity $v$ of the object with a relay with hysteresis. When the force applied to the mass is positive, the velocity of the object increases. When the velocity reaches a preset threshold value $v_0$, the applied force is switched from $F_0$ to $-F_0$. Then the velocity decreases. When the velocity reaches the lower threshold value $-v_0$, the applied force is switched to $F_0$ again. When the actuator is controlled in these ways, a limit cycle appears as shown in Fig. 2.

The period of the limit cycle is designated as $T$. When the applied force is $F_0$, the equation of motion is given by

$$m\ddot{x} = F_0. \tag{1}$$

Solving Eq.(1) with $\dot{x}(0) = -v_0$ leads to

$$2v_0 = \frac{F_0}{m} \cdot T_1, \tag{2}$$

where $T_1$ is the period when $F(t) = F_0$. From Eq.(2), we get

$$T_1 = \frac{2v_0}{F_0} m. \tag{3}$$

The period when $F(t) = -F_0$ is obtained in a similar way as

$$T_2 = \frac{2v_0}{F_0} m. \tag{4}$$

The period of the limit cycle $T$ is given by.

$$T = T_1 + T_2. \tag{5}$$

From Eq.(3) to Eq.(5), we get

$$m = \frac{F_0}{4v_0} T. \tag{6}$$

Therefore, the mass of the measurement object is determined by measuring $T$.

An advantage of this method over the other relay feedback method using displacement feedback is the ease of period measurement as mentioned in Introduction. A simple counter is applicable because $T_1 = T_2$ as shown by Eqs.(3) and (4).

### 3. Experimental Apparatus

Figures 3 and 4 shows a schematic drawing and a photo of an apparatus developed for experimental study. It has two VCM’s. One of them is for acting force on the object and the other for generating disturbance. Each VCM has a mover with a coil wound about a shaft.
The shaft is guided to move in a straight line by a linear bearing. The movers are connected with a mechanical coupling. A cylindrical weight is attached to the mover of the VCM for actuation as a part of mass to be measured as shown in Fig. 5. Thin long-head screws are attached to the weight for the fine adjustment of mass to be measured. One screw weighs about 14[g].

The maximum displacement and output force of the VCM’s are 15[mm] and 9.8[N], respectively. They are driven by power amplifiers with current output. The force generated by a VCM is given by

\[ F = K_i i , \]  

where

\[ i \] : current in the coil,

\[ K_i \] : current-force coefficient.

In actual measurement, a spring-like element must be added for avoiding the drift of the mover. In this work, the VCM for disturbance is controlled to operate as a spring by feeding back the displacement of the mover. Its stiffness is designated by \( k_s \).

A velocity sensor was fabricated in which a permanent magnet moves inside a windings to induce back electromotive force in the coil. The permanent magnet is attached to the
coupling. The voltage across the coil $e$ is proportional to the velocity of the permanent magnet. It is given by $$e = K_b v,$$ \hspace{1cm} (8)

where

$v$ : velocity of the permanent magnet,

$K_b$ : back electromotive force coefficient.

The control algorithm described in the previous chapter is implemented with a DSP-based digital controller. The signal given by Eq. (8) is inputted to the controller through an A/D converter. The controller generates binary command signal ($\pm I_0$) so as to simulate an on-off relay with hysteresis whose width is given by $\pm v_0$ (see Fig.2). This signal is sent to the power amplifier through a D/A converter. Thereby, the electromagnetic force is switched to be $\pm F_0$ alternatively so that the measurement object oscillates as shown in Fig.2. The period $T$ of this oscillation is measured with a digital oscilloscope.

4. Experimental Results

4.1 Operation of the Relay Feedback System

Figure 6 shows the velocity of the mover and the coil current when $F_0$ and $k_s$ are 2.20[N] and 1.56[kN/m], respectively. The switching velocity $v_0$ is set as

(a) $v_0 = 40$[mm/s],

(b) $v_0 = 50$[mm/s].

It is observed that the relay feedback system behaves as expected in "Measurement Principles". The period of the limit cycle increases as $v_0$ is set to be larger.
4.2 Selection of parameter

The effects of the parameter $v_0$ on measurement accuracy are experimentally studied. Figure 7 shows measurement results for nine samples from 297.0[g] to 409.0[g] when $F_0$ and $k_s$ are fixed to 2.20[N] and 1.56[kN/m] and $v_0$ is set as

(a) $v_0 = 30$[mm/s],
(b) $v_0 = 40$[mm/s],
(c) $v_0 = 50$[mm/s],
(d) $v_0 = 60$[mm/s]

In this figure, the period of the limit cycle is plotted for each measurement mass. The obtained data were fitted to a straight line by the least-squares method and the deviation from this line was estimated in each case. For example, the fitted line for (d) is

$$T_m[g] = 6.49ms^5.11 - \times$$

The deviation from the line was smallest in the case of (d) where the value of $v_0$ is
largest. This result is same as Mizuno et al. has got in the previous work\(^{(11)}\). In the following, therefore, \( F_0 \) and \( v_0 \) were set as \( F_0 = 2.20 \text{[N]} \) and \( v_0 = 60 \text{[mm/s]} \).

### 4.3 Measurement errors

Another measurement was again carried out to check the effectiveness of the calibration based on Eq.(9). Nine samples from 297.0\([\text{g}]\) to 409.0\([\text{g}]\) are measured again. Figure 8 compares the original estimation based on Eq.(6) with the estimation calibrated according to Eq.(9). It indicates that appropriate calibration is necessary for accurate measurement.

Figure 9 presents the relative error of the calibrated estimation \( m_c \), which is calculated according to

\[
e = \frac{m_r - m_c}{m_r} \times 100 \text{ [\%]}. \tag{10}
\]

The maximum and the average of their absolute values are 0.7\[\%\] and 0.4\[\%\], respectively. The accuracy is almost same as Mizuno et al. has got in the previous work\(^{(11)}\) on the same measurement method. Compared with the other relay feedback method using displacement feedback\(^{(10)}\), however, it is not superior in measurement accuracy. One of the main reasons is that the output of the fabricated velocity sensor is more noisy than that of the photo interrupters that was used in the displacement-feedback measurement system to detect the switching positions\(^{(10)}\). The accuracy will improve by modifying the velocity sensor with less noise or replacing it by a high-performance velocity sensor.

### 4.4 Effect of stiffness

One of the reasons for the difference between the actual mass and the original estimation shown in Fig.8 is that a spring-like element was added in actual measurement. It is necessary for avoiding the drift of the mover. To estimate the effects of such an element on measurement accuracy, measurements are carried out for various values of \( k_s \). Figure 10 shows the measurement results. It is found that the estimated values become smaller as stiffness is higher.

### 4.5 Effect of damping

Another possible factor in measurement error is damping and friction in the linear bearing for suspending the movers. To estimate the effects of damping on measurement accuracy, the VCM for disturbance is controlled to actuate as a damper by feeding back the velocity of the mover. Its damping coefficient is denoted by \( c_s \). Measurements are carried out for various values of \( c_s \) as shown in Fig.11. It is found that the effect of damping
becomes noticeable when added damping exceeds some level. Roughly speaking, the estimated values become larger as damping increases.

5. Conclusion

This paper studied on mass measurement using a relay feedback system. The system
uses a relay with hysteresis and switches force acting on the object to be measured in relation to the velocity of the object. An experimental apparatus was fabricated which had two VCM's, one of which was for moving an object and the other was for generating disturbances. The effect of the threshold value of velocity for switching was studied experimentally. Under appropriate conditions, the measurement errors were within 0.7%. The effects of stiffness and damping on measurement accuracy were also examined experimentally by simulating spring and damper with the VCM for disturbance. It was found that the estimated values became smaller as stiffness was higher while they became larger as damping increased.

References