Aerodynamic Forces Acting on and Lateral Translational and Rotational Motions of Intermediate Cars Travelling in a Tunnel*

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Abstract
The aerodynamic forces acting on representative cars of high-speed trains travelling in a tunnel have been studied theoretically, as well as the translational and rotational motions of these cars. The aerodynamic forces are simplified into one-dimensional sinusoidal ones. Each of the cars, coupled to others and supported by springs and dampers, has two degrees of freedom for translational and rotational motions. The results show that (a) the calculated aerodynamically-induced lateral force and yawing moment agree well with observations from running experiments; (b) the steady-state mode shapes agree well with those obtained previously from running experiments; and (c) the wavelength of the travelling aerodynamic force controls the phase differences among the cars of the train.

Key words: High-Speed Train, Aerodynamic Force, Vehicle Dynamics, Translational Motion, Rotational Motion

1. Introduction
Since 1986, fluid-structure interactions of high-speed trains have been studied in Japan as one of the issues affecting ride quality(1),(2). As trains travel at higher speeds, the vibration amplitude becomes greater, especially in tunnel sections. It has been noted that the yawing and lateral vibration of trains in a tunnel section is more pronounced than when the same train is travelling in the open environment (non-tunnel), and that the vibration amplitude gradually increases from the head towards the tail of a train. Initially, track irregularity was supposed to be a major cause of the vibrations, but little correlation between track irregularity and the vibrations was found for trains travelling in tunnels; this indicated that track irregularity was not the cause of the vibrations(3).

A mechanism for the vibrations based on aerodynamic effects of the tail car was proposed by Suzuki et al.**(4). However, the aerodynamic forces on intermediate cars were not considered. Later, the effect of aerodynamic forces acting on intermediate cars was investigated**(5)–(7). Vibrations and pressure fluctuations on the sides of a car were measured simultaneously, both in the open and in tunnel sections, and aerodynamic forces on the car were calculated from the pressure data. It was found that: (i) the aerodynamic force in tunnel sections is much greater than that in the open; (ii) the aerodynamic force and the vibration in tunnel sections gradually increases from the head towards the tail of the train; and (iii) lateral and yawing vibrations of the car have a close correlation with the aerodynamic force acting on the car. Velocity and pressure fluctuations on the sides of a high-speed train were also measured simultaneously, using hot-film probes and pressure gauges, by Sakuma et al.**(6). A strong correlation between pressure and velocity fluctuations indicated the existence of large-scale coherent structures.
moving downstream at a speed equivalent to about 80% of the train speed.

Numerical simulations and wind tunnel experiments for the flow around a high-speed train travelling in a tunnel have been conducted by several researchers(8) – (12). These investigations indicated that the flow on the sides of intermediate cars of a high-speed train contains vortical structures generated near the under-corners of those cars facing the tunnel wall. It was also shown that these vortices are responsible for the pressure fluctuations. Fixing fins on the under-corners of the car bodies has been proposed as a means of reduction of the aerodynamic force acting on the sides of the cars(10) – (12).

Fujimoto(13) measured the vibrations on two adjacent cars of a Japanese high-speed train, both in the open and in tunnel sections, and calculated the phase difference between the vibration of these cars. It was found that, for trains in a tunnel, both lateral and yawing motion of the back car lead those of the front car by 90 degrees. This was not observed when the train was travelling in the open. In addition, they obtained the motion of five adjacent cars over a period of vibration; with increasing time, for trains travelling in a tunnel, motion of the back car was found to propagate to the front car. The cause of this motion, however, was not clarified. In this paper, the mechanism of this motion will be clarified.

2. Theoretical model of the dynamics

2.1. Travelling aerodynamic force along the train

Since periodic pressure fluctuations in the space between the side of the train and the tunnel wall generate the aerodynamic force acting on the sides of the cars as explained in the previous section, the aerodynamic force can be modelled as shown in Figure 1. Figure 1 shows a schematic of the sinusoidal aerodynamic force travelling along five adjacent cars, from the head toward the tail of the train. The travelling aerodynamic force, per unit length of the train, is assumed to be a sinusoidal wave, and is given by

\[ F_{EX}(x, t, U) = F_E(U) \cos(kx - \omega_E t) \]

where

\[ L_j = 2 \sum_{k=1}^{j-1} l_k + l_j \]

is the length from the head of the train to the middle point of the \( j \)th car, \( F_E(U) \) is the amplitude of the aerodynamic force acting on the cars, which is directly proportional to the square of the air velocity(8), \( k = 2\pi/\lambda_E \) is the wavenumber of the aerodynamic force, \( f_E \) the frequency, \( \omega_E = 2\pi f_E \) the circular frequency, \( V_E = f_E \lambda_E = \omega_E/k \) is the travelling speed of the aerodynamic force, and \( \xi \) is the local coordinate on the \( j \)th car which is related to the \( x \) coordinate by

\[ x = 2 \sum_{k=1}^{j-1} l_k + l_j + \xi = L_j + \xi. \]

Hence, the lateral force and yawing moment acting on the \( j \)th car are given by
Lateral force \( Q_y(j) = \int_{l_j}^{l_{j+1}} F_{EX} d\xi \) = \( A_1 \cos \left( \frac{2\pi}{\lambda E} L_j - \omega_E t \right) \),

where
\[
A_1 = \frac{\lambda E F_E}{\pi} \sin \left( \frac{2\pi l_j}{\lambda E} \right) \tag{4}
\]

Yawing moment \( Q_\alpha(j) = \int_{l_j}^{l_{j+1}} \xi F_{EX} d\xi \) = \( A_2 \sin \left( \frac{2\pi}{\lambda E} L_j - \omega_E t \right) \),

where
\[
A_2 = \frac{\lambda E F_E}{\pi} \left\{ l_j \cos \left( \frac{2\pi}{\lambda E} l_j \right) - \frac{\lambda E}{2\pi} \sin \left( \frac{2\pi}{\lambda E} l_j \right) \right\}. \tag{5}
\]

2.2. Train model

Simulation of translational and rotational motions of train dynamics commonly includes seventeen and more degrees of freedom for each car and interaction between wheels and rails. Since the main cause of the vibrations of trains travelling in tunnels is the aerodynamic force acting on the sides of the cars, a simplified model of an actual train subjected to only forced vibration of the aerodynamic force is employed, that is, the interaction between wheels and rails is excluded. An actual vehicle with two bogies is simplified to a body supported only on two sets of translational springs and dampers; i.e., the bogies are modelled by springs and dampers. These cars are coupled by springs and dampers and they can perform translational and rotational oscillatory motions. The system under consideration is shown in Figure 2. It consists of \( N \) rigid cars that can only perform lateral translational \( y(j) \) and yawing \( \alpha(j) \) oscillatory motions of small amplitude. Each car is attached to effectively the rails or the “ground” via two sets of translational springs and dampers \( (k_f, k_b, c_f, \) and \( c_b; “f” \) for front, “b” for back). Rotational and translational springs and dampers are also considered interconnecting the cars \( (k_q, k_n, c_q, \) and \( c_n) \).

2.3. The equations of motion

The kinetic energy of the \( j \)th car, \( T_{sj} \), is
\[
T_{sj} = \frac{1}{2} m_j y_j(t)^2 + \frac{1}{2} I_c j \alpha_j(t)^2, \tag{6}
\]

where \( m_j \) is the mass of the \( j \)th car and \( J_c \) is its mass-moment of inertia about the centre of mass.
The dissipation energy of the \( j \)th car, \( D_{sj} \), is

\[
D_{sj} = \frac{1}{2} c_f (y_{c,j} - \beta l_j \dot{\alpha}_j)^2 + \frac{1}{2} c_b (y_{c,j} + \beta l_j \dot{\alpha}_j)^2 + \frac{1}{2} c_{aj} (\dot{\alpha}_j - \dot{\alpha}_{j-1})^2 + \frac{1}{2} c_{bj} (y_{c,j} + \beta l_j \dot{\alpha}_j - (y_{c,j-1} + l_{j-1} \dot{\alpha}_{j-1}))^2 + \frac{1}{2} c_{aj+1} (\dot{\alpha}_{j+1} - \dot{\alpha}_j)^2 + \frac{1}{2} c_{aj+1} (\dot{y}_{c,j+1} - (y_{c,j} + l_j \dot{\alpha}_j))^2,
\]

where \( \beta \) is the displacement coefficient for the supporting spring from the centre of the car as shown in Figure 2 and \( l_j \) is the half length of the \( j \)th car.

Finally, the potential energy of the \( j \)th car, \( V_{sj} \), is

\[
V_{sj} = \frac{1}{2} k_f (y_{c,j} - \beta l_j \dot{\alpha}_j)^2 + \frac{1}{2} k_b (y_{c,j} + \beta l_j \dot{\alpha}_j)^2 + \frac{1}{2} k_{aj} (\dot{\alpha}_j - \dot{\alpha}_{j-1})^2 + \frac{1}{2} k_{bj} (y_{c,j} - \beta l_j \dot{\alpha}_j - (y_{c,j-1} + l_{j-1} \dot{\alpha}_{j-1}))^2 + \frac{1}{2} k_{aj+1} (\dot{\alpha}_{j+1} - \dot{\alpha}_j)^2 + \frac{1}{2} k_{aj+1} (\dot{y}_{c,j+1} - (y_{c,j} + l_j \dot{\alpha}_j))^2.
\]

The total kinetic energy of the system, \( T = \sum_{j=1}^{N} T_{sj} \), \( D = \sum_{j=1}^{N} D_{sj} \), \( V = \sum_{j=1}^{N} V_{sj} \), equation (4), and equation (5) are substituted into Lagrange’s equations

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial \dot{q}_j} = Q_q, \quad q_1 = y_{c,j}, \quad q_2 = \alpha_j, \quad j = 1, 2, ..., N, \quad (9)
\]

which yields the equations of motion:

\[
\begin{align*}
& m_j \ddot{y}_{c,j}(t) - c_{nj} \dot{y}_{c,j-1}(t) + \left( c_{f,j} + c_{bj} + c_{nj,j} \right) \dot{y}_{c,j}(t) - c_{nj+1} \dot{y}_{c,j+1}(t) \\
& - c_{nj-1} \dot{\alpha}_{j-1}(t) + \left( -\beta c_{f,j} + \beta c_{bj} - c_{nj} + c_{nj+1} \right) \dot{y}_{c,j}(t) + c_{nj+1} \dot{l}_j \dot{\alpha}_{j+1}(t) \\
& - k_{nj} \dot{y}_{c,j-1}(t) + \left( k_{f,j} + k_{bj} + k_{nj} + k_{nj+1} \right) \dot{y}_{c,j}(t) - k_{nj+1} \dot{y}_{c,j+1}(t) \\
& - k_{nj-1} \dot{\alpha}_{j-1}(t) + \left( -\beta k_{f,j} + \beta k_{bj} - k_{nj} + k_{nj+1} \right) \dot{y}_{c,j}(t) + k_{nj+1} \dot{l}_j \dot{\alpha}_{j+1}(t) \\
& = A_1 \cos \left( \frac{2\pi t}{T} - \omega_E t \right), \\
& J_0 \ddot{\alpha}_j(t) + c_{nj} \dot{\alpha}_{j-1}(t) + \left( -\beta c_{f,j} + \beta c_{bj} - c_{nj} + c_{nj+1} \right) \dot{\alpha}_j(t) \\
& - c_{nj+1} \dot{\alpha}_{j+1}(t) + \left( -c_{aj} + c_{nj+1} \dot{l}_j \right) \dot{\alpha}_{j-1}(t) \\
& + \left( \beta^2 c_{f,j} + \beta^2 c_{bj} + c_{aj} + c_{nj+1} \right) \dot{\alpha}_j(t) + \left( -c_{aj+1} + c_{nj+1} \dot{l}_j \right) \dot{\alpha}_{j+1}(t) \\
& + \left( -\beta k_{f,j} + \beta k_{bj} - k_{nj} + k_{nj+1} \right) \dot{\alpha}_j(t) \\
& - k_{nj+1} \dot{\alpha}_{j+1}(t) + \left( -k_{aj} + k_{nj+1} \dot{l}_j \right) \dot{\alpha}_{j-1}(t) \\
& + \left( \beta^2 k_{f,j} + \beta^2 k_{bj} + k_{aj} + k_{nj+1} \right) \dot{\alpha}_j(t) + \left( -k_{aj+1} + k_{nj+1} \dot{l}_j \right) \dot{\alpha}_{j+1}(t) \\
& = A_2 \sin \left( \frac{2\pi t}{T} - \omega_E t \right).
\end{align*}
\]

The equations of motion are solved by a fourth-order Runge-Kutta numerical scheme. The convergence of this computation has been validated by employing different time increments, \( \Delta t \); we employ \( \Delta t = 0.01 \) in the computation. Typical system parameters of actual high-speed trains in Japan are employed as given in Table 1. The number of cars in the train is set at \( N = 5 \). (It has been confirmed that similar results can be obtained for other numbers of cars.)
3. Analytical results

3.1. Features of the force and comparison with a running experiment

In Section 2.1, since periodic pressure fluctuations in the space between the side of the train and the tunnel wall generate the aerodynamic force acting on the sides of the cars, the aerodynamic force is assumed to be sinusoidal. In this section, this assumption is examined. Features of the force and then comparison with a running experiment are discussed.

The amplitudes of both forces, \( Q_{\gamma j} \) and \( Q_{\alpha j} \), are a function of the wavelength \( \lambda_E \), as seen in equations (4) and (5) and illustrated in Figure 3. The vertical axes are arbitrarily scaled with respect to the absolute values for a wavelength of 30 meters. Both forces periodically become zero because of their dependence on the trigonometric sine and cosine functions. The lateral force, \( Q_{\gamma j}(t) \), becomes zero when \( \lambda_E = 25.0 \), the length of the car, as seen in Figure 3, and thus no lateral force acts on the cars. On the other hand, the yawing moment becomes zero at a wavelength of 17.5 m. Therefore, it is expected that the response of the cars to a sinusoidal aerodynamic force for particular wavelengths, such as 17.5 and 25 m, will be purely lateral translational or rotational motion. As shown in equations (4) and (5), \( Q_{\gamma j} \) is proportional to \( \cos \left( 2\pi L_j / \lambda_E - \omega t \right) \), while \( Q_{\alpha j} \) to \( \sin \left( 2\pi L_j / \lambda_E - \omega t \right) \). Because the cosine function leads the sine by 90 degrees, the lateral force \( Q_{\gamma j} \) leads the yawing moment \( Q_{\alpha j} \) by 90 degrees or one-quarter cycle.

Next, this phase relationship between \( Q_{\gamma j} \) and \( Q_{\alpha j} \) will be verified by comparing these forces with the result of a running experiment. Figure 4 shows correlation coefficients between the lateral force and the yawing moment acting on a car in a high-speed train, which is obtained from actual running experiments. Note that the peak of the pressure fluctuations is at 3.7 Hz in the running experiments. In addition, the time lag between the lateral force and the yawing moment obtained from the present model is 90 degrees or one-quarter cycle. Hence, the time lag of one-quarter cycle corresponding to 3.7 Hz of the running experiments is given by \( T/4 = 1/(4f) = 1/(4 \times 3.7) = 0.07 \) sec, which is quite close to the time lag in Figure 4 (0.08 sec). For this reason, it is shown that the lateral force and yawing moment calculated from the one-dimensional sinusoidal forces result in good agreement with running experiments.

3.2. Steady-state response to travelling forces with wavelengths of \( \lambda_E = 17.5, 25, 30, \) and 33.3 meters

In Section 3.1, the analytical model of the one-dimensional sinusoidal force representing the pressure fluctuations travelling along the side of a single car in a train has been validated by comparing with the results of the running experiment. In this section, the effect of the travelling forces on the lateral translational or rotational motion of cars in the train is examined. Note that wavelengths of the aerodynamic forces, \( \lambda_E \), can be estimated ranging approximately from 10 m to 40 m, which is of the order of the length of a single car, 25 m, from the results of running experiments of travelling speeds and frequencies of the pressure fluctuations. Thus,
the translational and rotational responses and mode shapes of the train are obtained varying several different wavelengths in that range. In Section 3.1, it was shown that for a wavelength of 17.5 m the yawing moment is zero, while the lateral force is zero for a wavelength of 25 m. Thus, in this section, first, the steady-state response to travelling aerodynamic forces with wavelengths of 17.5 and 25 m is illustrated; and then, the response due to forces with wavelengths of 30 and 33.3 m is obtained; these are examples of typical wavelengths of the aerodynamic force acting on high-speed trains\(^6\),\(^8\). The steady-state mode shapes of the train are also obtained for these examples.

Figure 5 illustrates the steady-state response of the train to a travelling aerodynamic force with a wavelength of 17.5 m. Figures 5(a) and (b) show the lateral translational displacements and the angular of the centre of all five cars, respectively. The steady-state lateral translational displacement of each car has the same magnitude; however, there is a phase difference of \(0.43T\) in the order of \(j = 1 \to 2 \to 3 \to 4 \to 5\) as seen the notes below the waves in Figure 5(a). On the other hand, as seen in Figure 5(b), the rotational motion is approximately zero for all time. Hence, cars subject to a travelling aerodynamic force of wavelength 17.5 m display almost pure translational motion, and no rotational motion. Figure 5(c) shows the train mode shape in its steady state, confirming that translational motion is dominant for all cars. (The axis of ordinate is an arbitrary scale representing lateral translational displacement in Figure 5(c).) Remember that this result can be expected from Figure 3, because the yawing moment for a wavelength of 17.5 m is zero, while the lateral force has a peak value.

Figure 6 illustrates the response of the train to a travelling aerodynamic force with a wavelength of 25 m. In this case, the lateral aerodynamic force is zero, and so the expectation is that there will be no translational displacement; Figure 6(a) shows no lateral displacements of the centre of each of the five cars. Figure 6(b) gives angular displacements of the cars. All cars display almost pure rotational motion with a phase difference of \(T/4\) between two adjacent cars, which can be verified by the modes in Figure 6(c).

Figures 7(a) and (b) illustrate the response of the train in the lateral and rotational directions, respectively, to a travelling aerodynamic force with a wavelength of 30 m. Following a similar procedure to that described earlier in this section, it can be shown that the steady-state responses of the cars are of the same magnitude but with a phase difference of \(T/6\) between their motions in the order of \(j = 5 \to 4 \to 3 \to 2 \to 1\) as seen the notes below the waves in Figure 7(a) and (b). All cars display both translational and rotational motion.

Figure 8 shows the response of the train in the lateral and rotational directions, respectively, to a travelling aerodynamic force with a wavelength of 33.3 m. As seen in Figure 8, the displacements of each car behave in almost the same way, with some phase difference of \(T/4\) between them in the order of \(j = 5 \to 4 \to 3 \to 2 \to 1\). Figure 8(c) illustrates the mode shape of the train in its steady state every \(T/4\). The motion of the tail-end cars travels toward the head of the train as a wave in the order of \(j = 5 \to 4 \to 3 \to 2 \to 1\), even though the force travels from the head of the train towards the tail of the train. The reason for this motion will
be discussed in Section 4.

3.3. Comparison with previous work

The steady state mode shapes obtained from the present study are compared to those of previous work obtained from running experiments. Figures 9(b) and (c) illustrate the motion of five adjacent cars in a high-speed train in unconfined flow (no-tunnel) and in tunnel sections obtained from running experiments by Fujimoto(13). It is seen that the mode shapes of a real train in unconfined flow (no-tunnel) are very different to those obtained when the train is in a tunnel section. In addition, the motion of the tail-end cars travels toward the head of the train as a wave in the order of $j=5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$. Note that Figure 9(a) is from Figure 8(c). Compare the result for $t=0$ in Figure 9(a) and $t=0$ in Figure 9(c); also $t=T/4$ in Figure 9(a) and $t=T/4$ in Figure 9(c); et seq. From these comparisons, it is apparent that agreement between the results of the present numerical study and previous experiments in a tunnel section is good. Therefore, it is shown that the present model of one-dimensional sinusoidal forces well explain the result of mode shapes obtained by running experiments.

4. Discussion

4.1. Phase relationship between two adjacent cars

In the foregoing, the phase relationship between $Q_{y_j}(t)$ and $Q_{\alpha_j}(t)$ for a single car was given. Here, the phase relationship between two adjacent cars, Car $j+1$ and Car $j$, in the train is examined. Since the length of all cars is $2l$, using equation (2) we have

$$L_{j+1} = L_j + 2l.$$  

(12)

Substituting (12) into (4) with $L_{j+1}$ replacing $L_j$, we obtain

$$\cos \left( \frac{2\pi}{\lambda_E} L_{j+1} - \omega_E T \right) = \cos \left( \frac{2\pi}{\lambda_E} (L_j + 2l) - \omega_E T \right) = \cos \left( \frac{2\pi}{\lambda_E} L_j - \omega_E T + \frac{2\pi}{\lambda_E} 2l \right).$$  

(13)

Because the phase expression in equation (5) is the same as that in (4), equation (13) also gives an indication of the phase relating the yawing moment between adjacent cars. This equation indicates that both the lateral force and the yawing moment acting on the front car, Car $j$, lead the back car, Car $j+1$, by the phase difference $(2\pi/\lambda_E)2l$. For this reason, the front car, Car $j$, ...
is expected to move first in the lateral and rotational directions, and then, after a phase interval of \((2\pi/\lambda_E)2l\), the back car, Car \(j + 1\) will move.

4.2. Wavelength of travelling sinusoidal forces and phase differences among intermediate cars of the train

The results of steady-state responses obtained in Section 3 will be discussed here by using equation (13).

In Section 4.1, it was shown that the front car, Car \(j\), is expected to move first in the lateral and rotational directions, respectively, and then, after a phase interval \((2\pi/\lambda_E)2l\), the next car, Car \(j + 1\) will move.

Substituting \(\lambda_E = 17.5\) and \(2l = 25\) into \((2\pi/\lambda_E)2l\), we have \((2\pi/17.5)25 \approx 2\pi(1.43)\) radians. Thus, the front car, Car \(j\), leads the next car, Car \(j + 1\), by \(2\pi(1.43)\), which are observed and noted above the waves in Figure 5(a). Note that, since \(2\pi(1.43) = 2\pi(1 + 0.43)\),

**Fig. 6** Response to a travelling aerodynamic force with a wavelength of 25 meters:
(a) steady-state translational response; (b) steady-state rotational response; (c) mode shape.

**Fig. 7** Response to a travelling aerodynamic force with a wavelength of 30 meters:
(a) steady-state translational response; (b) steady-state rotational response; (c) mode shape.
it appears that the phase difference between two adjacent cars is $0.43(2\pi)$, as illustrated and noted below the waves in Figure 5(a). When the wavelength of the aerodynamic force is shorter than a car length, $\lambda_E < 2l$, the phase lag of Car $j+1$ to Car $j$ is larger than $2\pi$ and thus the phase lag can be considered the one with $2\pi$ less. Therefore, the steady-state motion of the head-end cars can be considered to travel toward the tail of the train as a wave in the order of $j = 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5$, as shown in Figure 5(a).

Substituting $\lambda_E = 25$ and $2l = 25$ into $(2\pi/\lambda_E)2l$, we have $(2\pi/25)25 = 2\pi$. When the wavelength of the aerodynamic force is equal to that of the car, $\lambda_E = 2l$, the front car, Car $j$, ...
leads the next car, Car $j + 1$, by $2\pi$, which means all cars move in-phase with each other as shown in Figure 6(b).

Substituting $\lambda_j = 30$ and $2l = 25$ into $(2\pi/\lambda)2l$, we have $(2\pi/30)25 = 2\pi(5/6)$ radians. Thus, the front car, Car $j$, leads the next car, Car $j + 1$, by $2\pi(5/6)$, which are observed and noted above the waves in Figure 7(a). Note that, since $2\pi(5/6) = 2\pi(1 − 1/6)$, it appears that the back car, Car $j + 1$, leads the front car, Car $j$, by $\delta = 1/6(2\pi)$, as illustrated and noted below the waves in Figure 7(a).

Using a similar reasoning for $\lambda_j = 33.3$, it can be shown that the front car, Car $j$, leads the back car, Car $j + 1$, by $2\pi(3/4)$, which can be observed in Figure 8(b). Note that the motion of Car 1 and Car 5 are identical because of the phase difference $2\pi(3/4)$. Since $2\pi(3/4) = 2\pi(1 − 1/4)$, it appears that the back car, Car $j + 1$, leads the front car, Car $j$, by $\delta = 1/4(2\pi)$ as illustrated in Figure 8(b).

As mentioned above, when the wavelength of the aerodynamic force is longer than a car length, $\lambda_j > 2l$, the phase lag (say, $\Delta$) of Car $j + 1$ to Car $j$ is smaller than $2\pi$ and thus the phase lag can be considered with $2\pi \times \Delta$. Therefore, the steady-state motion of the tail-end cars can be considered to travel toward the head of the train with a phase lag of $\Delta$ as a wave in the order of $j = 5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$, even though the force travels from the head towards the tail of the train.

5. Conclusions

Since the main cause of the vibrations of trains travelling in tunnels is the aerodynamic force acting on the sides of the cars, a simplified model of an actual train subjected to only forced vibration of the aerodynamic force is employed to clarify the mechanism of motions of trains in tunnels, apart from large-scale numerical computations of a train with a few hundred degrees of freedom in a flow field. Employing the simplified model enables us relatively easily to examine the equations of motion to understand the mechanism of motions of trains in tunnels.

It was shown that the lateral force and yawing moment calculated from the one-dimensional sinusoidal forces well explain the results obtained by running experiments. The effects of the wavelength, $\lambda_j$, of the travelling aerodynamic force on the mode shapes of a finite-length train were clarified; the wavelength controls the phase differences between cars in the train. The front car moves first in the lateral and rotational directions, and then after a phase interval given by $(2\pi/\lambda_j)l_{car}$, the back car moves. Both the lateral force and yawing moment acting on the front car lead those on the back car by the same phase difference. For this reason, it appears that, for certain wavelengths, the steady-state motion of the tail-end cars can be considered to travel toward the head of the train as a wave, even though the force travels from the head of the train towards its tail.

The mode shapes obtained for the train in its steady-state are qualitatively in good agreement with those of previous work obtained in running experiments. These results reinforce the idea that the main cause of the vibration of actual high-speed trains running through tunnels is forced vibration by the travelling aerodynamic force along the train. In addition, even though a large number of simplifying idealizations were introduced, the present model may be sufficient to examine qualitatively the response of a train to this aerodynamic force.

Note that a general approach for the study of the dynamics of trains and train-like systems of flexibly interconnected rigid bodies with elastic supports subjected to fluid dynamic forces and moving in a tunnel will be given\(^{(16)-(17)}\).

References


