Vibration Analysis of a Railway Carbody Using a Shell Model*

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Abstract

This paper models a railway carbody as a non-circular cylindrical shell with simply-supported ends, where the shell does not have end plates or other equipment attached. Transfer matrix method (TMM) was applied to the analysis of three-dimensional elastic vibration problems on this carbody. A 1/12 size carbody model was made for experimental studies to verify the validity of the numerical simulation. This model has end plates and was placed on soft sponge at both ends to simulate the freely-supported condition. Modal analysis was applied to the experimental model, and natural frequencies and mode shapes of vibration were measured. Comparing the results by TMM and experiments, the natural frequencies and mode shapes of vibration for lower modes show good agreement in spite of the differences in boundary conditions. The effect of stiffening members installed on the experimental model was also investigated.

Key words: Elastic Vibration, Transfer Matrix Method, Cylindrical Shell, Modal Analysis

1. Introduction

Recent railway carbodies have become lightweight, shell-like structures. The panels of the carbody may deform elastically, for example roof and floor panels vibrate not only in-phase but also out-of-phase with each other⁽¹⁾. These elastic vibrations are usually observed around frequency regions that are sensed by humans. Therefore reducing the elastic vibration is necessary to improve riding comfort. It is well known that the mode shape of vibration of a carbody like the 1st mode of a free-free beam has a significant effect on passenger riding comfort. Accordingly, elastic vibration problems on railway carbodies have been treated as vibrations of beams^{(2), (3)}. However, to simulate the complex elastic vibration of railway carbodies, it is necessary to treat simulation models as three dimensional structures. Finite element (FE) methods are commonly used for this purpose⁽⁴⁾. But detailed FE model has difficulties in the modeling process and it needs long computational time. Therefore the FE model is not well suited for simulation of elastic vibrations of railway carbodies. To get around this, this paper proposes a simple shell model to simulate the elastic vibration. In this paper, railway carbodies are modeled as non-circular cylindrical shells. We applied the transfer matrix method (TMM) to a shell model with simply-supported ends^{(5), (6)}. The computation times of TMM are much shorter than with

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FEM due to the smaller size of the matrix of computation, where the matrix is 8-by-8.

Further, a 1/12 size carbody model for experimental studies was made to verify the validity of the numerical simulation. The length of the model is 1670 mm and it is made of stainless steel. The experimental results show good agreement with those of TMM. Characteristics of the deformation around the roof obtained from the experiments are also similar to those in TMM.

Additionally, the freely-supported carbody model with end plates was analyzed by using the finite element method (FEM), and FEM showed good agreement with the experimental results. Comparing the results by TMM and FEM, the natural frequencies and mode shapes of vibration for lower modes show good agreement with each other in spite of the differences in boundary conditions.

The experimental model uses connecting parts to connect the floor with the side panels and to maintain the form of the model. The connecting parts also work as stiffening members. Therefore it is necessary to investigate the effect of the connecting parts, and a TMM model was revised and numerical simulation was performed to investigate the effect of the stiffening members. These results allow an understanding of the effect of the stiffening members on the frequencies.

2. Analytical method

The model for the simulation is shown in Fig. 1. The carbody is modeled as a non-circular cylindrical shell. A cross-section of the shell is expressed as an arbitrary function $r=R_0f(\theta)$ and the shell is made of an isotropic material.

2.1 Equations of non-circular cylindrical shell

The cylindrical coordinates (x, s, z) are from the middle surface as shown in Fig. 1 and the displacement in each direction is defined as (u, v, w) . The Goldenveizer-Novozhilov equations of the shell are written as^{(7), (8)}

$$\begin{aligned}\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{sx}}{\partial s} + \rho H \omega^2 u &= 0, \\ \frac{\partial N_{ss}}{\partial s} + \frac{\partial N_{xs}}{\partial x} + \frac{Q_s}{R} + \rho H \omega^2 v &= 0, \\ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_s}{\partial s} - \frac{N_{ss}}{R} + \rho H \omega^2 w &= 0,\end{aligned}\quad (1)$$

where ρ is the density, R is the radius of curvature at the middle surface, and ω is the circular frequency in rad/s. The shearing forces are given by

$$Q_x = \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{sx}}{\partial s}, \quad Q_s = \frac{\partial M_{ss}}{\partial s} + \frac{\partial M_{xs}}{\partial x}, \quad (2)$$

and the Kelvin-Kirchhoff shearing force is

$$S_s = Q_s + \frac{\partial M_{sx}}{\partial x}. \quad (3)$$

The components of the membrane force are given by

$$\begin{aligned}N_{xx} &= E_p \left\{ \frac{\partial u}{\partial x} + \nu \left(\frac{\partial v}{\partial s} + \frac{w}{R} \right) \right\}, \quad N_{ss} = E_p \left(\frac{\partial v}{\partial s} + \frac{w}{R} + \nu \frac{\partial u}{\partial x} \right), \\ N_{xs} &= \frac{E_p(1-\nu)}{2} \left\{ \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x} + \frac{H^2}{6R} \frac{\partial}{\partial x} \left(\frac{v}{R} - \frac{\partial w}{\partial s} \right) \right\}, \quad N_{sx} = \frac{E_p(1-\nu)}{2} \left(\frac{\partial u}{\partial s} + \frac{\partial v}{\partial x} \right),\end{aligned}\quad (4)$$

and those of the moment are

$$\begin{aligned}M_{xx} &= D \left\{ -\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial}{\partial s} \left(\frac{v}{R} - \frac{\partial w}{\partial s} \right) \right\}, \quad M_{ss} = D \left\{ \frac{\partial}{\partial s} \left(\frac{v}{R} - \frac{\partial w}{\partial s} \right) - \nu \frac{\partial^2 w}{\partial x^2} \right\}, \\ M_{xs} &= M_{sx} = D(1-\nu) \frac{\partial}{\partial x} \left(\frac{v}{R} - \frac{\partial w}{\partial s} \right),\end{aligned}\quad (5)$$

Figure 1 Schematic view and cross-section of the carbody.

Figure 2 Angle at each joint of the cross-section.

Figure 3 A folded plate.

in terms of the displacements u , v , and w in the axial, circumferential, and transverse directions, respectively. The bending slope is defined as

$$\psi = \frac{v}{R} - \frac{\partial w}{\partial s}. \quad (6)$$

The quantities E_p and D are the extensional and flexural rigidities, respectively, expressed as $E_p = EH/(1-\nu^2)$ and $D = EH^3/\{12(1-\nu^2)\}$ in terms of Young's modulus E , Poisson's ratio ν , and the thickness H . For the simply-supported shell ($w = M_{xx} = N_{xx} = v = 0$), the following separation of variables was introduced by considering with Eqs. (1)–(6):

$$\begin{aligned} u &= H\bar{u} \cos(m\pi x/L), \quad (v, w) = H(\bar{v}, \bar{w}) \sin(m\pi x/L), \quad \psi = \frac{H}{R_0} \bar{\psi} \sin(m\pi x/L), \\ (N_{xx}, N_{ss}, Q_s, S_s) &= \frac{D}{R_0^2} (\bar{N}_{xx}, \bar{N}_{ss}, \bar{Q}_s, \bar{S}_s) \sin(m\pi x/L), \\ (N_{xs}, N_{sx}, Q_x) &= \frac{D}{R_0^2} (\bar{N}_{xs}, \bar{N}_{sx}, \bar{Q}_x) \cos(m\pi x/L), \\ (M_{xx}, M_{ss}) &= \frac{D}{R_0} (\bar{M}_{xx}, \bar{M}_{ss}) \sin(m\pi x/L), \\ (M_{xs}, M_{sx}) &= \frac{D}{R_0} (\bar{M}_{xs}, \bar{M}_{sx}) \cos(m\pi x/L), \end{aligned} \quad m=1, 2, \dots \quad (7)$$

where m is the axial half-wave number, and the quantities $\bar{u}, \bar{v}, \dots$ marked with an overscore are all dimensionless variables. The separation of variables indicates boundary conditions. The variables expressed by $\sin(m\pi x/L)$ become zero at both ends ($x=0, L$) of the shell. Accordingly, the terms $v, w, \psi, N_{xx}, N_{ss}, Q_s, S_s, M_{xx}$, and M_{ss} are equal to zero at both ends. As a result, the shell model is not constrained in the axial direction. For simplicity of analysis, the following dimensionless parameters are also introduced ($l, h)=(L, H)/R_0, c$

$=R/R_0$, $\lambda^2 = \rho H R_0^2 \omega^2 / E_p$. Here R_0 is the radius of a circle with the same circumferential length as the shell, expressed as

$$R_0 = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{r^2 + (dr/d\theta)^2} d\theta, \quad (8)$$

and λ is the dimensionless frequency parameter. Applying the separation of variables to the basic equations of the shell and eliminating the variables Q_s , Q_{ss} , N_{xx} , N_{xs} , M_{xx} , M_{xs} , and M_{sx} which are not differentiated with respect to s from Eqs. (1)–(6), the equations can be written as a matrix differential equation

$$\frac{d}{d\theta} \{ \mathbf{z}(\theta) \} = g(\theta) [\mathbf{U}(\theta)] \{ \mathbf{z}(\theta) \}, \quad (9)$$

$$[\mathbf{U}(\theta)] = \begin{bmatrix} 0 & U_{12} & 0 & 0 & 0 & 0 & 0 & U_{18} \\ U_{21} & 0 & U_{23} & 0 & 0 & 0 & 0 & U_{27} \\ 0 & U_{32} & 0 & U_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{43} & 0 & U_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & U_{54} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{63} & 0 & 0 & 0 & 0 & 0 \\ 0 & U_{72} & 0 & 0 & 0 & 0 & 0 & 0 \\ U_{81} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (10)$$

sym.

$$\begin{aligned} U_{12} &= -m\pi/l, U_{18} = -h/\{6(1-\nu)\}, U_{21} = \nu m\pi/l, \\ U_{23} &= -U_{32} = -1/c, U_{27} = h/12, U_{34} = -1, \\ U_{43} &= -\nu(m\pi/l)^2, \\ U_{45} &= 1/h, U_{54} = 2(1-\nu)h(m\pi/l)^2, \\ U_{63} &= -(1-\nu^2)h(m\pi/l)^4 + 12\lambda^2/h, U_{72} = -12\lambda^2/h, \\ U_{81} &= -12(1-\nu^2)(m\pi/l)^2/h + 12\lambda^2/h. \end{aligned} \quad (11)$$

Here $g(\theta)$ is the shape property function

$$g(\theta) = \sqrt{\{f(\theta)\}^2 + \{df(\theta)/d\theta\}^2}, \quad (12)$$

and $\{ \mathbf{z}(\theta) \}$ is the state vector

$$\{ \mathbf{z}(\theta) \} = \{ \bar{u}, \bar{v}, \bar{w}, \bar{\psi}, \bar{M}_{ss}, -\bar{S}_s, \bar{N}_{ss}, -\bar{N}_{sx} \}^T. \quad (13)$$

The state vector $\{ \mathbf{z}(\theta) \}$ can be expressed as $\{ \mathbf{z}(\theta) \} = [\mathbf{T}(\theta)] \{ \mathbf{z}(0) \}$ by using the transfer matrix $[\mathbf{T}(\theta)]$ of the shell. Substituting this relation into Eq. (9), the following equation is obtained

$$\frac{d}{d\theta} [\mathbf{T}(\theta)] = g(\theta) [\mathbf{U}(\theta)] [\mathbf{T}(\theta)]. \quad (14)$$

The matrix $[\mathbf{T}(\theta)]$ is obtained by integrating Eq. (14) numerically with the starting value $[\mathbf{T}(0)] = [\mathbf{I}]$ (identity matrix).

2.2 Joining condition (point matrix)

The cross-section of the model, as shown in Fig. 1, is used to represent the carbody model. The cross-section of the shell was divided into sections corresponding to floor, side, and roof panels. Figure 2 shows angle at each joint of the non-circular cylindrical model. The slope of the circumferential profile is discontinuous at the points b and d of the cross section. To connect properly at these points in the numerical simulation, a point matrix was introduced.

Next a folded plate, the shape as shown in Fig. 3, is considered. Two cranked plates are joined together at point $\theta = \theta_j$. The connection angle is α , and the notation of the displacement for each plate is (u^-, v^-, w^-) and (u^+, v^+, w^+) . The quantities marked with the superscripts $-$ and $+$ denote $\theta = \theta_j - 0$ and $\theta = \theta_j + 0$, respectively. At the joint, the following equilibrium relation must be satisfied:

$$\begin{aligned} & \{u^+, v^+, w^+, \psi^+, M_{ss}^+, -S_s^+, N_{ss}^+, -N_{sx}^+\}^T \\ &= [\mathbf{P}_j] \{u^-, v^-, w^-, \psi^-, M_{ss}^-, -S_s^-, N_{ss}^-, -N_{sx}^-\}^T, \end{aligned} \quad (15)$$

here $[\mathbf{P}_j]$ is a point matrix

$$[\mathbf{P}_j] = \begin{bmatrix} \mathbf{p} & \mathbf{0} \\ \mathbf{0} & \mathbf{p} \end{bmatrix}, \quad [\mathbf{p}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & -\cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (16)$$

The side panel is perpendicular to the floor panel at the point b in this model, i.e. $\alpha = \pi/2$. For this reason, the joining conditions at the point b satisfy $u^+ = u^-$, $v^+ = -w^-$, $w^+ = v^-$, $\psi^+ = \psi^-$, $M_{ss}^+ = M_{ss}^-$, $-S_s^+ = -N_{ss}^-$, $N_{ss}^+ = -S_s^-$, and $-N_{sx}^+ = -N_{sx}^-$. In this model, the circumferential profile is also discontinuous at the point d. The connection angle α is defined numerically because the angle depends on the shape properties of the shell model.

Accordingly, the state vector corresponding to each section as shown in Fig. 2 is expressed as

$$\begin{aligned} \{\mathbf{z}(\theta)\} &= [\mathbf{T}(\theta)] \{\mathbf{z}(0)\} & : 0 \leq \theta < \theta_b, \\ \{\mathbf{z}(\theta)\} &= [\mathbf{T}(\theta)] [\mathbf{P}_b] [\mathbf{T}(\theta_b)] \{\mathbf{z}(0)\} & : \theta_b \leq \theta < \theta_c, \\ \{\mathbf{z}(\theta)\} &= [\mathbf{T}(\theta)] [\mathbf{T}(\theta_c)] [\mathbf{P}_b] [\mathbf{T}(\theta_b)] \{\mathbf{z}(0)\} & : \theta_c \leq \theta < \theta_d, \\ \{\mathbf{z}(\theta)\} &= [\mathbf{T}(\theta)] [\mathbf{P}_d] [\mathbf{T}(\theta_c)] [\mathbf{P}_b] [\mathbf{T}(\theta_b)] \{\mathbf{z}(0)\} & : \theta_d \leq \theta \leq \pi. \end{aligned} \quad (17)$$

The slope of the circumferential profile is continuous at point c. Therefore the third expression in Eq. (17) is the same as the second expression.

2.3 Frequency equation

The shell has symmetrical properties, and the mode shapes of vibration can be classified into symmetrical and antisymmetrical types. Considering boundary conditions,

$$\bar{v} = \bar{\psi} = 0, \quad \bar{S}_s = \bar{N}_{sx} = 0 \quad : \text{for symmetrical type} \quad (18)$$

$$\bar{u} = \bar{w} = 0, \quad \bar{N}_{ss} = \bar{M}_{ss} = 0 \quad : \text{for antisymmetrical type} \quad (19)$$

gives frequency equations,

$$\begin{bmatrix} T_{21} & T_{23} & T_{25} & T_{27} \\ T_{41} & T_{43} & T_{45} & T_{47} \\ T_{61} & T_{63} & T_{65} & T_{67} \\ T_{81} & T_{83} & T_{85} & T_{87} \end{bmatrix}_{\pi} \begin{Bmatrix} \bar{u} \\ \bar{w} \\ M_{ss} \\ \bar{N}_{ss} \end{Bmatrix}_0 = \mathbf{0} \quad : \text{for the symmetrical type} \quad (20)$$

$$\begin{bmatrix} T_{12} & T_{14} & T_{16} & T_{18} \\ T_{32} & T_{34} & T_{36} & T_{38} \\ T_{52} & T_{54} & T_{56} & T_{58} \\ T_{72} & T_{74} & T_{76} & T_{78} \end{bmatrix}_{\pi} \begin{Bmatrix} \bar{v} \\ \bar{\psi} \\ -\bar{S}_s \\ -\bar{N}_{sx} \end{Bmatrix}_0 = \mathbf{0} \quad : \text{for the antisymmetrical type.} \quad (21)$$

Equations (20) and (21) include a frequency parameter λ in the elements of the transfer matrix. The natural frequencies of the non-circular cylindrical shell are determined by calculating the eigenvalues of Eqs. (20) and (21), and the circumferential mode shapes are determined by calculating eigenvectors corresponding to the eigenvalues.

3. Numerical results for a small model

Application of the TMM previously discussed is demonstrated by solving the vibration problem on the 1/12 size carbody model.

3.1 Cross sectional shape

The function $r = R_0 f(\theta)$ was used to express the cross-section of the model. We divided half of the cross-section into four parts as shown in Fig. 1 and 2. The function is expressed as

Figure 4 Cross-sectional mode shapes (Numerical simulation).

Figure 5 Three dimensional mode shapes (Numerical simulation).

Table 1 Shape properties of the model.

Width	w_i	246.0 mm
Height	h_e	209.5 mm
Radius (Roof)	R_r	400.0 mm
Radius (Connection)	R_t	5.0 mm
Length	L	1670.0 mm
Thickness	H	0.5 mm

Table 2 Material properties of the model.

Young's modulus	E	200 GPa
Poisson's ratio	ν	0.30
Density	ρ	7800 kg/m ³

$$\begin{aligned}
 f(\theta) &= h_e / 2 \cos \theta & \text{for } 0 \leq \theta < \theta_b, \\
 f(\theta) &= w_i / 2 \sin \theta & \text{for } \theta_b \leq \theta < \theta_c, \\
 f(\theta) &= x_{r1} \cos \theta + y_{r1} \sin \theta + \sqrt{(x_{r1} \cos \theta + y_{r1} \sin \theta)^2 - (x_{r1}^2 + y_{r1}^2 - R_t^2)} & \text{for } \theta_c \leq \theta < \theta_d, \\
 f(\theta) &= (R_r - h_e / 2) \cos \theta + \sqrt{(R_r - h_e / 2)^2 \cos^2 \theta - \{(R_r - h_e / 2)^2 - R_r^2\}} & \text{for } \theta_d \leq \theta \leq \pi,
 \end{aligned} \tag{22}$$

where $x_{r1} = (h_i - h_e / 2)$, $y_{r1} = -(R_t - w_i / 2)$.

3.2 Numerical results

Shape properties of the simulation model and the material properties are listed in Tables 1 and 2, respectively. It is sufficient to calculate the transfer matrix in half of the cross section because the model has structural symmetry. The calculation range between $\theta = 0$ to π was divided into 200 sections for the numerical integration. Then the transfer matrix $[T(\theta)]$ was calculated and using Eqs. (20) and (21), it was possible to obtain natural frequencies and mode shapes of vibration. With these procedures, three dimensional vibration shapes that cannot be obtained by using the beam model can be obtained.

Mode shapes of the cross-section are shown in Fig. 4. In the figure, the solid blue line indicates the mode shape of the model and the dotted line indicates the original shape. Mode shapes are normalized by the maximum value of displacement. Figure 5 shows the

Figure 6 Model for the experiments.

Figure 7 Shape properties of the aluminum angle joint (cross-section).

three dimensional mode shapes of the lower three natural frequencies obtained by TMM. These three modes correspond to the symmetrical type. In this model, natural frequencies of the antisymmetrical type were calculated as higher than 45 Hz.

The analytical model comprises the three main parts; the floor, sides and roof. Only the roof part has a curvature in this model. The curvature has the effect to stiffen the part, and the deformation of the floor and side panels are larger than that of the roof.

4. Experiment

We made a 1/12 size carbody model for experimental studies to verify the validity of the numerical simulation. Figure 6 shows a photo of the experimental model. The modal analysis was applied to the experimental model, and natural frequencies and mode shapes of vibration were measured.

4.1 Experimental set-up

The detailed specifications of the experimental model are the same as those of the numerical simulation in Table 1. The experimental model is made of 0.5 mm thick SUS304 stainless steel and it is 1670 mm long. The model is composed of two main parts: One is a roof-side unit made by press forming of a stainless steel plate, and the other is a floor plate. These two parts are joined with angle joints of aluminum. The shape properties of the angle joint shown in Fig. 7 are 20 mm × 20 mm × 2 mm (thickness) and its length is 1620 mm. The angle joints have screw holes at 100 mm intervals. These joints are screwed on to the floor plate and roof-side unit by M3 screws. The model has end plates and for the experiments is placed on soft sponge at both ends to simulate free support.

Modal analysis was applied to the experimental model, and the impact excitation test was carried out with hammer. An impact hammer (type 086M36, PCB Piezotronics Inc., USA) with a soft rubber tip was used to impact the model. The model was manually excited at 21 input points distributed on the floor, side and roof panels as shown in Fig. 8. An accelerometer (type 352C22, PCB Piezotronics Inc., USA) was used to measure the response of the impacts. The measurements were carried out on the floor at F6 as shown in Fig. 8 (inside of the model). An average of ten measurements for each input point was used. A multi-channel data station with a FFT analyzer function (type DS-2104, Ono Sokki Co., Ltd, Japan) was used for the data acquisition.

Figure 8 Excitation points of the model.

Figure 9 Mode shapes (Modal analysis).

4.2 Experimental result

Figure 9 shows the mode shapes of the experimental model. Crosses (+) indicate the excitation points on the model, and lines correspond to the center line of each panel: the floor, sides, and roof, respectively. The experimental model has a tendency to vibrate with higher frequencies than those of the numerical simulation. The natural frequencies of the experiment and the numerical simulation have slight differences in the 1st mode, and the differences of the natural frequencies grow larger in the 2nd and 3rd modes. These phenomena may be caused by the joint parts installed on the experimental model. The joint parts play the role of stiffening members, and the rigidity of the experimental model is higher than that of the numerical simulation. For this reason, the experimental model tends to vibrate at higher frequencies than those calculated by the numerical simulation. The mode shapes obtained from the experiment show similar characteristics to those of the numerical simulation, shown in Fig. 5. The deformation of the roof is smaller than that of the side and floor parts because the roof part has a curvature: $R_r = 400.0$ mm. Further, the roof part tends to vibrate in a different shape. In the first mode, floor and roof vibrate in opposite directions (out-of-phase), and the axial wave number of the roof, floor, and sides was one. In the second and third modes, the wave number of the floor and sides was the same as that of mode number 2 and 3, respectively. However the wave number of the roof was different from that of the other parts.

5. Finite element simulations

We also analyzed the freely-supported carbody model with end plates by using the finite element method (FEM). The conditions are similar to actual railway carbodies, and commercial FEM software ANSYS 11.0 was used to simulate the vibration of the model. The shape properties are the same as those of the numerical simulation in Table 1, and the material properties are the same as those of the numerical simulation in Table 2. In the finite

Figure 10 Mode shapes (ANSYS).

element model, the plates at both ends are modeled with the same thickness as the other panels. To simulate the same conditions of the experimental model, the boundary condition at both ends is set to free. A SHELL63 element type and a length of mesh of 0.05 m was adopted.

Figure 10 shows the elastic mode shapes of vibration obtained from the finite element simulation. In the first mode, the floor and the roof vibrate out-of-phase. The floor and the side parts tend to vibrate with larger deformations than the roof. In the process of the formulation of the TMM, a simply-supported condition is necessary to separate the variables. The freely-supported carbody model with end plates was adopted in the numerical simulation by using the FEM and the experiment. The characteristics observed from the finite element simulation are similar to those of the TMM model despite the differences in boundary conditions. Comparing the natural frequencies obtained by TMM (Fig.5) and ANSYS (Fig.10) there is good agreement in spite of the differences in boundary conditions. The carbody model has shape properties that are long in the axial direction. For this reason, the boundary condition at the two ends do not influence the mode shapes and frequencies greatly. It may be concluded that the use of TMM to simulate this type of shell structure is acceptable.

6. Effect of longitudinal length of the model

The mode shapes and natural frequencies obtained from TMM correspond closely to those of FEM in spite of the difference in boundary conditions as mentioned in the previous section. The carbody model is long along the longitudinal coordinate, and the radius R_0 calculated by Eq. (8) is smaller than the length of the shell. As a result, it is possible that the influence of the boundary condition at both ends is small, and that it may be neglected in this model. The effect coming from the length of the shell model was investigated next. The length of the shell model, L_0 , is 1670 mm, and the radius of a non-circular cylindrical shell R_0 with the same circumference as the cylindrical shell is equal to 139.15 mm. Both TMM and commercial FEM software ANSYS 11.0 were applied to obtain the natural frequencies. The end plates installed on the experimental model are not considered in the TMM simulation model. The simulation model is simply-supported at both ends while the ANSYS model has end plates at both ends. The end plates are modeled with the same properties as the shell part. The thickness of the end plates is also the same as the shell part. In the ANSYS simulation, the model is freely-supported at both ends. The differences in the dimensionless natural frequencies were calculated. Natural frequencies were calculated for length parameters between $0.25L_0$ and $2.0L_0$. In the calculation of the differences, $L=L_0$ (=1670 mm) was set to the original length of the shell. The differences are calculated by

$$\Delta = (f_a - f_i) / f_b \quad (23)$$

where f_a and f_i are the natural frequencies obtained by ANSYS and the natural frequencies obtained by TMM, respectively, f_b is the natural frequencies obtained by the original model of TMM ($L=L_0$).

The differences in the dimensionless natural frequency of the TMM model and ANSYS

Figure 11 Differences in the dimensionless frequency ($L_0=1670$ mm).

model is shown in Fig.11. The curves correspond to the differences of the lower three modes with axial half-wave numbers $m=1, 2$, and 3 (symmetrical type vibration). When the length of the shell becomes longer, the differences in the dimensionless frequencies become smaller. Accordingly, when the shell is longer than the radius R_0 , the influence of the boundary condition at both ends becomes smaller. Under such conditions, it is acceptable to choose this type of simply-supported simulation model.

In the vibration problem of circular cylindrical shell, many researchers have already investigated about the effect of boundary conditions, length of the shell, and others. Axial constraint has predominant effects on the natural frequency as the cylinder becomes relatively long^{(9), (10)}. By contrast, the displacement u along the edges at both ends of the shell model in these TMM simulation, FEM simulation, and experiment are not constrained in the axial direction. Therefore it is not appropriate to compare with the results previously obtained from these papers.

7. Effect of stiffening members

Aluminum angle joints to connect floor panel with the roof-side units were installed on the experimental model as shown in Fig.12(A). These joints function as stiffening members, and with the joints, the rigidity of the experimental model is higher than that in the numerical simulation discussed in §3. Considering the simplicity of the modeling process with TMM, the TMM model was revised to analyze the effect of the stiffening member. The cross-section of the revised TMM model is shown in Fig.12(B). In the modeling, the middle plane of the stiffening member with the stainless steel plate is set to the middle plane in the previous TMM model. The joining condition discussed in §2 is used to model the stiffening members.

7.1 Equivalent material properties

Equivalent material properties are necessary to simulate the effect of the stiffening members. Steel plate and aluminum angle joint are coupled at the joint. Considering the coupled structure, it is possible to calculate equivalent properties as for a combined plate made of the different materials. The structural properties are shown in Fig. 13. In this figure, h_{st} , h_{al} , and h are the thickness of the steel part, aluminum part, and the total thickness, respectively, and variable b corresponds to the width. The equivalent Young's modulus is calculated by

$$E_{eq} = (E_{st}I_{st} + E_{al}I_{al}) / I_{eq} \quad (24)$$

where I_{st} and I_{al} are the geometric moments of inertia of the steel part and the aluminum part

$$I_{st} = \{(h_{st} - h/2)^3 + (h/2)^3\} b/3, \quad (25)$$

$$I_{al} = \{(h/2)^3 - (h_{st} - h/2)^3\} b/3, \quad (26)$$

Figure 12 Model of the angle joint (cross-sectional shape).

Figure 13 Cross-section of the combined structure.

Table 3 Natural frequencies. [Hz]

No.	TMM	ANSYS	TMM (Angle joint)	Experiment
1	25.7	26.8	26.7	25.2
2	29.0	30.5	31.6	31.4
3	31.5	31.7	35.0	36.1
4	35.2	34.8	39.3	40.7

respectively. And equivalent geometric moment of inertia I_{eq} is

$$I_{eq} = bh^3 / 12 . \quad (27)$$

Here, E_{st} and E_{al} correspond to Young's modulus for each material. Substituting these relations into Eq. (24), the following equivalent Young's modulus is obtained

$$E_{eq} = \frac{4}{h^3} \left[E_{al} \left\{ \left(\frac{h}{2} \right)^3 - \left(h_{st} - \frac{h}{2} \right)^3 \right\} + E_{st} \left\{ \left(h_{st} - \frac{h}{2} \right)^3 + \left(\frac{h}{2} \right)^3 \right\} \right] . \quad (28)$$

The equivalent density ρ_{eq} is calculated by

$$\rho_{eq} = (\rho_{st} h_{st} + \rho_{al} h_{al}) / h \quad (29)$$

where ρ_{st} and ρ_{al} are the densities of the steel and aluminum parts. The equivalent Poisson's ratio ν_{eq} is calculated by

$$\nu_{eq} = (\nu_{st} h_{st} + \nu_{al} h_{al}) / h \quad (30)$$

where ν_{st} and ν_{al} are the Poisson's ratios corresponding to the steel and aluminum parts. These equivalent material properties are not related to the width b .

7.2 Numerical results

Considering the material properties of the shell model listed in Table 2, values of E_{st} =200 GPa, E_{al} =70 GPa, ρ_{st} =7800 kg/m³, ρ_{al} =2700 kg/m³, ν_{st} =0.30 and ν_{al} =0.34 were used. Additionally, considering the thickness of the shell listed in Table 1 and the shape properties of the angle joint shown in Fig. 7, h_{st} =0.5 mm and h_{al} =2.0 mm values were employed.

Table 3 shows the natural frequencies obtained from the TMM, ANSYS 11.0, the stiffened model using TMM (Angle joint), and the experiments. The lower four modes are

listed in this table. The natural frequencies obtained by the stiffened model are higher than those of the original TMM. As mentioned in §4, the experimental model has a tendency to vibrate with higher frequencies than that of TMM. Comparing the results with the stiffened model and the experiments show that they are in good agreement and it may be concluded that the differences in the natural frequencies are caused by the joints.

8. Conclusions

This paper applied the transfer matrix method (TMM) to the analysis of three-dimensional elastic vibration problems on a railway carbody. In this method, the carbody is modeled as a non-circular cylindrical shell with simply-supported ends. Natural frequencies and mode shapes of vibration are obtained by the method. To verify the validity of the numerical simulation, a 1/12 size carbody model for experimental studies was made. Applying the modal analysis to the experimental model, the natural frequencies and mode shapes of vibration for the lower modes show good agreement. The characteristics of the deformation around the roof obtained by the experiment are also similar to those obtained by TMM. A freely-supported carbody model with end plates by using the finite element method (FEM) was also analyzed. Comparing the results by TMM and FEM, the natural frequencies and mode shapes of vibration for the lower modes show good agreement in spite of the differences in boundary conditions. This is because the shell model is long in the axial direction. And the influence of the boundary conditions at both ends becomes small. This validates the choice of the simply-supported model for simulation. Further, numerical simulation was performed to investigate the effects of angle joints working as stiffening members. The natural frequencies obtained by the stiffened model show good agreement with those of the experiment using the modal analysis.

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