Modal Control of Vibrations of a Machine Structure by Using Electromagnetic Exciters*

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Abstract
The present study aims at the design of the active control of machine tools’ vibrations. The formulation of modal control for linear lumped parameter vibratory systems is offered. The electromagnetic exciters are developed to serve as active dampers for machine structures. A simple L-shaped beam structure is the object for the experiments and is mounted with two electromagnetic exciters. The behavior of the whole system is investigated by the model that is composed of the lumped masses and springs with 4-degree of freedom. The control system is designed through the investigation both by the root locus method and by the "MATLAB Simulink" simulation in which the elapsing responses, \( \ddot{c}_1(t) \), \( \ddot{c}_2(t) \), \( \ddot{c}_3(t) \), \( \ddot{c}_4(t) \) of all four modes and the acceleration waveforms, \( \dddot{w}_1(t) \), \( \dddot{w}_2(t) \), \( \dddot{w}_3(t) \), \( \dddot{w}_4(t) \) are computed. The best condition for the values of control parameters results by the simulation. In the experiment, the preparatory study for the vibration control of the L-shaped structure has been performed in advance to the investigation.

Key words: Modal Control, Vibration, Electromagnetic Exciter, Machine Structure, MATLAB Simulink

1. Introduction
The present study aims to apply the modal control technique to the design procedure for the active vibration control of machine tools which may induce chattering phenomena during the machining operations.

The modal control technique\(^{(1)}\) has been served for control for a class of one-dimensional linear distributed parameter systems such as the temperature control of heat conducting substances, and it has been preceded by the contribution\(^{(2)}\) of the distillatory process control problem based on the modal aspect. This control method has presented in a book\(^{(3)}\) as an illustrative example of the temperature control.

The author have studied the modal control of the flexural bending vibrations caused both in elastic beams and in flat plates as the one- and the two-dimensional distributed parameter control systems\(^{(4),(5)}\). Regarding the two-dimensional case, the design formulation of modal control has been made capable to realize basically by the idea of "Decreasing of order of dimension"\(^{(6)}\) which indorses the enlargement of the order of dimension of the domain space to be higher for applying modal control. As regards the modal control for lumped spring-mass vibratory systems, the basic formulation has been presented by the author\(^{(7)}\). The general formulations of the modal control for the lumped vibratory systems are given lately in the literatures\(^{(8),(9)}\).
The purpose of the present study is to find the means how to control the harmful vibrations on the machine structure with the help of the modal design technique and the electromagnetic exciters\(^{(10)}\). The accomplishment of the equipment of vibratory damper system is the ultimate end of the research study.

In the present study, the L-shaped beam structure is served as an example of the vibratory machine structure. To suppress the harmful vibration on machine structure, two electromagnetic exciters have been mounted with as the active dampers. The developed electromagnetic exciters are suitable to work as the vibratory dampers. They are of small size and of light mass and are easier to carry on the maintenance. The preliminary experimental study has been attempted to verify how the control system works for the vibrations induced in the machine structure. The acceptances have been measured as the frequency responses with random force stimulations in kind. The modal control system has been realized by the formulation for the lumped mass model with 4-degree of freedom, and the best values of parameters for control have been obtained by means of the Simulink simulation\(^{(11)(12)}\). In advance to the simulation for the whole modal control, the preparatory calculations have been performed to assure the stable control by means of the root locus method with \(K\) (gain value) and \(T\) (time constant) as the parameters.

2. Modal control of machine structure’s vibrations

(1) Vibration analysis of the L-shaped beam structure.

The L-shaped beam structure is composed of the column (member A) and the over-arm (member B) fastened with the right angle to each other by bolts as shown in Figure 1.
The members A and B are made of steel, and both have same sizes, that is, the lengths of 0.560m and the rectangular hollow cross-sections of 0.125m \( \times \) 0.075m with thicknesses of 6mm. The vibrations in the horizontal and vertical directions on to the vertical plane are the only matters of consideration. The two exciters (vibration dampers) are considered as the simple spring-mass elements when they are not activated by the exciting currents for the windings of coils. Thus, the vibratory model of the whole system is of 4-degree of freedom as shown in Figure 2. The numerical data of the vibratory model are shown in Table 1.

Table 1. The numerical data.

<table>
<thead>
<tr>
<th>l_1</th>
<th>M_1</th>
<th>l_2</th>
<th>M_2</th>
<th>l_c</th>
<th>m</th>
<th>K_0</th>
<th>k_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.598(m)</td>
<td>5.27(kg)</td>
<td>0.478(m)</td>
<td>5.52(kg)</td>
<td>0.240(m)</td>
<td>1.11(kg)</td>
<td>( 0.539 \times 10^6 \text{(Nm/rad)} )</td>
<td>( 2 \times 10^6 \text{(N/m)} )</td>
</tr>
<tr>
<td>( 0.295 \times 10^6 \text{(Nm/rad)} )</td>
<td>( 2 \times 10^6 \text{(N/m)} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The bending rigidity \( EI \) is assumed to be infinite for members A and B.

The fundamental equation of vibration is:

\[
Mw(t) + Sw(t) = g(t)
\]

where, \( w(t) = [w_1(t), \ldots, w_4(t)]^T \) is the displacement vector, \( g(t) = [g_1(t), \ldots, g_4(t)]^T \) is the force vector for the control of object machine structure, \( M \) the mass matrix, \( S \) the stiffness matrix. The above equation results in the eigenvalue problem when the right side term \( g(t) \) of the equation vanishes. Then the set of four modes, that is, natural frequencies \( \omega_i \) and the modal vectors \( \mathbf{x}_i \) are derived correspondingly \( (i=1, \cdots, 4) \). Accordingly, the modal matrix \( \mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_4] \) is composed of four modal vectors lined in row wise.

Figure 3 shows the calculated modal shapes with natural frequencies \( \omega_i/(2\pi) \) for all modes.

Figure 3. The Modal Shapes and the Natural Frequencies.

(2) Formulation of modal control system with 4-degree of freedom.

The displacement vector \( W(t) \) is expressed in terms of modal vectors \( \mathbf{x}_i \) as the modal expansion:
\[ w(t) = c_1(t)x_1 + \cdots + c_4(t)x_4 = Xc(t) \]  
(2)

where, \( c(t) = [c_1(t), \cdots, c_4(t)]^T \) is the modal component signal vector. So,

\[ c(t) = X^{-1}w(t) \]  
(3)

Similarly, the force vector \( g(t) \) is:

\[ g(t) = e_1(t)x_1 + \cdots + e_4(t)x_4 = Xe(t) \]  
(4)

where, \( e(t) = [e_1(t), \cdots, e_4(t)]^T \) is the error signal vector in the control system. \( g(t) \) is expressed by the sum of the partial driving forces \( g_i(t) \), that is,

\[ g(t) = \sum_{i=1}^{2} g_i(t) = \sum_{i=1}^{2} p_i f_i(t) = Pf(t) \]  
(5)

where, \( f(t) = [f_1(t), f_2(t)]^T \) is the time dependent factor vector of the driving force for control. In the present design, the spatial distributions of the partial driving forces are \( p_1 = [1 \ 0 \ -1 \ 0]^T \) and \( p_2 = [0 \ 1 \ 0 \ -1]^T \). The modal expansion of \( p_i \) is:

\[ p_i = k_1^i x_1 + \cdots + k_4^i x_4, \quad (i = 1, 2) \]  
(6)

and the matrix of the spatial distribution of driving forces is:

\[ P = [p_1 \ p_2] = XK \]  
(7)

where, \( K \) is composed of \( k_i^j \) etc., and is the \( 4 \times 2 \) rectangular matrix:

\[ K = X^{-1}P \]  
(8)

From Equations (5) and (7),

\[ g(t) = Pf(t) = XKf(t) \]  
(9)

Comparing Equations (4) and (9),

\[ e(t) = Kf(t) \]  
(10)

\[ \therefore f(t) = KLMe(t) \]  
(11)

where the \( 2 \times 4 \) rectangular matrix \( KL^M \) is the left minimal pseudo inverse matrix:

\[ KL^M = (K^T K)^{-1} K^T \]  
(12)

Therefore, the driving force signal \( f(t) \) is composed by the error signal \( e(t) \) according to the Equation (11).

Figure 4 shows the schematic of the modal control system for the vibratory machine structure with 4-degree of freedom, and it has such components as the operational circuits \( X^{-1}, KL^M \) and the modal feedback compensator \( \Lambda(s) \) (4 \( \times \) 4 diagonal matrix).
3. Electromagnetic exciters as active dampers

Figure 5 is the cutaway view showing the assembled structure of an electromagnetic exciter. The exciters are of small size, that is, 117mm(H) x 117mm(W) x 69mm(D), and of light mass, that is, 2.4 kg. The exciter consists of the following parts: (a) the housing made of aluminum alloy plates, (b) the slab made of soft iron, being fixed in the middle of the housing, (c) the pair of magnetic U-cores made of soft iron, each of which carrying double exciting coils, and their magnetic poles are facing to the fixed slab across the set air-gaps, (d) the pair of the H-shaped connecting elements made of brass, by which the cores on both sides being fastened each other and (e) the leaf springs made of phosphor bronze, suspending the cores, the coils and the connecting elements as an inertia mass in space so that the static equilibrium may be kept during the operations. The inertia mass is capable to vibrate relatively across the fixed slab by the amount ± 1 mm. It has been made possible by enlarging the set air-gaps 25% by the remade of the original exciters in purpose of increasing of the control forces. The natural frequencies of exciters are 41.6 Hz. The produced force becomes in magnitude asymptotically equal to the magnetic attractive force between the magnetic poles in the frequency range beyond the resonance. However, the control force available to operate are limited about to 300Hz or less than, and the maximum amplitudes are about 50Np-p.

4. Checking by the root locus method

In advance to the computer simulation, the root loci of the closed loop system for the vibration control have been investigated to clarify the effects of both the time constant $T$(ms) which stems from the impedance of exciting coils of the exciters and the overall loop gain $K$. A controlled system with the single mode having the natural frequency $\omega_n$ is assumed, and the control force is generated by the acceleration signal of vibration through the feedback loop. For the frequency range beyond the exciter’s resonance, the magnitude of the generated force is almost equal to the magnetic attractive force of the exciter. Then, the open-loop transfer function is expressed as the product of the form: $k_e / (Ts+1) \cdot k_e \omega_n^2 / (s^2 + \omega_n^2) \cdot k_a s^2$. The overall loop gain $K$ is put equal to $k_e k_a \omega_n^2 k_a$. For the root locus method, the characteristic equations with the varying parameters $K$ and $T$ are given as follows:

$$K \frac{s^2}{(Ts+1)(s^2 + \omega_n^2)} + 1 = 0 \quad (K \text{ varies}) \quad (13)$$

$$T \frac{s(s^2 + \omega_n^2)}{(K+1)s^2 + \omega_n^2} + 1 = 0 \quad (T \text{ varies}) \quad (14)$$

The root loci of the closed loop system with the acceleration feedback are depicted for the varying parameters $K$ and $T$ as shown in Figures 6(a) and 7(a). In the diagrams of the root loci, the locations of the marks × and ○ indicate poles and zeroes of the open loop systems and they are correspond to the values of the parameter $K$ or $T$ equal to 0 and infinity correspondingly. With the assumption of $\omega_n=615$ rad/s (natural frequency of the
4th mode), fifteen combinations of computing conditions are examined, that is, related to Equation (13) the values of $T$ are from 0.1ms to 10ms, and for Equation (14) the values of $K$ are 5, 10 and 20. A pair of the best sets of the parameters’ values, that is, ($K=4.1$, $T=5$ ms) and ($K=10$, $T=10$ ms) is attained, and they are obtained from the view point of the vibration decrements. For the former set of the parameters’ values, the characteristic roots are ($-210 \pm j 286$, $-600$), and for the latter ($-173$, $-460 \pm j 80$), and the damping ratios $\zeta$ are 0.592 and 0.985 respectively. For the above characteristic roots, the transient waveforms of the system are as shown in Figures 6(b) and 7(b). It is seen that the performance of control is affected remarkably by the value of $T$ as much as $K$.

5. Simulation by the MATLAB Simulink

To estimate the control parameters quantitatively, the computer simulation, MATLAB Simulink, has been performed. The modal control system has the feedback compensator matrix $\mathbf{\Lambda}(s)$ which is located in the feedback loop of modal domain and it involves such transfer functions as $\lambda_1(s)$, $\lambda_2(s)$, $\lambda_3(s)$ and $\lambda_4(s)$ in its diagonal. In addition, the PID compensators are implemented of which transfer functions for individual exciters are:
\[ G_i(s) = K_{pi} \left\{ 1 + \frac{1}{T_i s} + T_{di} s \right\}, \quad (i = 1, 2) \]  

The PID compensators must be able to stabilize enough the motion of the inertia mass inside each exciter, and then it could be expected to realize the control of vibrations of machine structure even if with their amplitudes in large.

The whole control system using electromagnetic exciters is composed as shown in Figure 8. It includes the analyzer matrix \( X^{-1} \), the synthesizer matrix \( K^{LM} \), the linearizing circuit, the PID compensators, the modal compensator matrix \( \Lambda(s) \) and the power amplifiers.

![Figure 8. The Schematic of the Whole Control System Using Electromagnetic Exciters.](image)

Figure 8 shows the computer program for the Simulink simulation for the whole system. The simulations for a variety of combinations of the parameters have carried out with the impulsive disturbance. The modal responses of the 1st to 4th mode and the simulated waveforms of the four points on the structure have been observed. The acceleration waveforms of all modes with no control actions are shown in Figure 10(a). It is noticed that each elapsing waveform shows the sustaining sinusoidal with the frequency which coincides with that of its resonance for each mode, namely, 30Hz, 40Hz, 44Hz and 95Hz respectively. The parameters for the simulation of the best response are as follows:

![Figure 9. The Simulink Simulation Program.](image)
The Modal Responses for the Impulsive Disturbance.

$T = 5 \text{ ms}$, $\lambda_1 = -100$, $\lambda_2 = 70$, $\lambda_3 = 20$, $\lambda_4 = 100$; $K_{P1} = -2000$, $T_{I1} = 12.3s$, $T_{D1} = 0.003s$, $K_{P2} = 100$, $T_{I2} = 12.3s$, $T_{D2} = 0.306s$. For the 1st mode, the negative value of $\lambda_1$ results from the inversion of the phase angle at the resonance of the exciters. The polarities of $K_{P1}$ and $K_{P2}$ for the PID compensators depend on the definition of the polarity of each spatial distribution $p_1$ and $p_2$. The waveforms of the best control are shown in Figure 10(b) below. In this case, the remarkable improvements have been attained for the 1st and 4th mode as the successful result of the vibratory modal control. On the other hand, it is considered that the vibratory behaviors $\tilde{c}_2(t)$ and $\tilde{c}_3(t)$ for the 2nd and 3rd mode are inherently due to the resonances of the exciters themselves, and the internal masses of both exciters vibrate inevitably. For PID compensators, the input signals are the relative displacements $w_3 - w_1$ and $w_4 - w_2$ from the exciters, and they are predominantly shared by the vibration components of the 2nd and 3rd mode. As the normal orthogonality property of eigenmode holds on the modal control, such vibrations of the exciters could able to be cancelled by the adequate band eliminating networks.

6. Conclusion

The purpose of the present study is to establish the design method of control of machine vibrations and also to utilize the electromagnetic exciters as vibration dampers for machine tools. The followings are enumerated as the strong points:

1. The present modal control is the primary study for a class of lumped multi-degree of freedom vibratory mechanical systems as the plan.
2. The electromagnetic exciters which are of small size and of light mass are developed and are adopted for the active vibration control.
3. Although not expected in advance, the computer simulation results in success even for the control of the 1st mode of which natural frequency is located under the exciters’ ones.

References

(3) Takahashi, Y., et al., Control and Dynamic Systems, (1970), pp446-452, Addison-Wesley
(7) Sato, T., Printed document, Graduate School of Tokyo Metr. Univ., (In Japanese), (Mar. 1976)
(9) Meirovitch, L., *Dynamics and Control of Structures*, (1990), John Wiley & Sons