Observations of Nonlinear Phenomena in Rotordynamics*

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Abstract
Observations, analysis and understanding of nonlinear rotordynamic phenomena observed in aircraft gas turbine engines and other high-speed rotating machinery over the course of the author’s career are described. Included are observations of sum-and-difference frequency response; effects of roller bearing clearance; relaxation oscillations; subharmonic response; chaotic response; and other generic nonlinear responses such as superharmonic and ultra-subharmonic response.

Key words: Nonlinear, Rotordynamics, Asynchronous, Relaxation Oscillation, Chaotic, Subharmonic, Superharmonic

1. Introduction
My active, full-time career from June 1951 to December 1993 was focused on design and development of aircraft gas turbines engines – first with the Westinghouse Aircraft Gas Turbine Division in Philadelphia, PA, USA, in Derby, UK (in partnership with Rolls Royce Ltd.) and Kansas, MO, USA and from 1957 to 1993 with GE Aircraft Engines in Lynn, MA, USA. In my semi-retirement from 1994 to the present, I have been involved at the Gas Turbine Laboratory at MIT with development of micro-gas-turbines – high-speed, high-power-density turbomachinery in millimeter/micron dimensional scale. These associations have had a profound influence on the nature of my involvement in the study and understanding of nonlinear rotordynamic phenomena. My experience was rich in opportunities to observe unusual and occasionally unprecedented rotordynamic behavior in complex high-speed rotating machinery, but involved little opportunity to perform carefully designed and instrumented experiments on idealized models. It inspired me to focus my efforts on problems of practical significance and to orient my findings more toward qualitative understanding in terms of simplified models of the phenomena that might be applied generally to the correction of identified problems and to the adoption of preventative design approaches – rather than to refined analytic formulations that would replicate the observed phenomena in enormous complex systems with exquisite precision.

Since the starting point of most of my investigations has been with observations of unusual phenomena, I have chosen to define nonlinear phenomena in a phenomenological context and more expansive way than its more common association with systems whose mathematical models involve sets of nonlinear differential equations. For the purposes of this survey, I have defined nonlinear phenomena as systems’ dynamic responses which are asynchronous in frequency with their rotors’ rotational speed and/or which have non-steady amplitude of response at constant rotational speed or involve responses which are discontinuous in amplitude with changes in rotational speed – that is, jump phenomena.

Early in my career, as my interest grew in rotordynamics, my interest centered on rotordynamic instabilities. The then-current view of rotordynamic instability was focused primarily on fluid film bearings. But large aircraft gas turbine engines used rolling element
bearings exclusively. I was therefore surprised to find occasional incidents of instability that required understanding and corrective action. Indeed, there were actually four different mechanisms that attracted my attention:

Hysteretic whirl — Where destabilizing forces are generated by rubbing action in the interface between rotor parts and sub-assemblies that are not sufficiently clamped to prevent relative motion;

Trapped liquids in the rotor — Where destabilizing forces are generated by fluids which are inadvertently trapped in cylindrical cavities within the rotor;

Dry friction whip — Where the Coulomb friction generates destabilizing forces at the contact zone of rotor/stator rubs in close-clearance seals or turbomachinery blade tips;

The tip clearance effect in turbomachinery — Where destabilizing forces are generated in radially-deflected turbomachinery rotors by the effect of local clearance’s change on the efficiency of the local turbomachinery’s blading.

The next phase of my career in rotor dynamics developed around incidents involving a broad range of nonlinear phenomena in aircraft gas turbine engines and other high-speed rotating turbomachinery which required understanding, explanation and correction. These included observations of sum-and-difference frequency response; effects of roller bearing clearance; relaxation oscillations; subharmonic response; chaotic response; and other generic nonlinear responses such as superharmonic and ultra-subharmonic response.

2. Sum and Difference Frequencies Noted in Rotordynamic Response

My first exposure to a distinctly nonlinear phenomenon was being confronted with data indicative of unexplained and puzzling evidence of “sum and difference frequencies”. When operating a system that had two distinct frequency signatures (say from the synchronous unbalance-excited frequency of each of two concentric rotors), one would expect the linear combination of the two harmonic waves to result in a so-called “beating” waveform who’s spectrum included only the two fundamental generating frequencies. But in some cases, as shown typically in Fig. 1a, the waveform was somewhat distorted, and the spectrum included components, one of whose frequencies was the sum of the fundamental frequencies and another of whose frequencies was the difference of the fundamental frequencies. I was successful in demonstrating the most likely origin of these “sum and difference frequencies” by postulating that the original “beating” wave form had been
truncated by some mechanical amplitude limit in the path of the signal’s generation or transmission, or some amplitude saturation in the data acquisition or data reduction system, as shown in Fig. 1b(6). Although the manifestation was benign, identifying its origin enabled the diagnosis of issues connected with the signal truncation.

3. Effects of Roller Bearings with Clearance

In observation of vibration response of operating engines, I frequently observed hysteresis in the signal – that is, a marked difference between the amplitude on accelerating through a critical speed and on decelerating through the same speed range, as shown in Fig. 2. Although the phenomenon was well identified in the technical literature at the time as associated with a rightward leaning critical peak resulting from a support stiffness which hardened with increasing amplitude of rotor deflection, a colleague and I undertook to study the situation in greater detail in the specific context of roller bearing clearance(7).

We modeled the system analytically as a Jeffcott rotor with tri-linear stiffness in the bearings, and studied it over a range of the normalized variables of the ratio of bearing clearance to unbalance; the ratio of rotor natural frequency and the stator natural frequency taken as independent systems; and system damping ratio. One unexpected finding is shown in Fig. 3a – a large whirling amplitude of the stator at high supercritical speeds where it is orbiting the rotor like a hula-hoop. We labeled it “stator whirl”, but it was never observed in practice nor could we find any reference to it in the technical literature, probably because the condition could not be arrived at in any straightforward way. It required the rotor, operating at high speed and out-of-contact with the stator, to be artificially displaced to contact the stator.

To expand our understanding, we modeled the system again on a newly-available analogue computer with results (shown in Fig. 3b) which, for the most part, resembled the

Figure 2. Observation of hysteresis in engine vibration response through a critical speed.

Figure 3. Analytic and analogue computer models of hysteresis in vibration amplitude response due to bearing clearance(7).
analytic results. An important exception was the presence of an asynchronous response in the transition zone between “stator whirl” and conventional supercritical operation. The finding had no immediate practical significance but was important in suggesting to us the possibility of using analogue computation for other investigations of nonlinear rotordynamic phenomena at a time (the mid-1960s) when high-speed, large-scale digital computation was not yet available to us.

4. Relaxation Oscillations in Rotordynamics

I was quite surprised on an occasion in the early 1960s to be confronted with a trace of engine vibration where the amplitude varied as an approximate square wave with a period of about 5 seconds (~1,000 rotations) over the course of time as shown in Fig. 4.

I immediately recognized the amplitude waveform as resembling relaxation oscillations which are generally illustrated in textbooks by the van der Pol equation. The van der Pol equation sets up system damping which varies as a function of amplitude and involves an unstable (negative damping) zone at low deflections and stable zones (with positive damping) at large deflections in either direction from a nominal rest position. Although I had evidence of similar behavior, I could not identify any physical rationale for assigning the van der Pol type of damping variation to our rotordynamic system. I had to hypothesize a much more convoluted rationale. I first established that, because of the effect of bearing clearance in imposing rightward leaning critical peak, the rotor had two stable states at the speed at which the bi-stable amplitude variation was noted (with no intermediate stable state). As noted in Fig. 3, the two states could be observed, one on an engine acceleration and the other on an engine deceleration. I then deduced a mechanism for the change from one state to the other in the course of operation by referring to changes in the bearing clearance induced by thermal changes in the bearing clearance associated with the temperature of the rollers – when the clearance was large, the rollers would skid and generate heat and experience an increase in temperature and diameter over the course of time (say, 2.5 seconds) tending to close the clearance until full contact was made; when the rollers were in full contact with the races, the rollers would be cooled by oil spray and their temperature and diameters would decrease over the course of time (say, another 2.5 seconds) tending to open the clearance until contact was lost. I was able to effectively demonstrate this hypothesis by systematically varying the period of the relaxation oscillation observed on engine test by varying the oil temperature and ultimately by eliminating the relaxation oscillation by modifying the roller bearing to effectively eliminate the out-of-contact operation.

5. Subharmonic Response in Rotordynamics

Having been encouraged by my ability to simulate nonlinear rotordynamic behavior of a rotor centered in a bearing clearance using the analogue computer, I undertook an analogue computer study of a rotor that was located eccentric to the stator centerline. The computed response is shown in Fig. 5a as the waveform recorded during a slow acceleration from a low subcritical speed to a high speed more than double the critical speed. I was able to identify the second peak as being the 2nd order subharmonic pseudocritical response where the rotor, operating at a speed approximately twice the critical
speed, was contacting the stator and bouncing at its natural frequency, energized by the motion of the unbalance on every other rotation of the rotor.

I was quite gratified when, shortly after this discovery on the analogue computer model, I was given data from a vibration episode on an aircraft gas turbine engine which had the distinct attributes of the same phenomenon – 2nd order subharmonic pseudo-critical response as shown in Fig. 6. To confirm my understanding, I requested the waveforms be recorded at pertinent speeds which enabled me to make the desired comparisons shown in Fig. 5b(9).

Several years thereafter, an asynchronous vibration response on an aircraft gas turbine engine was conclusively identified as actually episodes of 8th and 9th order subharmonic response(10) as shown in Fig. 7. Not fully appreciated at the time was the fact that the dominant frequency tracked the rising natural frequency of the system in the transition zone between the two subharmonic episodes. I subsequently appreciated that this anomaly was an indication of chaotic response in the transition zone and that the response, though chaotic, did have a dominant response frequency equal to the natural frequency of the system.
6. Chaotic Response in Rotordynamics

Sometime in the mid-1980s, I was presented with a photograph (Fig. 8) of the orbit of a vibrating aircraft gas turbine engine rotor that resembled nothing I had ever encountered before – what appeared to be a completely disordered trajectory. Not being able to offer an explanation or understanding of the data, I stashed it away in the bottom drawer of my desk and promised myself to retrieve it whenever I came across a possible explanation of this unprecedented behavior.

An opportunity to do that came in 1988 when I received as a gift a copy of James Gleick’s book, *Chaos; Making a New Science*. The book was not a technical text, but rather was an excellent graphic description of the new findings and understandings in the developing field of chaos. I very quickly recognized its relevance to that previously unexplained picture of a rotor trajectory and to other anomalies I had noted in previous experimental and analytic data – the unexpected appearance of an asynchronous dominant response at system natural frequency in the inter-order subharmonic responses (Fig. 7); and problems I had encountered in trying to obtain a closed trajectory in the analytic bilinear stiffness model that I used to study nonlinear rotordynamic phenomena.

Figure 8. Early observation of chaotic rotordynamic response.

Figure 9. Early analytic simulation of chaotic response in a high speed rotor.
With this new appreciation of chaotic phenomena I renewed my analytic studies \(^{(12)}\) of inter-order subharmonic responses using the bilinear stiffness model, and was able to evoke evidence in graphic form of chaotic waveforms (Fig. 9a); chaotic Poincaré iterates (Fig. 9b); chaotic rotor orbits (Fig. 9c); and bifurcations, quasi-periodic response, and chaotic response zones in the response curve of the system (Fig. 9d). My only disappointment was that the rotor orbits (Fig. 9c) were chaotic in one dimension only, whereas the picture of the rotor orbit observed in the aircraft gas turbine engine (Fig. 8) was clearly chaotic in two dimensions. This was inevitably due to the fact that, in process of simplification of the analytic model, I had made assumptions to simplify the analytic model so that there was a separation of the dependent variables and the system of equations was nonlinear in only one dimension. But I had established a firm foundation for the role of chaotic response in nonlinear rotodynamics.

7. Other Generic Nonlinear Rotodynamic Responses Identified in the Bilinear Stiffness Model

**Superharmonic response** – The literature of nonlinear dynamics was full of references to superharmonic response where a system being excited at a frequency approximately \(1/N\) times the system’s natural frequency will respond with a pseudo-critical peak amplitude at an asynchronous frequency precisely \(N\) times the exciting frequency. But I expected that I would rarely see superharmonic response in the operation of high-speed rotating machinery because the exciting force due to unbalance is only \((1/N)^2\) of the exciting force at critical speed (although the damping force is only \((1/N)\) of the damping force at critical speed). Moreover, the induced bouncing action is energized on every \(N\)th bounce rather than on every bounce, as is the case for the synchronous critical speed response. Nevertheless, a rare sighting on an aircraft gas turbine engine was indeed experienced in 1992\(^{(13)}\). Figure 10 shows a comparison of a spectrum data and a waterfall chart of a slow acceleration for both the engine response and an analytic model of the phenomenon. It also displays the then-

![Figure 10. Comparison of 2nd order superharmonic response noted on an aircraft gas turbine engine with an analytic simulation of 2nd order superharmonic response\(^{(13)}\).]
newly developed capability for generation of waterfall charts from the analytically generated response.

Ultra-subharmonic response – In the mid 1990s, I had the good fortune to collaborate with a colleague from the French aircraft engine company SNECMA on the analysis and understanding of an observation he had made of ultra-subharmonic responses in an aircraft gas turbine engine vibration response. In the prior text of this essay, I have dealt with asynchronous excitation of systems’ critical frequency when the rotational speed was approximately $N$ times the critical speed with the response frequency $(1/N)$ times the rotational frequency (that is, subharmonic response), and when the rotational speed was approximately $(1/N)$ times the critical speed with the response frequency $(N)$ times the rotational frequency (that is, superharmonic response). In the case of ultra-subharmonic response, we are dealing with asynchronous excitation of systems’ critical frequency when the rotational speed is approximately $(J/K)$ times the critical speed with the response frequency $(K/J)$ times the rotational frequency. The asynchronism results from the rotor bouncing with cyclic regularity $K$ times for every $J$ rotations resulting in a pseudo-critical peak in response amplitude. In particular, the data indicated that I was dealing with a cluster of ultra-subharmonic response peaks around the 2nd order subharmonic response where the individual responses occurred at rotational speeds (relative to the critical speed) approximately equal to:

$$J/K = 1/1, 3/2, 5/3, 7/4, 9/5, 11/6, 13/7, \ldots, 2/1, \ldots, 15/7, 13/6, 11/5, 9/4, 7/3, 5/2, 3/1$$

The response frequencies (relative to the rotational frequency) at each point were then equal to:

$$K/J = 1/1, 2/3, 3/5, 4/7, 5/9, 6/11, 7/13, \ldots, 1/2, \ldots, 7/15, 6/13, 5/11, 4/9, 3/7, 2/5, 1/3$$

The engine data, shown in Fig. 11, illustrates the 2nd order subharmonic center-point (response frequency at 1/2 times the rotational frequency when operating at 2 time the critical speed) along with ultra-subharmonic responses at $J/K = 3/2$ of the critical speed and at $9/4$ of the critical speed – all of which responses are at an asynchronous frequency approximately equal to the critical frequency. Further study indicated that such clusters were possible around every subharmonic order as well as at speeds halfway between every subharmonic order.

![Figure 11. Comparison of ultra-subharmonic response noted on an engine with an analytic simulation of ultra-subharmonic response](image)
A new class of asynchronous response frequencies – More recently, I and a colleague\(^{(15)}\) were involved in the early stages of development of high-speed gas bearings for high power density micro-devices. A micro-rig with a rotor 4.2 mm in diameter and 450 µm thick was designed to enable the development of such a bearing. Since it was anticipated that the micro-device would not be amenable to the instrumentation necessary for a detailed development, a preliminary program was undertaken to build and test a large-scale 26x “macro-rig” for that purpose. The early development was conducted by D.J. Orr\(^{(16)}\). Much of the early operation was conducted with considerable side-load to stabilize the bearing. When stimulated by the inevitable residual unbalance in the rotor, the eccentricity of the rotor resulted in generation of a complex spectrum of nonlinear responses. The presence of superharmonic, synchronous critical and subharmonic responses at critical frequency were duly noted, but some more subtle asynchronous responses at lower frequencies in both the sub-and supercritical regimes, illustrated in Figs. 12a and 12b, were not fully addressed at the time.

More recently, I noted a similar pattern of rotordynamic response in the course of subcritical operation of an experimental turbomachinery component which was experiencing a local rub between the rotor and stator of an interstage seal. Excerpts of experimental waterfall diagrams recorded during accelerations of the rotor are shown in Fig. 12c assembled in appropriate positions in a large-scale Campbell diagram. The resemblance between the observations in the two vehicles suggested that a systematic general mechanism was at work, and inspired application of an analytic model to gain insight into the nature of its origin\(^{(17)}\).

Study of the data from the two vehicles and exploration of spectra data from the analytic models suggested that the systematic response of the natural frequency over the subcritical, transcritical, and supercritical speed regimes was associated with the subharmonic, superharmonic, and ultra-subharmonic responses which had already been identified. A more intensive exploration of the individual spectra of those responses indicated that, in addition to generating a strong frequency component at the natural frequency, at least two other frequency components were generated at frequencies below the natural frequency. These lower frequencies, when arrayed on a Campbell diagram over the entire speed range, generated the cross-like patterns which had been observed. A typical set of the observed frequency points is shown on Fig. 12c, superimposed on the experimental data.

Figure 12. Sightings of a new class of nonlinear asynchronous response in a micro-rotor and a macro-scale turbomachinery component at both subcritical and supercritical speeds\(^{(17)}\).
8. Concluding Remarks

Even with the progress I have made in identifying, understanding, and resolving many troublesome nonlinear rotordynamic vibration problems, I am also very proud of an unpublished finding which I happened on in the course of my other studies. I refer to it as the “The happy face of chaos” – an unusual Poincaré section derived in the course of solving the equations of motion of a bilinear stiffness model of an eccentrically placed rotor with bearing clearance. I am pleased to present it here in Fig. 13.

In concluding, I would like to express my gratitude to Prof. Yukio Ishida of Nagoya University for the invitation to make this presentation in this distinguished journal.

References


