Coupled Interval Genetic Algorithm Technique with the Finite Element Method for the Interval Optimization of Structures*

Ting-Nung SHIAU**, Chung-Hao KANG***, De-Shin LIU** and Wei-Chun HSU****

**National Chung Cheng University, 621 Chia Yi, Taiwan, R.O.C
E-mail: imetls@ccu.edu.tw
***National University of Tainan, 700 Tainan, Taiwan, R.O.C
****Wu Feng Institute of Technology, 621 Chia Yi, Taiwan, R.O.C.

Abstract
This paper combines a verified interval optimization method with the FEM for designing structures, which is denominated as the Hybrid Interval Genetic Algorithm (HIGA). This algorithm can neglect formulated equations and interval analysis, and while determining the optimum interval parameters. Furthermore, it can also maximize the design scope. In this paper, this algorithm is implemented for both a truss and frame structure. The interval optimizations include the static and dynamic responses of these structures. The results show that the algorithm which combines the IGA with the FEM can determine the feasible interval design parameters of structures with allowable objective errors.

Key words: Interval Genetic Algorithm, Hybrid Interval Genetic Algorithm, Interval Optimization, Finite Element Method, ANSYS, Truss Structure, Frame Structure

1. Introduction
In engineering, the techniques of optimization designs have been independently investigated by several savants. Most optimization methods focus on deriving the exact optimal parameters for an optimal design. However, it usually is not easy to manufacture exact design parameters because of the measurement inaccuracies or errors within the manufacturing process itself. Furthermore, exact manufacturing is expensive. In order to subdue this problem, the interval optimization is proposed.

The interval analysis was initially developed in the mid-1960s; several savants independently had the idea of bounding errors by doing computations with intervals. The statistical approach to interval analysis first appeared in research by Moore(1), (2). He transformed a preliminary concept into a viable tool for error analysis, and then extended it to measure the effect of errors from all sources. Recently, the interval analysis method has been used to deal with practical engineering problems, such as static displacement, eigenvalues, and dynamic response analysis within interval parameters (Chen(3); Qiu and Wang(4), (5)). Interval analysis is an issue that has been studied in conjunction to both interval optimization and methods using interval analysis for solving global optimization problems (Hansen(6)). In Hansen’s book, he showed that these methods could identify global optimums and provide bounds on their values. An interval algorithm has been described (Wolfe(7)) that could be used to create bounds for the solutions of a constrained optimization problem. This paper combines a verified interval optimization method with the FEM for designing structures, which is denominated as the Hybrid Interval Genetic Algorithm (HIGA). This algorithm can neglect formulated equations and interval analysis, and while determining the optimum interval parameters. Furthermore, it can also maximize the design scope. In this paper, this algorithm is implemented for both a truss and frame structure. The interval optimizations include the static and dynamic responses of these structures. The results show that the algorithm which combines the IGA with the FEM can determine the feasible interval design parameters of structures with allowable objective errors.
global optimization problem in which the problem functions were assumed to be continuous. The interval analysis method was utilized (Ichida (8)) to determine the maximum global amount of a multimodal multivariable function. Furthermore, the interval analysis was used in the optimization of mechanical structures (Andrzej (9)) and interval optimization for uncertain structures (McWilliam (10); Chen et al. (11)-(14); Jiang et al. (15)).

As the above statements indicate, it is easy to understand the relationship between the system performance and system parameters by using the interval analysis. But the system equations of motion or interval differential formulations sometimes are not easy to be determined, especially with complicated systems. To account for this problem, Shiau et al. (16), (18) presented the Interval Genetic Algorithm (IGA), which they referred to as the genetic algorithm (GA). The GA comprises a search strategy based upon the natural rules of the genetic evolution that was first developed by Holland (19). Several savants described the GA in detail (Holland (20); Goldberg (21); Davis (22); Gen and Cheng (23)). With the IGA, the interval analysis can be neglected in the process of optimization, while also achieving the maximization optimum interval design scope. The description of the IGA is presented in the following sections. In this paper, our technique combines the IGA with the FEM applied to the interval optimization of structures.

2. Description of the IGA

The IGA first appeared at the paper presented by Shiau et al. (16) With this algorithm, the interval analysis can be neglected in this optimization process, and the feasible interval set of design parameters that conforms to the optimization can be determined. The procedure for implementing the IGA is presented in the following simple example. Fig. 1 is the flow chart depicting the IGA. The procedure itself involves five steps:

![Flow chart of the Interval Genetic Algorithm](image)

**Fig. 1 Flow chart of the Interval Genetic Algorithm**

**Step 1:** Calculate the exact optimum design variables for the optimization program.

The IGA in this paper is based on the exact design variables after optimum design. The optimization algorithms of the pre-work in the IGA can be decided upon by means of
engineering experiences that are not described in this paper. The exact optimum design parameters and optimum value of the objective are defined as $\bar{x}_i$ and $f$. In the first step of the IGA, the objective error $E_{error}$ is defined as

$$E_{error} = \left| \bar{f} - f(x_1, x_2, \ldots, x_i) \right| / \bar{f} \times 100\% \quad i = 1, 2, \ldots, n \quad (1)$$

where $x_i$ is the design parameter of the optimization program, $n$ is the number of design parameters, and $f(x)$ is the corresponding objective function. Additionally, the value of the allowable objective error $E_{error}$ is designated. The following is a described two-dimensional example:

Maximization : $f(x, y) = 3(1 - x)^2 e^{-x^2 - (y + 1)^2} - 10 \left( \frac{x}{5} - x^3 - y^5 \right) e^{-x^2 - y^2}$

Subject to $-2 \leq x, y \leq 2$

Fig. 2(a) is the contour map of Eq. (2). Using the optimization approach, the solution to Eq. (2) is obtained; that is, the optimal $x$ and $y$ are -0.010656 and 1.580400, respectively.

**Step 2: Define and calculate the operators of the IGA.**

The interval of the design parameters for the optimization program is defined as

$$\bar{x}_i - \Delta x_i^l \leq x_i \leq \bar{x}_i + \Delta x_i^u \quad i = 1, 2, \ldots, n \quad (3)$$

where $\Delta x_i^u$ and $\Delta x_i^l$ are the upper and lower bounds of $x_i$, respectively, and the design variables of the IGA are defined as those upper bounds and lower bounds.

In this step, the objective function $f(x)$ is iteratively calculated with random design variables $x_i$ in the interval set of Eq. (3). The corresponding objective errors can be
derived in which one operator is defined as
\[
H(x) = \begin{cases} 
10^m & \text{when } E_{\text{error}} \leq E_{\text{error}} \\
-E_{\text{error}} & \text{when } E_{\text{error}} > E_{\text{error}} \\
-10^m & \text{when design constraints are not satisfied}
\end{cases}
\] (4)

where the parameter $m$ is selected as a haphazard positive constant that allows the function $H(x)$ with a feasible objective error to keep the same value. For the described example, $m$ is selected as 4. Two corresponding operators are defined as
\[
A_{\text{design}} = \prod_{i=1}^{n} p_i (\Delta x_{\text{non},i}^u + \Delta x_{\text{non},i}^l) 
\] (5)
\[
G(x) = (\text{Average } [H(x)]) + 10^n \times A_{\text{design}} 
\] (6)

$A_{\text{design}}$ is the non-dimensional design size, and $G(x)$ is the objective function of the IGA, where $n$ is the number of $x_i$; $\Delta x_{\text{non},i}^u$ and $\Delta x_{\text{non},i}^l$ are the non-dimensional bounds and $p_i$ is the weight of each design parameter.

Once the objective error $E_{\text{error}}$ is smaller than the allowable error $E_{\text{error}}$, $H(x)$ becomes large. Therefore, the difference of Eq. (4) among the objective errors is feasible or not will be increased. In addition, during the optimization process, the non-dimensional design size $A_{\text{design}}$ is increased with the maximizing of Eq. (6). Fig. 2(b) is the contour map of $H(x)$ for the described example in $E_{\text{error}} = 5\%$, which shows the behavior of $H(x)$ with the different error $E_{\text{error}}$. The non-dimensional design size of Eq. (5) for the described example is defined as:
\[
A_{\text{design}} = \frac{\Delta x^u + \Delta x^l}{(2 - (-2))} \times \frac{\Delta y^u + \Delta y^l}{(2 - (-2))} 
\] (7)

Step 3: Estimation of $H(x)$, generation and population.

Two estimations are evaluated in step 3. First, the objective error $E_{\text{error}}$ of the interval parameters is considered. The purpose of the IGA is to identify the feasible interval design parameters for the optimization problem. Thus, all the objective errors $E_{\text{error}}$ of the optimum interval parameters must be smaller than the allowable error $E_{\text{error}}$. If the average value of all $H(x)$ in step 2 is equal to $10^n$, it is granted that all $E_{\text{error}}$ of the interval parameters are smaller than $E_{\text{error}}$. Second, the pre-set numbers of generation and population are ascertained to see whether they are completed or not.

Step 4: Typical evolutionary rules of Genetic Algorithms.

In this step, the typical GA evolutionary rules are applied. The upper and lower bounds of $x_i$ are chosen as the design variables and $G(x)$ in Eq. (6) is the objective function of the GA in step 4. Successive populations are produced, primarily by the operations of estimation, selection, encoding, crossover, and mutation. The detailed description of the GA can be found in the following references (Holland(19), (20), Goldberg(21), Davis(22), Gen and Cheng(23)). Estimation is the operator that appraises the merit of the objective function in the evolutionary process. The selection operator determines those members of the population that survive to participate in the production of members for the next population. The roulette-wheel is chosen as the selection method in this paper. Also, a binary code-encoding operator is used in such a way that the design variables are coded as a binary string in which each bit represents a gene. Crossover recombines the traits of the selected members in the hope of producing an offspring with a better fitness level than its parents. After crossover, the next process is mutation. Very rarely do mutations yield radically-improved designs with almost all yielding unsuitable configurations. In this study, the single-point bit flip mutation is used, which changes the coded gene value based upon a predetermined mutation probability.
Step 5: Estimation of convergence.

The convergence analysis for the number of random sampling points, generation number and population number are analyzed in the final step of the IGA. Smaller random sampling points, generations and that fail to solve complex systems are possible. If those numbers are not enough, the optimum interval set will have a larger non-dimensional design size than a sufficient one, and the interval set will include some sets in which objective errors $E_{error}$ are infeasible.

<table>
<thead>
<tr>
<th>Allowable Error</th>
<th>Random Number</th>
<th>Interval of $x$</th>
<th>Interval of $y$</th>
<th>$A_{design}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>250</td>
<td>-0.23256 ~ 0.15693</td>
<td>1.48562 ~ 1.72565</td>
<td>5.84308029×10^{-3}</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>-0.17852 ~ 0.14262</td>
<td>1.42255 ~ 1.69255</td>
<td>5.41923750×10^{-3}</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>-0.16074 ~ 0.11901</td>
<td>1.43268 ~ 1.68561</td>
<td>4.42232297×10^{-3}</td>
</tr>
</tbody>
</table>

The convergence analysis for the number of random sampling points for the described example with $E_{error} = 5\%$ is shown in Fig. 2(c) and Table 1. Also shown in Fig. 2(c) are the interval sets with random numbers of 250 and 350, which cover some sets in which $E_{error}$ is larger than 5\%. With this kind of convergence analysis, the number of random sampling points should be selected as 500. The numbers of generation and population are selected as 50 and 100 using the same method. Table 2 shows the results of the interval optimization for this example with $E_{error} = 5\%$ and 10\%. Fig. 2(d) shows that the interval sets of designed allowable objectives can easily be determined.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>Maximum Value</th>
<th>Optimum Design Variables</th>
<th>Allowable Objective Error</th>
<th>Interval Parameter of $x$</th>
<th>Interval Parameter of $y$</th>
<th>$A_{design}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10(x^2-y^2) e^{-x^2-y^2}$</td>
<td>8.116490</td>
<td>$x = -0.010656$ , $y = 1.580400$</td>
<td>5 %</td>
<td>-0.156742 ~ 0.118254</td>
<td>1.445187 ~ 1.684837</td>
<td>4.11892639×10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10 %</td>
<td>-0.247187 ~ 0.213007</td>
<td>1.406981 ~ 1.728185</td>
<td>9.23853404×10^{-3}</td>
</tr>
</tbody>
</table>

3. The technique of HIGA combines the IGA with FEM

In the original IGA, the objective error $E_{error}$ is calculated from the formulated equation. In fact, it is easy to describe and formulate the system equations for an uncomplicated system, but complicated systems sometimes are not easy to achieve. Furthermore, a simplified system equation sometimes is difficult to determine the accurate solutions. To overcome those difficulties, the Finite Element Method (FEM) analysis is applied to calculate the corresponding objective function $f(x)$ with random design variables $x_i$ in the interval set, and the software ANSYS version 10.0 is applied for the FEM analysis in this paper. This hybrid algorithm is denominated as the Hybrid Interval Genetic Algorithm (HIGA). In the HIGA, the FEM analysis is used instead of the middle block in the step 2 of the IGA flow chart. The random design variables $x_i$ are input into the ANSYS for FEM analysis, and the corresponding objective $f(x)$ is the output value of the ANSYS. With this kind of FEM analysis, operators $H(x)$ and $G(x)$ can be accurately and successfully determined.

4. Numerical results and discussion

In this paper, the HIGA is applied to an interval optimization design of structures where both the static response of a truss structure and the dynamic response of a frame are taken...
into consideration.

(i) Example 1: Application in the displacement for truss structure

This considered example consists of the truss structure shown in Fig. 3, which is in reference to an example considered by McWilliam [10]. Where $P$ defines the applied external constant force $20 \, kN$, $A_i$ is the cross-sectional area of the $i$th member, the length $L$ takes a value of $1.0 \, m$, and the Young’s modulus and density for each member are assigned values of $2.1 \times 10^4 \, N/m^2$ and $7800 \, kg/m^3$. The uncertainty exists in the cross-section area of each member. The cross-sectional areas in these members are assigned interval values between $1.0 \times 10^{-3} \, m^2$ and $2.0 \times 10^{-3} \, m^2$.

\[ P \]

\[ A \]

\[ L \]

Fig. 3 A truss structure of example 1

In the preceding part of this example, the optimum design objective is the minimization of the displacements of node 1 and node 2, which is expressed as

\[ \text{Minimisation: Displacement}_{\text{node 1}} + \text{Displacement}_{\text{node 2}} \]

\[ \text{Subject to: } 10 \, \text{cm}^2 \leq A_i \leq 20 \, \text{cm}^2 \quad i = 1, 2, \cdots, 6 \]

Table 3 The interval optimization design for the node displacements of a truss structure in example 1

<table>
<thead>
<tr>
<th>Minimisation Objectives</th>
<th>Displacements of node 1 &amp; node 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowable Objective Error</td>
<td>Optimal Design</td>
</tr>
<tr>
<td></td>
<td>$A_1$</td>
</tr>
<tr>
<td>5%</td>
<td>$20 , \text{cm}^2$</td>
</tr>
</tbody>
</table>

In this paper, the node displacements are calculated by using the FEM analysis. As shown in Table 3, the optimal design of Eq. (8) can be determined with an optimization approach. The optimization algorithm can be decided upon by means of engineering experiences, such as a genetic algorithm, augmented Lagrange multiplier method, goal programming method or others, which are described elsewhere (Vanderpaats [24]). In this paper, the hybrid genetic algorithm (Shiau et al. [17]) is presented. The allowable error in Table 3 is selected as $5\%$, and the non-dimensional design size $A_{\text{design}}$ of Eq. (5) is rewritten as

\[ A_{\text{design}} = \prod_{i=1}^{6} \frac{(\Delta A_i^o + \Delta A_i^l)}{(2.0 \times 10^{-3} - 1.0 \times 10^{-3})} \]

where $\Delta A_i^o$ and $\Delta A_i^l$ are the upper bound and lower bound of $\Delta A_i$, respectively. Table 3
shows the interval optimization results of this example using the HIGA. In the optimum interval cross-section area set, both of the displacements of node 1 and node 2 can keep the objective errors $E_{error}$ under 5%. Not only the interval combination of the allowable error can be determined effectively, but also the corresponding design scope of the feasible interval parameters is at its maximum.

In Table 3, All of the $A_i$ are the same with design side constraint $20 \text{ cm}^2$. For this reason, increasing the cross-section area of each member is one of the best ways to reduce the displacement of any node. Therefore, reducing the structure weight at the same time is another important issue. The optimization design program is rewritten as

Minimization : $W_{structure}$ and Displacements of node 1 & node 2

Subject to : \(10 \text{ cm}^2 \leq A_i \leq 20 \text{ cm}^2 \; i = 1,2,\cdots 6 \) (10)

where $W_{structure}$ is the weight of the structure. For this multi-objective optimization, the weighting objectives method (Charnes and Cooper(25)) is applied to solve this Pareto optimum problem. The values of weight coefficients for objective functions are the same. The optimum design results are shown in Table 4; results show that the reducing truss weight will increase the displacements of node 1 and node 2. Not all of the cross-section areas are close to the upper design side constraints. With the interval optimization, the optimum interval set is effectively determined. For this multi-objective interval optimization, most of the interval ranges of the cross-section areas are reduced except $A_4$. The non-dimensional design size is decreased, which is smaller than that in the preceding part of this example.

<table>
<thead>
<tr>
<th>Table 4 The interval optimization design for the node displacements and system weight of the truss structure in example 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Minimization Objectives</strong></td>
</tr>
<tr>
<td>Allowable Objective Error</td>
</tr>
<tr>
<td>Optimal Design</td>
</tr>
<tr>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_2$</td>
</tr>
<tr>
<td>$A_3$</td>
</tr>
<tr>
<td>$A_4$</td>
</tr>
<tr>
<td>$A_5$</td>
</tr>
<tr>
<td>$A_6$</td>
</tr>
</tbody>
</table>

(ii) Example 2: Application in the dynamic response for frame structure

In example 2, the interval optimization of dynamic response for structure is considered and the HIGA is presented to a frame structure as shown in Fig. 4; the frame structure refers to a similar example considered by Chen and Wu(12)~(14). Suppose the sine excitation $F(t)$ with frequency $\omega = 90 \text{ s}^{-1}$ at node 10 is along the $x$-positive direction, and the amplitude of the load is $3000 \text{ N}$. The Young’s modulus and density for each member are assigned values of $2.0 \times 10^{11} \text{ N/m}^2$ and $7800 \text{ kg/m}^3$, $b_{frame}$ and $h_{frame}$ are the length of each horizontal and vertical member, and all the members have the same cross-section area $A_{frame}$. In this example, the optimization objective is minimization for the steady state vibration amplitude for node 10 in $x$-direction, and the design parameters are $h_{frame}$, $b_{frame}$ and $A_{frame}$.
This optimization problem is expressed as

Minimization: \( W_{\text{structure}} \) and/or \( R_{\text{amp}} \)

Subject to:
\[
0.3 \text{ cm}^2 \leq A_{\text{frame}} \leq 0.6 \text{ cm}^2 \\
1 \text{ cm} \leq h_{\text{frame}} \leq 8 \text{ cm} \\
1 \text{ cm} \leq b_{\text{frame}} \leq 6 \text{ cm}
\]  \hspace{1cm} (11)

where \( W_{\text{structure}} \) is the weight of the structure and \( R_{\text{amp}} \) is the steady state vibration amplitude of node 10 in \( x \)-direction. The non-dimensional design size \( A_{\text{design}} \) is defined as

\[
A_{\text{design}} = \frac{(\Delta A^u_{\text{frame}} + \Delta A^l_{\text{frame}})}{(6.0 - 3.0) \times 10^{-3}} \times \frac{(\Delta h^u_{\text{frame}} + \Delta h^l_{\text{frame}})}{(8.0 - 1.0) \times 10^{-2}} \times \frac{(\Delta b^u_{\text{frame}} + \Delta b^l_{\text{frame}})}{(6.0 - 1.0) \times 10^{-2}}
\]  \hspace{1cm} (12)

where \( \Delta A^u_{\text{frame}}, \Delta A^l_{\text{frame}}, \Delta h^u_{\text{frame}}, \Delta h^l_{\text{frame}}, \Delta b^u_{\text{frame}} \) and \( \Delta b^l_{\text{frame}} \) are the upper bounds and lower bounds of \( A_{\text{frame}}, h_{\text{frame}} \) and \( b_{\text{frame}} \), respectively.

Two parts are comprised in this example; one is the single-objective design of \( R_{\text{amp}} \), and the other is the multi-objective design of \( W_{\text{structure}} \) and \( R_{\text{amp}} \). The values of weight coefficients for objective functions are the same. The optimal \( x \)-direction steady state response of node 10 can be calculated by using the FEM analysis, and Fig. 5 shows a response from 0.35 sec to 0.5 sec after the single-objective optimization. The results of the single-objective interval design and multi-objective interval design are separately shown in Table 5 and Table 6. With the optimum interval sets, the vibration amplitude and frame structure weight can keep 5% error of the optimum amplitude and weight, respectively.
In other words, the HIGA is effective in deriving optimal interval design parameters within the allowable error when minimizing the structure weight and/or node vibration amplitude of a frame structure.

As the results show in the above two examples, it proves that the technique of the HIGA combining the IGA with FEM is not only efficacious in dealing with interval static problems but can also successfully be implemented to handle the interval dynamic problems.

Table 5 The interval optimization design for the dynamic response of the frame structure in example 2

<table>
<thead>
<tr>
<th>Minimization Objectives</th>
<th>( A_{\text{frame}} )</th>
<th>Interval parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-direction amplitude of node 10 (cm)</td>
<td>0.58582531 ( \text{cm}^2 )</td>
<td>0.5788452 \text{ cm} ( \sim ) 0.5958453 \text{ cm}</td>
</tr>
<tr>
<td>h_{\text{frame}}</td>
<td>1.01567150 \text{ cm}</td>
<td>0.9711117 \text{ cm} ( \sim ) 1.0439518 \text{ cm}</td>
</tr>
<tr>
<td>b_{\text{frame}}</td>
<td>2.38367935 \text{ cm}</td>
<td>2.2566997 \text{ cm} ( \sim ) 2.5262136 \text{ cm}</td>
</tr>
</tbody>
</table>

Table 6 The interval optimization design for the dynamic response and system weight of the frame structure in example 2

<table>
<thead>
<tr>
<th>Minimization Objectives</th>
<th>Frame weight and x-direction amplitude of node 10 (mm)</th>
<th>Interval parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_{\text{design}}</td>
<td>0.15173020 \text{ mm}</td>
<td>0.15173020 \text{ mm} ( \sim ) 0.15931671 \text{ mm}</td>
</tr>
<tr>
<td>( A_{\text{design}} )</td>
<td>3.178438984834045 ( \times ) 10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusion

To overcome the difficulties inherent in the manufacturing of exact optimum design variables with an optimum design, the interval optimization scheme is proposed in engineering applications. In former interval optimizations, interval analysis was necessary and important; however the formulated system equations and the interval differential formulations sometimes are not easily determined, especially with complicated systems. The Interval Genetic Algorithm (IGA) was presented to counteract those problems. Furthermore, the paper combines the IGA with the Finite Element Method (FEM) and nominates it as the Hybrid Interval Genetic Algorithm (HIGA). This hybrid algorithm can exclude equation formulations and interval analysis, and determines the optimum interval parameters. Moreover, it can also maximize the design scope.

In this paper, the software ANSYS version 10.0 is applied for the FEM analysis and the HIGA is implemented with the interval optimization design. Two simple interval optimizations for structures are presented to prove this application. Results show that the HIGA can deal with interval structure static response problems. It also can be implemented for interval dynamic response problems of structures.

References

(2) Moore, R.E., Methods and applications of interval analysis, Philadelphia, SIAM, (1979).


