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Suppression of Friction-Induced Vibration of a Glass Plate by a Dynamic Absorber*

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Abstract
Friction-induced vibration often causes noise problems. When a plate-like object is rubbed by rubber, self-excited vibration is generated, which results in noise since plate vibration oscillates the air. For reducing vibration and noise, we investigate the characteristics of friction-induced vibration of a glass plate experimentally. Then, we study the effectiveness of a dynamic absorber mounted on the glass plate. The results demonstrate that the self-excited vibration is well suppressed by an absorber, and that absorbers with higher damping ratios are more effective than those with lower. The location of an absorber is also examined for reduction of vibration. Further, analytical study is performed to understand the mechanism of the vibration. We obtain an analytical model from observation and analyze the motion assuming bouncing vibration. Calculated results agree qualitatively with experimental ones.

Key words: Self-Excited Vibration, Friction, Dynamic Absorber, Sound Pressure, Bouncing

1. Introduction
Friction-induced vibrations occur in mechanical systems and have been studied by many workers. Adams(1) studies the dynamic characteristics of a flat body sliding on a rough surface with a simplified model that consists of a tensioned beam on an elastic foundation. He shows the effects of friction coefficient, foundation stiffness and asperity spring stiffness on stability. Ouyang and Mottershead(2) analyze the instability of the transverse vibration of a circular disk under rotating two sliders. They indicate that the damping of the disk and the sliders is effective in reducing the unstable region and an increase of friction coefficient expands the unstable region. Hochlenert et al.(3) analyze a beam moving through two brake pads. They conclude that the three-dimensional kinematics of a brake disk is necessary to understand disk brake squeal. Nakano et al.(4, 5) apply a dynamic absorber to suppress friction-induced vibrations generated in disk brake systems, and conclude that the damping of a dynamic absorber is necessary for suppressing squeal in a bicycle disk brake system whereas it is not necessary for suppressing low frequency squeal in a car disk brake system. Ono and Yamane(6) study the self-excited bouncing vibration of a flying head slider of a hard disk drive. They perform numerical simulation of a two-degree-of-freedom model assuming a simplified contact condition, which shows good agreement with experimental result.

Friction-induced vibration often causes noise problems. When a plate-like object is rubbed by rubber, self-excited vibration is generated, which results in noise since plate
vibration oscillates the air. An example is the noise induced by a closing window of an automobile. For reducing vibration and noise, we investigate the characteristics of friction-induced vibration of a glass plate experimentally. Then, we study the effectiveness of a dynamic absorber mounted on the glass plate. Analytical study is also performed to understand the mechanism of the vibration.

2. Experimental Apparatus

Figure 1 shows the experimental apparatus, which consists of a glass plate and a rubbing mechanism. The glass plate is suspended horizontally by four metal wires, which are inclined so that the glass plate moves only in one direction as shown in Fig. 1(a). The wires are located at nodal lines of the first vibration mode of the glass plate as shown in Fig. 2(a) since the first vibration mode is induced by rubbing. The dimensions and parameters of the glass plate are shown in Fig. 2(a) and Table 1. The rubbing mechanism consists of a rubber ball supported by a beam. The end of the beam is firmly fixed. When the glass plate moves horizontally, self-excited vibration occurs as well as noises since the rubber ball rubs
against the glass plate. Figure 2 shows positions of a dynamic absorber, rubbing area, an accelerometer and a microphone, by which the acceleration of the glass plate and the sound pressure were measured simultaneously. We used nine glass plates, whose first natural frequencies are almost the same, that is, 133 Hz, and their modal damping ratios are also the same, that is, 0.001. A dynamic absorber was attached to the middle of the long side of the glass plate. Figure 3 shows a schematic of a dynamic absorber consisting of two metal plates, a rubber block and a weight. Table 2 shows parameters of nine dynamic absorbers we used.

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<td>Dynamic absorber (DA)</td>
<td>Glass plate</td>
<td>Accelerometer</td>
</tr>
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<td>Rubbing area</td>
<td>DA</td>
<td>Microphone</td>
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(a) Top view (b) Front view

Fig. 2  Glass plate and sensors

Table 1  Parameters of glass plate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Length</td>
<td>450 mm</td>
</tr>
<tr>
<td>Width</td>
<td>250 mm</td>
</tr>
<tr>
<td>Thickness</td>
<td>4.85 mm</td>
</tr>
<tr>
<td>Mass</td>
<td>1.365 kg</td>
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<tr>
<td>Young’s modulus</td>
<td>71.6 GPa</td>
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<tr>
<td>Poisson’s ratio</td>
<td>0.23</td>
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Fig. 3  Schematic of a dynamic absorber
### Table 2  Parameters of dynamic absorbers (DA)

<table>
<thead>
<tr>
<th>No.</th>
<th>Mass [g]</th>
<th>Natural frequency [Hz]</th>
<th>Damping ratio $\zeta_{DA}$</th>
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<tr>
<td>#1</td>
<td>27.0</td>
<td>139</td>
<td>0.06</td>
</tr>
<tr>
<td>#2</td>
<td>27.0</td>
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</tr>
<tr>
<td>#3</td>
<td>25.5</td>
<td>131</td>
<td>0.02</td>
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<tr>
<td>#4</td>
<td>25.3</td>
<td>132</td>
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</tr>
<tr>
<td>#5</td>
<td>18.7</td>
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<td>0.001</td>
</tr>
<tr>
<td>#6</td>
<td>18.4</td>
<td>130</td>
<td>0.02</td>
</tr>
<tr>
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<td>18.4</td>
<td>133</td>
<td>0.02</td>
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<tr>
<td>#8</td>
<td>18.4</td>
<td>133</td>
<td>0.02</td>
</tr>
<tr>
<td>#9</td>
<td>18.4</td>
<td>132</td>
<td>0.02</td>
</tr>
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</table>

### 3. Results and Discussion

Figure 4 shows waveforms of acceleration of a glass plate and sound pressure in the case without a dynamic absorber, as well as their frequency spectra. The background noise in the experimental room was less than 30 dB. The similarity between the two waveforms suggests that the sound is caused by the vibration of the glass plate. The vibration grows up by rubbing and its frequency is 134 Hz, which is almost the same as the first natural frequency of the glass plate. Then mounting dynamic absorber #3 on the glass plate, we obtain waveforms and frequency spectra shown in Fig. 5. Comparing Figs. 4 and 5, we note that the dynamic absorber reduces vibration well.

Figures 6(a) and 6(b) show the effect of damping ratio $\zeta_{DA}$ of a dynamic absorber on the acceleration amplitude of a glass plate and sound pressure level, respectively. The acceleration amplitudes and sound pressure levels were measured after the vibrations were fully developed. We note that observed acceleration amplitude and sound pressure level are reduced with damping ratio $\zeta_{DA}$, that is, an absorber with higher damping ratio is more effective to reduce vibration in the range of experiment.

The effect of the location of a dynamic absorber was investigated. Figure 7 shows relationship between the location of a dynamic absorber and the decrease of sound pressure level. The abscissa represents the location of a dynamic absorber from the left end of the glass plate. The ordinate represents the decrease of sound pressure level from the value in the case without a dynamic absorber. The dynamic absorber was attached to only the left half region of each glass plate since the glass plates are symmetric. As shown in Fig. 7, dynamic absorber #6 mounted near an anti node, that is, on the center line of the plate is most effective. On the other hand, dynamic absorber #8 mounted near a node is ineffective.
Fig. 4  Vibration of a glass plate without a dynamic absorber

Fig. 5  Vibration of a glass plate with dynamic absorber #3
(a) Acceleration amplitude

(b) Sound pressure level

Fig. 6  Effect of damping ratio

Fig. 7  Effect of location of a dynamic absorber

(△: DA#6, □: DA#7, Δ: DA#8, ○: DA#9)
We observe the motion of a ball with respect to a glass plate that moves horizontally by a high-speed digital video camera. Figure 8 shows the motion schematically. Black and gray arrows in the figures represent the rotational velocity $\dot{\theta}$ and the vertical velocity $\dot{y}$ of the ball, respectively. For understanding, vertical displacement is exaggerated. The observed motion is as follows:

(i) Since the glass plate moves horizontally, frictional force rotates the ball in the clockwise direction. This rotational motion leads to the translational acceleration of the ball (1$\rightarrow$2).

(ii) After the ball leaves the glass plate, it stops rotating and then starts to rotate in the counter-clockwise direction. The vertical displacement increases until it reaches a maximum (2$\rightarrow$3$\rightarrow$4$\rightarrow$5).

(iii) Then the ball moves downward and rotates again in the clockwise direction (5$\rightarrow$6$\rightarrow$7$\rightarrow$8).

(iv) The ball moves downward after it falls on the plate (8$\rightarrow$1).

We understand that bouncing motion of a ball is induced by the friction that rotates the ball and bends the supporting beam.

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![Diagram of motion](image1)

**Fig. 8** Motion of a rubber ball

![Analytical models](image2)

(a) Contact state

(b) Bouncing state

(c) Dynamic absorber

**Fig. 9** Analytical models
From observation, we assume an analytical model that consists of a ball supported by a flexible beam and a glass plate as shown in Fig. 9. Motion of the glass plate is assumed to be a single-degree-of-freedom system since the plate vibrates at the first natural frequency. Their modal mass $m_0$, modal stiffness $k_0$ and modal damping $c_0$ are obtained experimentally as well as analytically\(^{(7)}\). By considering that the ball is bouncing, two models are used, which are switched whether the ball is in contact with the plate or not as shown in Figs. 9(a) and 9(b), where $z_0$ indicates the displacement of the glass plate (point P in the figure) from an equilibrium condition. In addition, the horizontal velocity $V$ of the glass plate is assumed to be constant.

The rubber ball is modeled as a rigid sphere of mass $m_1$ and radius $a$ supported by an inclined massless flexible beam of length $l$ and bending stiffness $EI$. The inclination of the beam to the vertical is denoted by $\phi$. For simplicity, the contact stiffness and contact damping is modeled by the linear stiffness $k_c$ and linear damping $c_c$. It is assumed that a damping force and a damping torque act on the rubber ball, and their damping coefficients are denoted by $c_1$ and $c_2$, respectively. The translational and angular displacements of the rubber ball from equilibrium conditions are denoted by $y$ and $\theta$, respectively. Further, we assume that a rubber ball in contact state does not slip on a glass plate. The static frictional force is simply considered in this assumption.

An analytical model of a dynamic absorber shown in Fig. 9(c) is modeled as a single-degree-of-freedom system consisting of mass $m_{DA}$, stiffness $k_{DA}$ and damping $c_{DA}$, which is mounted on the glass plate at point Q. The displacement of point Q can be described as $rz_n$, where $r$ is the ratio of the displacement at Q to that at P.

The equations of motion for the translational motion of the rubber ball, the glass plate and the dynamic absorber are derived as follows:

\[
m_1 \ddot{y} + c_1 \dot{y} + \frac{12EI}{l} y - \frac{6EI}{l} \theta - F_n \sin \phi - F_f \cos \phi = 0, \tag{1}
\]
\[
m_0 \ddot{z}_0 + (c_0 + r^2 c_{DA}) \ddot{z}_0 - r c_{DA} \ddot{z}_{DA} + (k_0 + r^2 k_{DA}) z_0 - r k_{DA} \ddot{z}_{DA} + F_n = 0, \tag{2}
\]
\[
m_{DA} \ddot{z}_{DA} + c_{DA} (\ddot{z}_{DA} - rz_0) + k_{DA} (z_{DA} - rz_n) = 0, \tag{3}
\]

where, $F_n$ and $F_f$ are normal and tangential (frictional) forces that act on the ball at the contact point. The equation of motion for the rotational motion of the rubber ball as well as $F_n$ and $F_f$ in Eqs. (1) and (2) need to be switched depending on states of the ball. When the ball is in the contact with the plate (Fig. 9(a)), they are given by

\[
\dot{\theta} = \frac{V - \dot{y} \cos \phi}{a}, \tag{4}
\]
\[
F_n = -c_1 (y \sin \phi - \dot{z}_0) - k_1 (y \sin \phi - z_0), \tag{5}
\]
\[
F_f = \frac{1}{a} \left[ J \ddot{\theta} + c_2 \dot{\theta} - \frac{6EI}{l^2} y + \frac{4EI}{l} \dot{\theta} \right], \tag{6}
\]

where, $J = (2/5)m_1 a^2$, and $\ddot{\theta}$ in Eq. (6) is obtained from Eq. (4). On the other hand, when the ball is bouncing (Fig. 9(b)), they are given by

\[
J \ddot{\theta} + c_2 \dot{\theta} - \frac{6EI}{l^2} y + \frac{4EI}{l} \dot{\theta} = 0, \tag{7}
\]
\[
F_n = -k_1 d_s, \tag{8}
\]
\[
F_f = 0, \tag{9}
\]

where, $d_s$ represents static deflection in the equilibrium condition.
Since the purpose of this analysis is to consider the basic features of the observed results, a qualitative discussion is given here. The Runge-Kutta method is employed for numerical simulation. Parameters we used for simulation are shown in Table 3. The values of \( m_0 \) and \( r \) are estimated by FEM analysis using a mesh of \( 10 \times 18 \) thin rectangular elements. In Table 3, the following parameters are introduced:

\[
\begin{align*}
\xi_0 &= \frac{c_0}{2m_0\omega_0}, \quad \xi_1 = \frac{c_1}{2m_1\omega_0}, \quad \xi_2 = \frac{c_2}{2J\omega_0}, \quad \xi_r = \frac{c_r}{2m_r\omega_r}, \\
\omega_0 &= \sqrt{\frac{k_0}{m_0}}, \quad \omega_b = \sqrt{\frac{3EI/l^3}{m_1}}, \quad \omega_c = \sqrt{\frac{k_c}{m_1}} \quad \text{(10)}
\end{align*}
\]

Some parameters in the table are difficult to identify exactly. We determine them approximately as follows. The contact stiffness \( k_c \) is determined based on the measured stiffness of the rubber ball under an initial load. The static deflection \( d_s \) is the ratio between the contact stiffness \( k_c \) and the measured force from the beam. The damping ratios, except for \( \xi_0 \), are determined from the measured amplitude of the friction-induced vibration in the case without a dynamic absorber.

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![Fig. 10](image)

Comparison of calculated and experimental results (without DA)
(●: by laser sensor; ○: by high-speed digital video camera; –: calculated)
Figure 10 shows time histories of motions of the rubber ball and the glass plate without a dynamic absorber after vibration is developed. Black lines represent the calculated result, and the blue and red points represent the experimental results measured by laser sensors and high-speed digital video camera, respectively. In the figure, the dispersion of the experimental data of $\theta$ is relatively large, because the accuracy of the measurement by the video camera is not high. Figure 10 shows that the phase relationship of the calculated waveforms agrees with that of the measured waveforms.

Figure 11 shows comparison of calculated and measured acceleration of the glass plate without a dynamic absorber and Fig. 12 with dynamic absorber #3. Here, Fig. 12(b) is the same as Fig. 5(a). From comparison of these figures, calculated results show good agreement with experimental ones.

Finally, the effect of damping ratio of a dynamic absorber is briefly investigated. In the calculation, the mass and natural angular frequency of a dynamic absorber are set to $m_{d1} = 10\, \text{g}$ and $\omega_{d1} = 133 \times 2\pi \, \text{rad/s}$, respectively. In Fig. 6(a), calculated results are also plotted, which show fairly good agreement with experimental ones. The plotted acceleration amplitudes are those of fully developed vibration. The calculated results also demonstrate that acceleration amplitude decreases as the damping ratio of a dynamic absorber increases.

4. Conclusions

From experimental study, we obtain the following conclusions:

(1) Self-excited vibration of a glass plate induced by rubbing with rubber is well suppressed by mounting a dynamic absorber on the glass plate.

(2) An absorber with higher damping ratio is more effective than that with lower.

(3) An absorber mounted near an anti node is effective while ineffective near a node.

Further, we obtain an analytical model from observation and analyze the motion of a rubber ball together with a glass plate assuming that the rubber ball bounces on the glass plate. Calculated results agree qualitatively with experimental ones.
References


