Identification of Nonlinear Parameters by Using M-sequence and Harmonic Probing Method

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Abstract
Many classes of nonlinear systems can be represented by Volterra kernels. The authors have recently developed a method for identification of Volterra kernels of nonlinear systems by using M-sequence and correlation technique. In this paper, the authors propose a new method for identification of nonlinear mechanical systems by use of Volterra kernels. The nonlinear mechanical systems are approximated to a nonlinear vibrating system which consists of a mass, a dumper, a linear spring and nonlinear and springs. Then, the parameters, which represent the nonlinear springs are calculated by use of the Volterra kernels. From the results of computer simulation and experiment, the parameters that represent the nonlinear characteristics of the nonlinear mechanical systems can be identified by the proposed method.

Key words : Nonlinear Vibration, Nonlinear Mechanical System, M-sequence, Volterra Kernel, Cross-Correlation Function, System Identification

1. Introduction

System identification is a technique to obtain the mathematical model of a physical system based on the input and the output data obtained by experiment and the structure of the system. Therefore, various techniques have been proposed for the identification of linear systems. On the other hand, accurate analysis and identification of the nonlinear systems are difficult, because the superposition principle does not consist in the nonlinear systems. However, since almost all existing control system are nonlinear systems, identification of nonlinear system has an important roll in the control engineering. The identification methods for nonlinear systems are devided into two categories: parametric method and non-parametric method. In the parametric method, various model structures are proposed, such as block-oriented model, nonlinear autoregressive exogenous (NARX) model, neural network model, fuzzy model.

Among the nonparametric methods, Volterra kernel representation is one of the most general method, because many classes of nonlinear systems can be represented by Volterra series. So, many researchers have tried the method of measuring the Volterra series. For example, Hooper and Gyptopoulos proposed to use of pseudorandom ternary signal for obtaining 2nd order Volterra kernel by using a property that odd order moment of the signal is zero. Barker et al. suggested a method for obtaining 2nd order Volterra kernel by using pseudorandom antisymmetric M-sequence. Shi and Hecox proposed a method for measuring Volterra kernels of the brainstem auditory evoked response by use of m-pulse. One of the authors has proposed a method to identify Volterra kernels of up to 3rd order by using M-sequence and cross-correlation function. In the proposed method, an M-sequence was
applied to a nonlinear system to be identified and the cross-correlation function between the input M-sequence and the system output was calculated. Then the obtained cross-correlation function includes not only the impulse response of the system but also all the cross-sections of Volterra kernels.

However, Volterra kernel representation has the defect that too many data are necessary to represent the nonlinear characteristics of the system. In order to solve this problem, we estimate parameters which represent the nonlinear characteristics of the system from the Volterra kernels identified by the M-sequence correlation method. The estimation method is based on the harmonic probing method(14) and nonlinear parameters are calculated from high-order Fourier transform of the Volterra kernels. In section 2, we review the method for identification of Volterra kernels of nonlinear systems by using M-sequence and correlation technique. The method for estimation of nonlinear parameters is shown in section 3. The proposed method was applied to a nonlinear vibrating system. The results of computer simulation are described in section 4. We also applied the proposed method to a mechanical system driven by an AC servo motor. The results of the experiment are shown in section 5. Finally, in section 6, we summarize the obtained results.

2. Identification of Volterra Kernels

Let u(t) and y(t) be the input and the output of the nonlinear system, respectively. In this paper, we identify a nonlinear system where the input u(t) and the output y(t) satisfy the following equation

\[ y(t) = \sum_{i=1}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} g_i(\tau_1, \tau_2, \tau_3) u(t - \tau_1) \cdots u(t - \tau_i) dt_1 \cdots dt_i. \] (1)

Here, \( g_i(\tau_1, \tau_2, \cdots, \tau_i) \) is the \( i \)-th order Volterra kernel. When an \( n \)-th degree M-sequence is applied to the nonlinear system, the cross-correlation function \( \phi_{uy}(\tau) \) between the input \( u(t) \) and the output \( y(t) \) can be calculated as following equation

\[ \phi_{uy}(\tau) = \frac{u(t - \tau) y(t)}{\int_{0}^{N \Delta t} \int_{0}^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) u(t - \tau_1) \cdots u(t - \tau_i) dt_1 \cdots dt_i} \]

\[ = \sum_{i=1}^{\infty} \int_{0}^{N \Delta t} \int_{0}^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) \phi_{uu}(\tau_1, \cdots, \tau_i) dt_1 \cdots dt_i. \] (2)

Here, \( \tau \) means time average over \( N \Delta t \), where \( N \) is a period of the M-sequence and \( \Delta t \) is the clock pulse cycle of the M-sequence. \( \phi_{uu}(\tau_1, \cdots, \tau_i) \) denotes the \( (i + 1) \)-th moment of the input \( u(t) \) and can be written as

\[ \phi_{uu}(\tau_1, \cdots, \tau_i) = \begin{cases} 1 & \text{(for certain } \tau \text{)} \\ -\frac{1}{N} & \text{(otherwise)} \end{cases} \] (3)

Then, the cross-correlation function \( \phi_{uy}(\tau) \) can be approximated as

\[ \phi_{uy}(\tau) \approx \Delta t g_1(\tau) + 2^i \sum_{j=1}^{m_i} (\Delta t)^2 g_2(\tau - d_{11}^{(j)} \Delta t, \tau - d_{12}^{(j)} \Delta t) + \cdots + 3^i \sum_{j=1}^{m_i} (\Delta t)^3 g_3(\tau - d_{11}^{(j)} \Delta t, \tau - d_{12}^{(j)} \Delta t, \tau - d_{13}^{(j)} \Delta t) + F_3(\tau). \] (4)

Here, \( F_3(\tau) \) is the 3rd order Volterra kernel having the same time coordinate. The integers \( d_{11}^{(j)} (1 \leq r \leq i) \) satisfy the next equation and \( m_i \) is the number of the group of the integers.

\[ u(t) u(t + d_{11}^{(j)} \Delta t) \cdots u(t + d_{1i}^{(j)} \Delta t) = u(t + d_{ii}^{(j)} \Delta t) \] (5)

The group of integers can be uniquely determined by the characteristic polynomial of the M-sequence. This property is called "shift and add property" of the M-sequence(16). From equation (4), the obtained cross-correlation function includes not only the impulse response of the system but also cross-sections of higher order Volterra kernels. So, by using the cross-correlation function, the 2nd and 3rd Volterra kernels can be reconstructed.
3. Nonlinear Parameter Estimation from Volterra Kernels

In this paper, we propose a new method for estimation of nonlinear parameters. The method is based on the harmonic probing method\(^{(14),(15)}\). In the proposed method, it is assumed that the nonlinear system to be identified is a nonlinear vibration system shown in Fig. 1. Here, \(m\) is a mass of an object, \(c\) is a damping coefficient, \(k_1\) and \(k_2\) are a linear and a nonlinear stiffness constant, respectively. Let \(f\) be an external force and \(y\) be the displacement from the neutral point. The equation of motion for the nonlinear vibration system is given by

\[
m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + k_1 y + k_2 y^2 = f. \tag{6}
\]

By changing the variables, the equation (6) can be transformed into

\[
\frac{d^2y}{dt^2} + 2\zeta \omega_n \frac{dy}{dt} + \omega_n^2 y + \frac{k_2}{k_1} y^2 = \frac{\omega_n^2}{k_1} u. \tag{7}
\]

Here, \(\zeta\), \(\omega_n\) and \(u\) are defined by

\[
\zeta = \frac{c}{2 \sqrt{mk_1}}, \tag{8}
\]

\[
\omega_n = \sqrt{k_1/m}, \tag{9}
\]

\[
u = f. \tag{10}
\]

Thus, this vibration system becomes a nonlinear system which satisfies equation (7).

By using the harmonic probing method\(^{(14),(15)}\), the nonlinear parameter \(k_2\) is obtained as follows. If the input \(u(t)\) of the nonlinear system consists of two sinusoidal signals having different frequency,

\[
u(t) = e^{i\omega_1 t} + e^{i\omega_2 t} \tag{11}
\]

Then, substituting equation (11) into equation (7), the output \(y(t)\) of the nonlinear vibration system can be expressed by

\[
y(t) = G_1(\omega_1) e^{i\omega_1 t} + G_1(\omega_2) e^{i\omega_2 t} + G_2(\omega_1, \omega_1) e^{2i\omega_1 t} + G_2(\omega_2, \omega_2) e^{2i\omega_2 t} + 2G_2(\omega_1, \omega_2) e^{i(\omega_1+\omega_2) t} + \ldots. \tag{12}
\]

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Table 1  Parameters of the nonlinear vibration system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>(m)</td>
<td>1.0</td>
</tr>
<tr>
<td>damping coefficient</td>
<td>(c)</td>
<td>1.0</td>
</tr>
<tr>
<td>linear stiffness constant</td>
<td>(k_1)</td>
<td>1.0</td>
</tr>
<tr>
<td>nonlinear stiffness constant</td>
<td>(k_2)</td>
<td>0.2</td>
</tr>
<tr>
<td>clock pulse cycle</td>
<td>(\Delta t)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

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Fig. 1  Nonlinear vibration system
Here, $G_i(\omega_1, \cdots, \omega_i)$ is the high-order Fourier transform of the $i$-th Volterra kernels. If we substitute equation (11) and equation (12) into the differential equation (7) and compare the coefficients of the term having $G_2(\omega_1, \omega_2)$, the next equation is obtained.

$$G_2(\omega_1, \omega_2) = -k_2 G_1(\omega_1) G_1(\omega_2) G_1(\omega_1 + \omega_2)$$  \hspace{1cm} (13)

Thus, the nonlinear parameter $k_2$ can be calculated from the 1st and 2nd order Fourier transform $G_1(\omega)$ and $G_2(\omega_1, \omega_2)$ as

$$k_2 = -\frac{G_2(\omega_1, \omega_2)}{G_1(\omega_1) G_1(\omega_2) G_1(\omega_1 + \omega_2)}.$$  \hspace{1cm} (14)

In this equation, the Fourier transform $G_1(\omega)$ and $G_2(\omega_1, \omega_2)$ are obtained from the Volterra kernels of the nonlinear system. So, once the Volterra kernels are identified by using the method shown in section 2, it is not necessary to perform further experiments to calculate the nonlinear parameter $k_2$.

4. Computer Simulation

In order to ensure that the nonlinear parameters can be estimated by the proposed method, a computer simulation was carried out. The parameters of the nonlinear vibration system used in the simulation are shown in Table 1. To identify the Volterra kernels of the nonlinear system, it is necessary to select suitable M-sequence with which the cross-correlation function $\phi_{uy}(\tau)$ contains Volterra kernel slices in a non-overlapped manner(16). In this paper, we chose 16-th degree M-sequence whose characteristic polynomial is given by

$$f(x) = x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^{9} + x^{8} + x^{3} + x^{2} + 1.$$  \hspace{1cm} (15)
Fig. 2 shows a part of the calculated cross-correlation function $\phi_{uy}(\tau)$ between the input $u(t)$ and the displacement $y(t)$ of the load. The obtained 1st and the 2nd order Volterra kernels are shown in Fig. 3 and Fig. 4, respectively. The nonlinear parameter $k_2$ was calculated using 30 different angular frequency pairs $(\omega_1, \omega_2)$. The obtained nonlinear parameter was $\hat{k}_2 = 0.198$, which was almost equal to the true value $k_2 = 0.2$.

We estimated nonlinear parameter by using different nonlinear parameter $k_2$. The relation between the estimated parameter $\hat{k}_2$ and true value $k_2$ is shown in Fig. 5. In this figure, the black point shows the estimated parameter $\hat{k}_2$ and the solid line shows the theoretical value. From Fig. 5, it is shown that the error between the estimated parameter $\hat{k}_2$ and the true value $k_2$ tends to become large according to increase of the nonlinear parameter $k_2$. However, it is clear that the nonlinear parameter $k_2$ can be estimated precisely by the proposed method when the nonlinear parameter is rather small.

When the proposed method is applied to an actual system, it is possible that a correct parameter can not be obtained because of the noise included in the measured signal. We also investigated the influence of the noise on the proposed method. The parameter $\hat{k}_2$ was estimated by the proposed method under various signal to noise ratio. The relation between the estimated parameter $\hat{k}_2$ and signal to noise ratio is shown in Fig. 6. In this case, the true value of the nonlinear parameter is $k_2 = 0.2$ and $k_2 = 0.4$. From the results shown in Fig. 6, it is clear that the nonlinear parameter $k_2$ can be estimated excellently even when signal to noise ratio is 0dB.

5. Experimental Results

In order to confirm that the proposed method is effective to nonlinear systems, we also apply the proposed method to a nonlinear mechanical system shown in Fig. 7. The system
Fig. 6 Relation between the estimated parameter $\hat{k}_2$ and S/N ratio

Fig. 7 Experimental apparatus

Fig. 8 Block diagram of the linear part of the apparatus

consists of an AC-servo motor, a coupling and a load. The AC motor generates a torque $T$ and the torque is transmitted to the load through a coupling. In this case, the input of the nonlinear system is the input command $u(t)$ to the AC motor and the output is the rotation angle $\theta(t)$ of the load measured by an encoder. There exists the nonlinear element, such as backlash of the coupling and friction, in the mechanical system.

Fig. 8 is a block diagram of the linear part of the system. In this figure, $P_G$ is the position gain of the amplifier and $J$ is the moment of inertia of the motor. The AC motor is controlled by a PD controller. $K_P$ and $K_I$ are the proportional gain and the integral gain of the controller, respectively. The parameters of the experimental apparatus are also shown in Table 2. The M-sequence used to identify the Volterra kernels is the same one used in the computer simulation.

The cross-correlation function between the input command $u(t)$ to the AC motor and the rotation angle $\theta(t)$ of the load is calculated. The obtained 1st Volterra kernel $g_1(\tau)$ and the 3rd order Volterra kernel $g_3(\tau_1, \tau_2, \tau_3)$ in case of $\tau_3 = 1$ are shown in Fig. 9 and Fig. 10, respectively. In this system, 2nd order Volterra kernel $g_2(\tau_1, \tau_2)$ was negligibly small.

In order to obtain linear and nonlinear parameters from the obtained Volterra kernels, it is necessary to construct differential equation between the input and output of the nonlinear system. The order of the differential equation can be determined by the gain $|G(j\omega)|$, which is
Table 2 Parameters of the experimental apparatus

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>moment of inertia of the motor</td>
<td>$J$</td>
<td>0.0014</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>damping coefficient</td>
<td>$c$</td>
<td>0.07071</td>
<td>Nm/s/ rad</td>
</tr>
<tr>
<td>torque constant</td>
<td>$K_T$</td>
<td>0.3677</td>
<td>Nm/A</td>
</tr>
<tr>
<td>backlash of the coupling</td>
<td>$B$</td>
<td>0.0126</td>
<td>rad</td>
</tr>
<tr>
<td>position gain</td>
<td>$P_G$</td>
<td>30.0</td>
<td>rad/s</td>
</tr>
<tr>
<td>proportional gain</td>
<td>$K_P$</td>
<td>-1906</td>
<td>rad/s</td>
</tr>
<tr>
<td>integral gain</td>
<td>$K_I$</td>
<td>212</td>
<td>rad/s</td>
</tr>
<tr>
<td>clock pulse cycle</td>
<td>$\Delta t$</td>
<td>0.002</td>
<td>s</td>
</tr>
</tbody>
</table>

Fig. 9 Obtained 1st order Volterra kernel of the experimental system

the Fourier transform of the 1st-order Volterra kernel $g_1(\tau)$. From the result shown in Fig. 11, it is obvious that the gain of the system decreases 60dB with the angular frequency increase from $100 \text{ [rad/s]}$ to $1000 \text{ [rad/s]}$. By using this result we determine the differential equation as

$$
\alpha_0 \frac{d^3 \theta(t)}{dt^3} + \alpha_1 \frac{d^2 \theta(t)}{dt^2} + \alpha_2 \frac{d\theta(t)}{dt} + \alpha_3 \theta(t) + k_3 \theta^3(t) = u(t).
$$

(16)

Here, $\alpha_i$ ($i = 0, \cdots, 3$) are parameters which determine the linear part of the dynamic characteristics of the system and they are identified by the least square method. The parameter $k_3$ is the nonlinear parameter and can be estimated by the proposed method mentioned in section 2. Since the experimental system does not have the 2nd order Volterra kernel, the parameter $k_3$ can be obtained as

$$
k_3 = \frac{G_3(\omega_1, \omega_2, \omega_3)}{G_1(\omega_1 + \omega_2 + \omega_3)G_1(\omega_1)G_1(\omega_2)G_1(\omega_3)}.
$$

(17)

The estimated parameters are shown in Table 3.

Fig. 10 Obtained 3rd order Volterra kernel of the experimental system
Table 3  Estimated parameters of the nonlinear system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>$0.2593 \times 10^{-1}$</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$0.1840 \times 10^1$</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$0.2422 \times 10^2$</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$0.8092 \times 10^3$</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$k_3$</td>
<td>$0.1533 \times 10^{-3}$</td>
<td>Nms/rad$^3$</td>
</tr>
</tbody>
</table>

In order to confirm the estimated parameters are correct, we compare the estimated output which is calculated from the parameters shown in Table 3 with the actual output. The result is shown in Fig. 12. In this case, the input of the nonlinear system is the M-sequence used for identification. In this figure, the solid line shows the actual output of the experimental system and "•" denote the estimated output. It is obvious that the estimated output shows a good agreement with the actual output.

6. Conclusion

In this paper, the authors propose to estimate parameters of nonlinear system from the Volterra kernels. The feature of the proposed method is that once the Volterra kernels are identified, it is not necessary to perform further experiments. The nonlinear parameters are calculated from the high-order Fourier transform of the Volterra kernels. We applied the proposed method to identification of a nonlinear mechanical system driven by an AC servo motor. From the results of the computer simulation and the experiment, the parameters that represent the nonlinear characteristics of the mechanical system can be identified by the proposed method.
References