Resonance by Parametric Excitation of Variable Damping

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Abstract
This paper proposes a new method for the utilization of variable damping in a vibration system. A single-degree-of-freedom base-excited model with a variable damper is considered. The coefficient of variable damping can be changed as that in the case of a sine wave, i.e. parametric excitation whose frequency can be arbitrarily selected. One of the external forces acting on the mass through the damper from the base is equivalent to the product of the damping coefficient and the input velocity. By multiplying the input sine wave by the frequency-controlled sine wave of variable damping, a new vibration that has a frequency different from the input frequency arises. Therefore, the oscillation of the damping coefficient at a suitable frequency can generate a new vibrational component that has the same frequency as the eigen-oscillation of the vibration system. As a result, the vibration amplitude increases because of resonance. In this paper, we confirm the growth of the vibration amplitude by the proposed parametric excitation of damping by simulation and experiment.

Key words: Variable Damping, Parametric Excitation, Resonance, Base-Excitation

1. Introduction

Recently, some variable dampers such as magnetorheological (MR) damper and electrorheological (ER) damper, whose coefficients can be easily changed, have been developed. Many researchers used variable damping for semi-active control to reduce the amplitude of vibration (1), (2). In this manner, variable damping has been used to reduce the amplitude of the vibration system and to dissipate the vibration energy. In this paper, we propose a new method for the utilization of variable damping. The purpose is not to decrease the vibration amplitude but to increase it, and it is equivalent to the resonance phenomenon; however, its input frequency is not equal to the natural frequency of the vibration system. In order to apply the proposed method to a multi-degree-of-freedom system, the system can be vibrated at a natural frequency by the input oscillation of another natural frequency. Therefore, in this method, there are possibilities to use the variable damper as an actuator at one frequency by using the vibration of another frequency. In this manner, by exchanging the vibration energy between multiple modes of oscillation, there are possibilities to develop a new semi-active dynamic absorber or an energy-harvesting device in the future. In this paper, we characterize the resonance phenomenon, because there are very few studies on the parametric excitation of damping.

We give a brief description of resonance by variable damping. First we consider a
base-excited model with a variable damper as an example. We change the damping coefficient like a sinusoidal wave, i.e., artificial parametric excitation whose frequency can be selected freely. One of the external forces acting on the mass through the damper from the base is equivalent to the product of the damping coefficient and the input velocity. By multiplying the input sinusoidal wave from the base excitation by the frequency controllable sinusoidal wave of the variable damper, a new vibration that has a frequency different from the input frequency arises. Therefore, the controllable oscillation of the damping coefficient at a suitable frequency can generate a new vibration that has the same frequency as the natural frequency of the vibration system. As a result, an increase in the amplitude of the vibration system can be expected, because of a phenomenon that is similar to resonance. We confirm the phenomenon by simulation and experiment.

2. Base-Excitation Model

In this study, a base-excited single-degree-of-freedom system is considered as shown in Fig. 1. A mass \( m \) is supported by a spring \( k \) and a variable damper \( c(t) \) is connected in parallel. In this figure, \( x \) and \( z \) are the absolute displacements of the mass and base, respectively.

![Fig.1 Single-degree-of-freedom vibration model](image)

The equation of motion of the system is obtained as follows.

\[
\ddot{x}(t) + c(t) \left( \dot{x}(t) - \dot{z}(t) \right) + k \left( x(t) - z(t) \right) = 0. 
\] (1)

We change the variables \( x, z, t \) to \( x^*, z^*, t^* \), respectively.

\[
x = Lx^*, z = Lz^*, t = Tt^*, \]

where, \( L \) is a representative length, and \( T = \sqrt{m/k} \) is a representative time. The dimensionless equation of motion is obtained by the transformation of variable as follows.

\[
\frac{d^2x^*(t^*)}{dt^{*2}} + 2\zeta(t^*) \frac{dx^*(t^*)}{dt^{*}} + x^*(t^*) = 2\zeta(t^*) \frac{dz^*(t^*)}{dt^{*}} + z^*(t^*), \]

where \( \zeta \) is the damping ratio that is represented as \( \zeta(t^*) = c(t^*) / 2\sqrt{mk} \). The input-related terms are transposed to the right side. The left side of the equation represents a vibration system with a variable damping ratio, and the right side has the controllable input term. The damping ratio \( \zeta(t^*) \) can be controlled by using an MR or ER damper. In a general vibration control problem, variable damping is used to reduce the amplitude and dissipation of vibration energy by changing the coefficient according to a control law \(^1\)(\(^2\)).

3. Control Law of Variable Damping Coefficient

The purpose of this study is to induce quasi-resonance outside the natural frequency by changing the damping coefficient. In this section, we explain how to vary the damping coefficient and the expected effect of variable damping. We consider the behavior of the spring mass damper model shown in Fig. 1. Further, we add a harmonic displacement to the
system from the base.

\[ z'(t^*) = \sin \omega t^*. \]

From this equation, the temporal differentiation of the displacement is considered as follows.

\[ \frac{dz^*(t^*)}{dt^*} = \omega \cos \omega t^*. \]

Then, we consider the controllable damping ratio \( \zeta(t^*) \) as a sinusoidal wave that has a frequency \( \omega \zeta \). In this case, \( \zeta(t^*) \) can be expressed as follows.

\[ \zeta(t^*) = \alpha \cos(\omega \zeta t^* + \beta) + \zeta_{\text{cons}}, \]

(4)

where \( \alpha \) is the amplitude; \( \beta \) the phase; and \( \zeta_{\text{cons}} \), the non-zero center amplitude (also known as DC offset). Each parameter is set for \( \zeta > 0 \).

One of the external forces acting on the mass represented by the left side of equation (3) through the damper is equivalent to the product of the damping ratio and the input velocity. The force generated by the variable damper is obtained as follows.

\[ f_{\text{damp}} = 2\zeta(t^*) \frac{dz^*(t^*)}{dt^*} = 2\left( \alpha \cos(\omega \zeta t^* + \beta) + \zeta_{\text{cons}} \right) \cdot \omega \cos \omega t^*, \]

(5)

where \( \omega \zeta \) is the frequency of the damping ratio \( \zeta(t^*) \) and \( \omega \) is the frequency of the velocity \( dz^*/dt^* \). From equation (5), the obtained damping force \( f_{\text{damp}} \) is the addition of three sinusoidal waves with three different frequencies \( \omega, \omega - \omega \zeta, \omega + \omega \zeta \).

As described above, the next effect is considered by multiplying the controllable sinusoidal damping ratio \( \zeta(t^*) \) at the base excited input.

- By using the damping ratio \( \zeta(t^*) \) with a suitable angular frequency, the oscillating component of the obtained damping force can contain arbitrary frequencies. If the oscillating component generated by the variable damper has the natural frequency of the vibration system, a resonance phenomenon is produced by the parametric excitation of the damping coefficient.

In the case of the parametric excitation of damping, the eigen-frequency of the vibration system is constant. Thus, it would appear that quasi-resonance produced by the damping parametric excitation is always possible over the natural frequency.

Next, we will confirm the resonance phenomenon by numerical simulation.

4. Simulation Results

By applying the proposed control law of the variable damping ratio to the single-degree-of-freedom system in Fig. 1, the generation of quasi-resonance is confirmed by numerical simulation. We use Matlab created by The Mathworks, Inc. for numerical analysis. We create a Simulink model referring to the equation of motion (3), as shown in Fig. 5. In addition, Ode45 based on an explicit Runge-Kutta formula was applied to the differential equation in numerical analysis, and we computed until the response of the vibration system became the steady state of vibration. In this simulation, we defined the frequency of base excitation as \( \omega \), frequency of the parametric excitation of the variable damper as \( \omega \zeta \), and absolute value of difference between the two as \( \lambda = |\omega - \omega \zeta| \).
4.1 Growth of Amplitude by Parametric Excitation of Variable Damper

First, we set the input frequency to $\omega = 5$, frequency of parametric excitation to $\alpha_\omega = 6$, and the amplitude, phase and DC offset of the parametric excitation to $\alpha = 0.25$, $\beta = 0$, and $\zeta_{\text{con}} = 0.25$, respectively. Figures 6-9 show the base excitation as the input, damping ratio of the variable damper, actual damping force generated by the variable damper, and response of the absolute displacement of the mass $m$, respectively.
By comparing Fig. 6 with Fig. 9, it is found that the maximum amplitude of the response is larger than that of the input because the natural frequency $\omega_n = 1$ of the vibration system was excited by the oscillating component $\lambda = \omega_c - \omega = 1$ of the newly generated wave. The phenomenon is similar to resonance; however, the main frequencies of the input and output are different. Therefore, we term this phenomenon “quasi-resonance.”

### 4.2 Influence of Frequency and Phase of Parametric Excitation

Next, the frequency of the base excitation was set to $\omega = 5$, and we changed the frequency of the variable damping ratio $\omega_c$ from 5 to 6. As a result of this simulation, the absolute displacement responses of the mass were obtained as shown in Fig. 10. In the case of changing the frequency of the variable damper, the controllable force $f_{damp}$ had the frequency components $\lambda = \omega_c - \omega = 1$ from 0 to 1; therefore, it is clear that a vibration corresponding to the frequency components arises.

In addition, when the damping ratio $\zeta(t')$ was vibrated with the frequency of the input, i.e. $\omega_c = \omega = 5$, a steady state deviation was obtained due to DC offset. Both the frequency parameters were substituted in equation (5); the third term of the equation was obtained as $a\omega_c \cos((\omega_c - \omega)t' + \beta) = a\omega_c \cos \beta$. Therefore, the amplitude of the steady state deviation varied depending on the phase $\beta$ of the damping parametric excitation. Figure 11 shows the steady state deviation when $\beta$ was changed from 0 to $\frac{3\pi}{2}$ and the other parameters were constant. Thus, the maximum steady state deviation is generated in the case of $\beta = \frac{\pi}{2}, \frac{3\pi}{2}$. Not only the steady state deviation but also all the waves
Oscillated by the variable damper have the absolute displacement response whose phase can be arbitrarily changed depending on $\beta$. Figure 12 shows an example of the response of the mass when $\beta$ was changed from 0 to $3\pi/2$. Then, we set frequencies, $\omega = 5$ and $\omega_c = 6$.

![Fig.10 Displacement outputs for angular frequencies of coefficient excitation](image1)

Fig.10 Displacement outputs for angular frequencies of coefficient excitation

![Fig.11 Steady state errors for phases of coefficient excitation](image2)

Fig.11 Steady state errors for phases of coefficient excitation

![Fig.12 Displacement outputs to phases of coefficient excitation](image3)

Fig.12 Displacement outputs to phases of coefficient excitation
4.3 Transmissibility of Proposed System

In this subsection, we compare the transmissibility between the proposed parametric excitation model with a base-excited input and the general passive vibration isolation system. The transmissibility of the parametric excitation model is a quasi-resonance curve that plots the absolute maximum values of the absolute displacement amplitude of the mass. The parameters of the parametric excitation of the variable damper were fixed as $\omega_\zeta = 5$, $\alpha = 0.25$, $\beta = 0$ and $\zeta_{\text{cons}} = 0.25$, and the displacement amplitude of the base excitation was fixed as 1.

![Fig.13 Transmissibility of proposed system](image)

When the frequency $\omega$ of the base excitation was changed from 0 to 10, the quasi-resonance curve was obtained as shown in Fig. 13. For the purpose of comparison, figure 14 shows the transmissibility of the passive base-excited model. In the case of the proposed parametric excitation, two local maximum values were obtained with the exception of the peak around the natural frequency $\omega_\zeta = 1$. Because the frequencies of the two local maxima were $\omega = 4, 6$, which were the points of $\lambda = |\omega - \omega_\zeta| = 1$, quasi-resonance was obtained at these points.

![Fig.14 Transmissibility of base excited model](image)
5. Shaking Table Test

In this section, we describe the confirmation of the growth of the vibration amplitude by variable damping on an experimental basis.

5.1 Experimental Setup

In this subsection, we explain the experimental setup for quasi-resonance. Figure 15 shows the experimental setup developed by us. The copper mass was set on the slider that was mounted on the shaking table, and the mass was connected to the shaking table through the two tension springs. Because the mass had a single –degree –of -freedom for the moving direction of the slider (we define the direction as the $x$ direction), the device was the single –degree –of -freedom vibration system. By placing a neodymium magnet close to the copper mass, the variable damping of the system is obtained as magnetic damping. The magnet can be driven in the $y$ direction perpendicular to the $x$ direction by the linear motor with a PID controller. The damping ratio of the vibration system was varied with the position of the magnet. When the magnet was oscillated as a sine wave, the proposed variable damping could be realized on the vibration system. Table 1 shows the specifications of the vibration system.

Table 1. Specification of the experimental setup

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>10.35</td>
<td>kg</td>
</tr>
<tr>
<td>$k$</td>
<td>730.21</td>
<td>N/m</td>
</tr>
</tbody>
</table>

5.2 Identification of Damping Ratio in Passive Vibration System

First, to clarify the specification of the experimental setup, a random vibration experiment was carried out by fixing the position of the magnet. The displacement $z$ of the shaking table and the mass displacement $x_a = x - z$ related to the shaking table were measured, and we set the input as $z$ and the output as $x = x_a + z$. The frequency responses of the vibration system were obtained as the ratio of the cross-spectrum between the input and the output to the power -spectrum of the input. Because of the fixing of the position of the magnet, the damping ratio of the system was constant. Therefore this system was a passive base-excited system. This variable damping device had a 24 [mm] range of movement. The center of the movable portion was set at 0 [mm], and we carried out random vibration experiments with 7 different magnet positions between -12 [mm] and 12 [mm]. The experimental results are shown in Fig. 16. From this figure, it is clear that the change in the damping ratio is associated with the position of the magnet.
Fig. 16 Transmissibility of base excited model

Fig. 17 Relation between position and $\zeta$

By applying the nonlinear least square method to the experimental results, the damping ratios of the vibration system were identified. Figure 17 shows the results. In this figure, the circles indicate the damping ratios of the experimental setup identified from the results. From this figure, the changes in the damping ratio are related to the positions of the magnet for the variable damping device. The rigid line in Fig. 17 shows the value estimated by the least square method. From these results, by measuring the current position of the magnet, the variable damping ratio can be directly estimated.

5.3 Generation of Quasi-resonance by Sinusoidal Experiment

Based on the fundamental experiment, a sinusoidal experiment was carried out for quasi-resonance by the parametric excitation of the damping coefficient. In this experiment, we fixed the frequency of the parametric excitation as $\omega_c = 6$, and the frequency of the shaking table was changed from 1 [Hz] to 8 [Hz]. The step size of the frequency was 0.1 [Hz].
We measured the absolute displacements of the shaking table and the relative displacement of the copper mass by using two laser displacement sensors. In the case of \( \omega = 7 \), the results of time-series data are shown in the following figures. Figure 18 shows the position of the magnet. From these results, it is found that the damping coefficient is changed like a sine wave, and the amplitude and frequency are 3.8 [mm] and 6 [Hz], respectively. Figure 19 shows the displacement of the shaking table. Because the shaking table was driven by open-loop control, we can observe another slow vibration caused by the vibration system mounted on the shaker; however, the vibration was mainly a sine wave whose frequency was 7 [Hz]. Figure 20 shows the absolute displacement of the copper
mass. The main frequency of the output was not only different from that of the input but also slower than it. This vibration was generated by quasi-resonance of the parametric excitation, and the frequency was 1 [Hz] in this particular case. Moreover, the vibration amplitude was larger than that of the input. It is clear that the increase in the amplitude occurred without the primary resonance frequency.

Finally, quasi-resonance curve are shown in Fig. 21. In this figure, the horizontal axis shows the frequency of the input that is frequency of the shaking table, and the vertical axis shows the ratio of the maximum value of the output to the maximum value of the input. In this figure, we can see two peaks without the primary resonance frequency. Therefore we have confirmed the generation of quasi-resonance by experiment. In addition, figure 21 shows the simulation results numerically calculated using the experimental specification. By comparing the two curves, the characteristic of quasi-resonance was sufficiently confirmed.

![Quasi-resonance curve](image)

5. Conclusion

This paper proposed a new method to generate quasi-resonance by using the variable damper in the single degree-of-freedom base-excited vibration system. The coefficient of the variable damper was varied as a sine wave, i.e. the proposed control law of variable damping is an artificial parametric excitation whose frequency can be freely selected. Because the damping force generated by the variable damper was equivalent to the product of the damping coefficient and the relative velocity between the base and the mass, the changing of the damping coefficient at a suitable frequency could generate a new vibration that had the same frequency as the natural frequency of the vibration system. As a result, the vibration amplitude increased because of phenomena that were similar to resonance. We confirmed the increase in the vibration amplitude by simulation and experiment. A steady response analysis of the proposed damping parametric excitation is a topic for future study.

References