Design for the Maximum Natural Frequency of Laminated Composite Plates by Optimally Distributed Short Fibers*

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Abstract
A new design method is proposed for vibration of functional fibrous composite plates, which imitate micro structures of natural materials such as bones and shells. The material has local anisotropy induced by optimally distributed short fibers which are defined in each element of the finite element method. To design such locally anisotropic plates, the present optimum design method combines a genetic algorithm method (GA) with a layerwise optimization (LO) concept. The LO concept reduces a multi-layer optimization into iterations of a single-layer optimization, and the GA is used for the single-layer optimization to determine fiber orientation angles in each element. The fundamental frequency of the plates is chosen as the objective function to be maximized. It was revealed that the present plates give higher fundamental frequencies than conventional plates reinforced by parallel straight fibers. Further optimally distributed short fibers indicated specific orientations even though no constraint was imposed on those directions.

Key words: Local Anisotropy, Composite Material, Genetic Algorithm, Finite Element Method, Optimum Design, Free Vibration, Layerwise Optimization

1. Introduction

Natural materials often have local anisotropy. For example, the internal structure of bone is a spongy structure composed of numerous voids, which enable to distribute anisotropy in the area where it is needed, and this allows bones to have both light weight and sufficient strength. If local anisotropy can be exploited in structural design, industrial objects with less weight but maintaining enough strength will be obtained and be used to lighten structures in such areas as automobiles, aircrafts and marine vessels. For the purpose of enhancing the composite material properties, the present study employs a concept of locally anisotropic structure to develop high performance composite plates by distributing short fibers optimally.

Conventional laminated fibrous composite plates are typically fabricated by stacking orthotropic layers, each of which is composed of reinforcing parallel straight fibers and matrix materials. Structural designers can make use of fiber orientation angles in layers to improve the overall mechanical properties, and tailoring of such composite plates to specific situations has been archived by varying the orientations of parallel fibers or thickness of the layers. Gürald and his co-workers*(1) used a genetic algorithm (GA) to design stacking...
sequences of laminated plates by directly assigning fiber orientation angles to the design variables. Using a gradient method, Fukunaga and his co-workers\textsuperscript{(2)} optimized lamination parameters which describe plate stiffnesses in a simple form, and determined the angles by geometrical features in feasible regions of lamination parameters. Todoroki and his co-workers\textsuperscript{(3)} employed a genetic algorithm (GA) and a "fractal branch and bound method" to design lamination parameters. Autio\textsuperscript{(4)} determined corresponding lay-ups to the optimum lamination parameters using a GA. Narita and Turvey\textsuperscript{(5)} proposed a layerwise optimization (LO) method which determines fiber orientation angles directly from the outermost layer to the innermost layer sequentially based on simple physical observations. The present authors\textsuperscript{(6), (7)} also proposed an effective method to derive the angles from lamination parameters.

Due to recent developments in tow-placement technologies, it becomes possible to fabricate composite materials with fiber orientations that vary continuously\textsuperscript{(8)}. This allows reinforcement of the composite plate along curvilinear fibers and distribution of the anisotropic properties flexibly in the plate. Further, the design space in optimization problem for fibrous composites can be dramatically expanded, when it is compared with conventional design problems on stacking sequence, and hence it is possible to improve the composite properties significantly.

Leissa and Martin\textsuperscript{(9)} presented a variable stiffness concept to improve the vibration and buckling performance of plates by using the Ritz method. Hyer and Lee\textsuperscript{(10)} used the finite element method (FEM) to analyze the buckling performance of variable stiffness plates with curvilinear fibers. They varied the fiber orientation angles from one element to another, and it was shown that such plates had higher failure loads than plates with parallel fibers. Gürdal and Olmendo\textsuperscript{(11)} proposed an analysis method for the in-plane response of variable stiffness plates by using a system of coupled elliptic partial differential equations, and successfully confirmed improvement in the buckling performance of the plates reinforced with curvilinear fibers. Setoodeh and his co-workers\textsuperscript{(12)} combined the gradient method with FEM and calculated distributions of the optimum lamination parameter that yield lower compliance, but they did not obtain actual curvilinear fiber paths for the laminated plate from the obtained parameters. The present authors\textsuperscript{(13)} proposed a method for determining distributions of optimum short fibers from the optimized lamination parameters by applying the method\textsuperscript{(6), (7)} to each element of the FEM, and it was shown that the plates with optimum short fibers give the lower maximum deflection than conventional plates. Furthermore, it was shown that the optimum short fiber distribution displays specific global preferential orientations. This suggested that the\textit{continuous} curvilinear fiber paths may exist which optimize the plate property, and they\textsuperscript{(14)} proposed an effective vibration analysis method for plates with continuous fiber shapes. It turned out, however, that application of the gradient method to the optimization in vibration and buckling causes heavy calculation efforts and uncertainties in the accuracy, especially for problems with a large number of design variables. To avoid such computational problems, Abdalla and his co-workers\textsuperscript{(15)} employed a generalized reciprocal approximation technique.

In the present study, a GA is employed to avoid above computational problems, and the fiber orientation angles are considered as design variables directly in each element of the FEM. The LO concept\textsuperscript{(6)} is also used with the GA to reduce computational efforts, since the direct assignment of the fiber orientation angles to design variables causes a rapid increase in the number of design variables as more layers are included. The GA is therefore applied sequentially from the outermost to the innermost layers, and this reduces the multi-layer optimization problem to repetitions of the single-layer optimization problem. The plate generated from the present optimization displays higher fundamental frequencies and more evident trends in the fiber orientations than plates optimized using lamination parameters\textsuperscript{(13)}.  

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2. Optimization problem and method

2.1. Formulation of the optimization problem

A conventional laminated plate with symmetric $N$ layers is shown in Fig. 1, where the plate dimensions are given by $a \times b \times h$ (thickness) in the $O$-xyz co-ordinate system. In each layer, the major and minor principal axes caused by parallel fibers are denoted by $L$ and $T$, respectively. Due to stacking symmetry around the middle surface, there is no coupling between bending and stretching. Thus, the vibration of plates is characterized only by bending stiffnesses. The differential equation governing vibration of symmetrically laminated plates is given by

$$
\frac{4}{3} \left( \frac{D_{11}}{\partial x^4} + 2D_{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_{22}}{\partial y^2} + \frac{4D_{16}}{\partial x} \frac{\partial^4 w}{\partial x \partial y} + \frac{4D_{26}}{\partial y} + \rho \frac{\partial^2 w}{\partial t^2} \right) = 0
$$

(1)

where $w(x,y,t)$ denotes deflection of the plate, $D_{ij}$ ($i,j = 1, 2, 6$) are the bending stiffnesses and $\rho$ is the average mass per unit area. Since the bending stiffnesses $D_{ij}$ are defined by material constants and fiber orientation angles, bending of the plate is dominated by the fiber orientation angles as long as homogenous material constants are considered. To realize local anisotropy, fiber orientation angles $[\theta_1/\theta_2/\ldots/\theta_N/2]$ for each element are given as independent design variables, and thus each element has different bending stiffnesses $d_{ij}^{(n)}$ ($n = 1, 2, \ldots, ne$ and $i, j = 1, 2, 6$) where $ne$ is the number of elements. An element stiffness matrix is calculated using $d_{ij}^{(n)}$ for each element. Then, the present optimization problem can be stated as

Maximize $\Omega$

Design variables $:\left[ \theta_1^{(n)} / \theta_2^{(n)} / \ldots / \theta_{N/2}^{(n)} \right]_k$ ($n = 1, 2, \ldots, ne$)  

Subject to $:-90^\circ < \theta_i^{(n)} \leq 90^\circ \quad (i = 1, 2, \ldots, N/2)$  

(2)

where $\theta_i^{(n)}$ is the angle of $n$th element in $i$th layer. The $\Omega$ is the normalized fundamental frequency defined as a frequency parameter

$$
\Omega = \frac{\rho}{\sqrt{D_0}}
$$

(3)

where $D_0 = E_I h^3/12(1-\nu_{LT}^2)$ is a reference stiffness, $E_I$ is modulus of the elasticity in the $T$ direction, $\nu_{LT}$ and $\nu_{TL}$ are major and minor Poisson ratios and $\omega$ is angular frequency. No artificial constraint is imposed on the fiber orientation angles.

Fig. 1  The symmetrically laminated $N$-layer composite plate.
2.2. Layerwise optimization

Design variables are assigned in each element of the FEM, and this optimization problem has a vast number of possible solutions. For example, when the plate is divided into 10 × 10 elements and 15° increment is used for the optimization in the range from -90° to 90° (giving 12 possible angles), the total number of possible solutions is 12^{100} even for a single-layer plate. In addition, the possible solutions increase exponentially with the increasing the number of layers. To deal with the vast number of combinations, a layerwise optimization (LO) approach is employed to reduce the possible number of solutions.

The LO approach was first presented by Narita and is based on a simple physical observation: the outer layer has a greater stiffness effect than the inner layer in the bending of laminated plates and therefore has a greater influence on the vibration behavior. For this, the optimum condition for bending vibration of laminated plates is assumed to be determined by optimizing each layer sequentially from the outermost to the innermost layer. In the algorithm, the inner layers are assumed initially to have no stiffness but have the same density as other layers while the outer layers are optimized.

The concept of the LO approach successfully reduces the multi-layer optimization into iterations of the single-layer optimization, because the optimization for each plate is repeated sequentially. The optimization problem Eq. (2) is now re-stated as

\[
\begin{align*}
\text{Iterate } i = 1 & \text{ to } N / 2 \\
\text{Maximizing: } & \Omega \\
\text{Design variables: } & \{\theta_1^{(i)}, \theta_2^{(i)}, ..., \theta_n^{(i)}\} \\
\text{Subject to: } & -90^\circ < \theta_i^{(i)} \leq 90^\circ
\end{align*}
\]

Iterations of this optimization starting from the outermost layer of a laminated plate may find an optimum fiber distribution for each layer. By simply adding further iterations, the plate with more layers can be optimized. Therefore, the advantages of the LO approach are both to reduce the calculation efforts and to make it insensitive to the increase in the number of layers. Further, the accuracy is improved by adding another set of iterations with remaining angles determined at the previous cycle.

2.3. Genetic algorithm

A genetic algorithm (GA) method is employed for the single-layer optimization because the GA method based on integer coding is easily applicable to multi-variable optimizations and allows the fiber orientation angles to be included directly in design variables. The GA based on integer coding was used by Riche and Haftka and is often used to represent the fiber orientation angles with fine increments. It is standard practice to use binary coding to represent the design variables, but this yields long gene strings (chromosome) when multi-variable optimization problems are considered. The long strings have disadvantages over short ones in the computational effort, and thus binary coded strings would not be suitable here. Integer coded strings are not affected by these problems. When \(k\) angles are considered, an integer ranging from zero to \(k-1\) is used to represent the possible angles for an element in a layer.

As for GA operators, a two-point crossover and a uniform mutation are used to obtain offspring with elitist tactics. Parents with higher fitness are selected by a roulette rule, and their genes between two crossing points selected randomly are exchanged to generate a child individual in the crossover procedure. In the mutation procedure, a randomly selected gene is converted another integer with low probability. This is conducted to keep the variety of genes. Some of the fittest individuals are selected from the parent generation and placed into next generation without the GA operations. This procedure named as elitist strategy is
implemented to save the gene which may contribute to the high fitness and to making the optimization search monotonically increase.

### 2.4. Finite element formulation modified in the present method

For the frequency analysis of laminated thin plates, the classical laminated plate theory (CLPT) is employed due to its simplicity together with the ACM element proposed by Adini, Clough and Melosh (17). The element is rectangular and has 12 degrees of freedom since each corner of the rectangle has three variables \((w, \partial w/\partial x \text{ and } \partial w/\partial y)\). The deflections for each element is assumed as

\[
w = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 y^3 + \alpha_9 xy^2 + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3
\]

where \(\alpha_i (i = 1, 2, ..., 12)\) are nodal unknowns. Although this element is a non-confirming element which may mean to form kinks along the boundary of element, it was confirmed that there are advantages in accuracy and calculation speed because the domain of integration is a simple shape and the integration can be done analytically.

For symmetrically laminated thin plates, the maximum strain energy stored in each element is given by

\[
U_e = \frac{1}{2} \iiint_A \{\kappa\}^T \left[ d_{ij}^{(n)} \right] \{\kappa\} dA
\]

where \(d_{ij}^{(n)}\) are local bending stiffnesses defined by the fiber orientation angles in each element, and \(\{\kappa\}\) is a curvature vector obtained by the second derivative of the deflection. The element stiffness matrix \([K_e]\) is obtained from Eq. (6).

The maximum kinetic energy stored in each element is given by

\[
T_e = \frac{1}{2} \omega^2 \iiint_A \rho w^2 dA
\]

The element mass matrix \([M_e]\) is obtained from Eq. (7).

The global stiffness matrix \([K]\) and mass matrix \([M]\) are provided by assembling \([K_e]\) and \([M_e]\). After imposing the boundary conditions on the global matrixes, the frequency parameter is determined by solving an eigenvalue equation.

\[
([K] - \omega^2 [M]) \{\delta\} = 0
\]

where \(\{\delta\}\) is the global deflection vector.

![Fig. 2 Plates with the same short fiber distributions but in different mesh sizes.](image)
Table 1 Frequency parameters from the present FEM and ANSYS for the plates in Fig. 2, and differences between two methods.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Mode</th>
<th>Present</th>
<th>ANSYS</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>112.0</td>
<td>143.7</td>
<td>28%</td>
</tr>
<tr>
<td>(a)4x4</td>
<td>2</td>
<td>182.6</td>
<td>274.8</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>261.1</td>
<td>427.2</td>
<td>64%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>111.9</td>
<td>116.0</td>
<td>3.7%</td>
</tr>
<tr>
<td>(b)8x8</td>
<td>2</td>
<td>180.9</td>
<td>193.1</td>
<td>6.8%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>269.3</td>
<td>299.6</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>112.8</td>
<td>111.2</td>
<td>-1.4%</td>
</tr>
<tr>
<td>(c)12x12</td>
<td>2</td>
<td>181.5</td>
<td>181.3</td>
<td>-0.12%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>272.8</td>
<td>278.2</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

3. Numerical result and discussion

3.1. The validity of the present FEM

The material is assumed for a graphite/epoxy (CFRP) composite with $E_L = 138$ GPa, $E_T = 8.96$ GPa, $G_{LT} = 7.1$ GPa, and $\nu_{LT} = 0.30$. To assess the validity of the present FEM for the locally anisotropic plate, the present results are compared with the results from a commercial FEM software (ANSYS). Figure 2 shows three kinds of plates with the same short fiber distributions which are modelled in different numbers of elements. The fibers in each element are denoted by single line, and all elements have a same fiber volume fraction. The numbers of elements are 4 × 4, 8 × 8 and 12 × 12 in (a), (b), and (c), respectively, and the bold lines represent boundaries where adjacent elements give discontinuity in fiber orientation. For the plate with (a) 4 × 4 elements, each element has different fiber orientation angles, and every boundary between the elements indicates the orientational discontinuity in fibers. The plates with (b) 8 × 8 and (c) 12 × 12 elements have finer elements obtained by dividing the element of the plate with (a) 4 × 4 into 2 × 2 and 3 × 3 divisions, respectively. Table 1 lists the lowest three frequency parameters given by the present method and ANSYS, and those differences (%) based on the values of the present method for (a) - (c) plates.

It is known from Table 1 that the differences between the present results and those of ANSYS become smaller as the number of elements increases, and the plate with (c) 12 × 12 elements gives quite close frequency parameters for both methods. There is the large difference between (a) 4 × 4 and (c) 12 × 12 elements for ANSYS, but the small difference for the present FEM. Hence, it was confirmed that the present method calculates the frequency parameters accurately for the plate with different fiber orientation angles in each element.

Fig. 3 Examples of boundary conditions used in the comparison.
Table 2 Contribution of the outermost layers to the frequency of laminated plates obtained by ten times trials, the average of the trials, and the standard deviation (S. D.).

<table>
<thead>
<tr>
<th>Trial No.</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89.31</td>
</tr>
<tr>
<td>2</td>
<td>88.65</td>
</tr>
<tr>
<td>3</td>
<td>87.46</td>
</tr>
<tr>
<td>4</td>
<td>89.84</td>
</tr>
<tr>
<td>5</td>
<td>89.32</td>
</tr>
<tr>
<td>6</td>
<td>91.13</td>
</tr>
<tr>
<td>7</td>
<td>89.51</td>
</tr>
<tr>
<td>8</td>
<td>90.35</td>
</tr>
<tr>
<td>9</td>
<td>89.43</td>
</tr>
<tr>
<td>10</td>
<td>90.93</td>
</tr>
<tr>
<td>Average</td>
<td>89.59</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.075</td>
</tr>
</tbody>
</table>

Table 3 The maximum frequencies of the present plates and conventional plates, optimum lay-ups for conventional plates, and differences between frequencies of both plates.

<table>
<thead>
<tr>
<th>B.C.</th>
<th>Present</th>
<th>Conventional</th>
<th>Lay-up</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex. 1</td>
<td>60.13</td>
<td>56.08</td>
<td>[45/-45/-45/-45]s</td>
<td>7.22%</td>
</tr>
<tr>
<td>Ex. 2</td>
<td>120.9</td>
<td>92.78</td>
<td>[90/0/0/0]s</td>
<td>30.3%</td>
</tr>
<tr>
<td>Ex. 3</td>
<td>19.08</td>
<td>16.39</td>
<td>[20/-45/20/20]s</td>
<td>16.4%</td>
</tr>
<tr>
<td>Ex. 4</td>
<td>43.24</td>
<td>31.87</td>
<td>[55/-50/20/-70]s</td>
<td>35.7%</td>
</tr>
<tr>
<td>Ex. 5</td>
<td>71.48</td>
<td>63.41</td>
<td>[55/-50/-55/55]s</td>
<td>12.7%</td>
</tr>
<tr>
<td>Ex. 6</td>
<td>71.05</td>
<td>64.72</td>
<td>[45/-45/-45/45]s</td>
<td>9.78%</td>
</tr>
</tbody>
</table>

3.2. Short fiber distributions and fundamental frequencies

The parameters used in the GA are \( S = 2000 \), \( g_e = 500 \), \( p_c = 0.7 \), \( p_m = 0.003 \), and \( p_e = 0.005 \) where \( S \) is the number of population, \( g_e \) is the number of generation, and \( p_c, p_m \) and \( p_e \) represent the ratios of crossover, mutation and the proportion of elite individuals who are inherited to the next generation without further operation, respectively. The 15° increment angle (i.e. 12 possible angles) is used in this optimization, and the maximum value of the integer becomes 11 as the first number being zero.

Stability of a GA depends on the numbers of individuals and generations. To confirm the sufficiency of the aforementioned numbers, the present GA is run ten times only for the outermost layer. Since the present GA searches in one layer individually based on the LO idea, trials for one layer are enough to confirm the GA stability. The obtained values are considered as contributions of the outermost layers to the frequencies of laminated plates, and listed in Table 2. The average frequency is 89.59 and the standard deviation (S. D.) is 1.075 which is about 1.2 % of the average. Thus it is confirmed that the numbers of individuals and generations are large enough, and the present GA gives stable results.

Optimization results are given for symmetric 8-layer plates with six boundary conditions shown in Fig. 3. Free, simply supported and clamped edges are designated by the letters F, S and C, respectively, and the letter P presents a point support. These letters are defined in the counterclockwise direction, starting from the left edges. Except for Ex. 6 in Fig. 3, all plates are divided into 10 × 10 = 100 elements, thus each plate has 100 design variables for each layer. The figures from Ex. 1 to 5 in Fig. 3 present a totally simply supported plate, a totally clamped plate, a plate with unsymmetrical boundary conditions including two free edges, a plate with a point supported at the free corner of Ex. 3, and a
plate with a mixed boundary at the lower edge, respectively. Ex. 6 shows L-shaped plate with all edges simply supported. The size of a corner cutout is 0.2a with the length of each square edge a, and thus, Ex. 6 has 96 elements.

Fig. 4  Optimally distributed short fibers in each layer of symmetric 8-layer square plate with totally clamped edges (Ex. 2).

Fig. 5  Overlapping views of optimum short fiber distributions for six examples of boundary conditions.
Table 3 presents the maximum frequency parameters from the present plates in the second column, those from conventional plates with optimally oriented parallel fibers which are obtained by using the LO method \(^\text{(18)}\) in the third column, the optimum lay-ups given by Ref. \(^\text{(18)}\) in the fourth column, and differences (\%) based on the value of conventional plates in the fifth column. Note that the frequency parameters were given for the plate with 20 × 20 element in Ref. \(^\text{(18)}\). However, the values in Table 3 are re-calculated using 10 × 10 elements, but still keeping the same lay-ups, to compare with the present results. It is shown from Table 3 that the present plates give higher frequency parameters for all boundary conditions than the conventional plates. Therefore, it is clear that the locally anisotropic plates with optimally oriented short fibers make it possible to design the fibrous composite plate with higher frequencies than the conventional plates.

Optimum short fiber distributions in each layer for the plate with totally clamped edges (Ex. 2) and overlapping views for the six examples (boundary conditions) are shown in Figs. 4 and 5, respectively. The outermost layer is defined as the first layer in Fig. 4, and fibers in the first to fourth layers are denoted by blue, green, red and orange lines, respectively, in Fig. 5. Fibers are placed to radiate toward to the center of plate in the outer two elements adjacent to the plate edges, and allocated concentrically in the inner elements in Fig. 4. Those orientations become weak as the layer goes to inner, and this result agrees with the physical observation which is the base of the LO concept: the outer layer has greater influence to the bending vibration than the inner one. The specific fiber orientations are clarified in the overlapping view (in Fig. 5, Ex. 2). These orientations may fit to the vibration mode of the totally clamped plate, which forms a relatively small peak in the center of plate. The plate gives 30.3 % higher frequency parameter than the conventional plate, and this improvement is the second largest among the six examples.

![Fiber distributions and overlapping view](image-url)

Fig. 6  Short fiber distributions for each layer and the overlapping view for symmetric 8-layer square plate with totally clamped edges calculated using the method in Ref. \(^\text{(13)}\).
The fibers in the totally simply supported plate (in Fig. 5, Ex. 1) form a diamond shape, and this shape may be appropriate pattern to constrain a relatively large peak in the center of plate. In the elements outside the diamond shape, fibers are composed of about 45° and -45°. These angles are the same configuration with the optimum lay-up of the conventional plate. The improvement is 7.22 % which is the lowest among the six examples. The fibers in Ex. 3 are allocated horizontally in the elements near left edge, and are composed of various angles in other elements. This gives 16.4 % improved frequency. It is indicated in Ex. 4 that the fibers flow from the clamped (left) edge and the lower right corner to the point supported (upper right) corner through the center of plate. This plate gives the largest improvement with the value of 35.7 %. Example 5 has mixed boundary condition on the lower edge, and also has the mixed fiber orientations of Ex. 1 and Ex. 2, giving 12.7 % improvement in frequency parameter. The skewed diamond shaped orientations due to the corner cutout appear in Ex. 6, and frequency parameter is obtained with 9.78 % higher.

From above discussion, it is concluded that the plate with optimally distributed short fibers indicates higher natural frequencies than the conventional plate with parallel straight fibers, and the plate has specific fiber orientations even though no additional constraint is imposed on the fiber orientation in this study. This suggests the possibility for existence of the continuous fibers with optimally curvilinear shapes.

3.3. Comparison with the previous method

Figure 6 shows optimum short fiber distributions for each layer and the overlapping view calculated by the previous method which the authors have proposed. The results are given for the totally clamped plate (Ex. 2). Although the second layer indicates specific orientations similar to that in Fig. 4, no specific orientation is given for the other three layers. The overlapping view does not give the clear specific orientations either. The improvement of frequency parameter is 12.2 % compared with the conventional plate, and the value is lower than that of the present method. Thus, it turns out that the present method determines optimum short fiber distributions more effectively than the previous one.

4. Conclusion

The short fiber distribution in each element of the FEM was determined to design the locally anisotropic structure in the laminated composite plate. The LO concept successfully reduced the multi-layer optimization problem into iterations of the single-layer problem, and the simple GA was employed to solve the single-layer problem by making use of advantages in dealing with a large number of design variables.

The results showed that the present plates give higher natural frequencies in the six examples of boundary conditions than the conventional plates with parallel straight fibers. Further, the optimized short fiber distributions indicated specific fiber orientations. This suggests the existence of the optimum curvilinear and continuous fiber shapes which reinforce the composite plate more effectively than the parallel straight fibers. Comparison of two sets of results from the present and the previous methods revealed that the present method extracts the optimum short fiber distribution more skilfully than the previous method.

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