A Method of Output Feedback Adaptive Nonstationary Control* (Its Application to Positioning Control of Vibration Systems)

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Abstract

One of the authors had previously proposed an adaptive nonstationary control (ANSC) method and described its applications for the positioning control of vibration systems. In that control method, the time-varying feedback gains were obtained by online computations of a time-varying Riccati equation taking the parameter variations of the controlled object into account. However, the previous paper discussed the state feedback control method. The improvement of this method to enable output feedback control is one of important issues for actual applications. Generally, the output feedback controller of the optimal control includes a Kalman filter as the observer. However, the use of the Kalman filter for the ANSC method implies a heavy computational load in actual implementations because the Kalman filter is designed by another time-varying Riccati equation. Therefore, this paper proposes a practical output feedback control method for ANSC that does not include Kalman filter. The proposed method corresponds to a two-degree-of-freedom control system; the system consists of an online simulator and a sliding mode controller. This structure enables us to reduce the computational load. The effectiveness of the method is verified theoretically as well as experimentally.

Key Words: Motion Control, Positioning, Vibration Control, Adaptive Control, Nonstationary Control, Output Feedback, Time-Varying Riccati Equation

1. Introduction

Positioning is one of the most important control techniques for mechanical structures including conveyance devices, robots, information precision machinery and so on. Recently, many mechanical structures have become light and/or large scale structures due to the expansion of their applications. At the same time, the parameters of control performance such as traveling period (seeking time) and settling accuracy have to satisfy rigorous specifications. Therefore, we cannot ignore the influences of vibration modes in controlled objects in many situations. Because of these considerations, positioning control problems

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for vibration systems have been increased^{(1), (2)}.

Nonstationary Optimal Regulator (NOR) is a well-known and effective control method used for the positioning of vibration systems^{(3), (4)}. NOR is obtained by the finite-horizon LQ optimal regulator problem, which is based on the vibration system model of a controlled object. NOR has a superior transient performance and produces vibrationless trajectories by a simple one-degree-of-freedom control system structure. In NOR designs, (i) a time-varying Riccati equation is solved by reversed time calculations and (ii) the responses of a controlled object are solved by progressive time calculations. If the responses in (ii) do not satisfy some necessary specifications, we tune the time-varying weights again and repeat (i). Both (i) and (ii) cannot be carried out simultaneously.

Recently, one of the authors proposed adaptive nonstationary control (ANSC) methods utilizing time-varying Riccati equations^{(5)–(7)}. In these methods, the Riccati equation is solved by progressive time calculations. Therefore, these methods are different from the optimal control method based on the LQ criterion function. ANSC cannot optimize the criterion function. However, the responses in these methods become similar to the NOR responses. The time-varying feedback gains are obtained by real-time computations in actual implementations. Therefore, ANS controllers can vary adaptively taking time-varying parameters of a controlled object into account. Reference (5) shows an application of the ANSC regulator to a positioning control problem of a vibration system.

However, Refs. (5)–(7) deal with only the state feedback control problems. The effectiveness of such a control technique for output feedback control problems has not been addressed yet. The output feedback NOR utilizes an observer based on the Kalman filter in the same manner as general optimal control problems⁽⁸⁾. However, the use of the Kalman filter for ANSC requires the online, real-time computation of another time-varying Riccati equation. Because of the computational load imposed by such computations, this technique may not be practical for many applications. Therefore, this paper proposes another practical output feedback ANSC method. The proposed method corresponds to a two-degree-of-freedom control system; the system consists of an online simulator and a sliding mode controller. This structure enables us to reduce the computational load.

For the application of the proposed method, in this paper, we choose the same control problem as that described in Ref. (5). The effectiveness of the proposed method is verified by numerical calculations and experiments.

2. Adaptive Nonstationary Control Method

2.1 A Nonstationary Control Method Using the Solutions of Time-Varying Riccati Equations

This subsection summarizes the NOR⁽⁴⁾ and the first author's previous method⁽⁵⁾. Here, we consider that a controlled object is described by the following single-input time-varying system state equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{b}(t)u(t) \quad (1)$$

where $\mathbf{x}(t)$ and $u(t)$ are the state variables and the control input, respectively. This model includes the vibration characteristics of the controlled object. NOR is equivalent to the LQ optimal regulator based on the following finite-horizon (time: $t \in [0, t_f]$) criterion function:

$$J = \int_0^{t_f} [\mathbf{x}^T(t)\mathbf{Q}(t)\mathbf{x}(t) + r(t)u^2(t)]dt. \quad (2)$$

The optimal control input is obtained by the following state feedback formula:

$$u(t) = -f_b(t)x(t), \quad f_b(t) = r^{-1}(t)b^T(t)P(t) \quad (3)$$

where $f_b(t)$ are time-varying feedback gains. $P(t)$ are the solutions of the time-varying Riccati equation:

$$-\dot{P}(t) = P(t)A(t) + A^T(t)P(t) - P(t)b(t)r^{-1}(t)b^T(t)P(t) + Q(t). \quad (4)$$

In NOR designs, –(i) the time-varying Riccati equation (4) is solved by reversed time ($t_f \rightarrow 0$) calculations under appropriate time-varying weights $Q(t)$ and $r(t)$ and the boundary condition (e.g., $P(t_f) = \mathbf{0}$) and (ii) the responses of the controlled object are solved by progressive time ($0 \rightarrow t_f$) calculations. The controlled object model should be definite during $t \in [0, t_f]$ in (i). If the responses in (ii) do not satisfy certain specifications, we tune the time-varying weights again and repeat (i). The calculations of (i) and (ii) can not be carried out simultaneously.

The first author had previously proposed an efficient time-varying gain-type controller design method, different from NOR, that uses the solutions of novel time-varying Riccati equations⁽⁵⁾. In this method, the following time-varying Riccati equation is utilized instead of Eq. (4):

$$\frac{dP'(\tau)}{d\tau} = P'(\tau)A(\tau) + A^T(\tau)P'(\tau) - P'(\tau)b(\tau)r'^{-1}(\tau)b^T(\tau)P'(\tau) + Q'(\tau) \quad (5)$$

where $Q'(\tau) = Q(t = \tau)$ and $r'(\tau) = r(t = \tau)$. The time-varying weights are given to Eq. (5) following time progress. The control input is obtained by

$$u(t) = -f'_b(t)x(t), \quad f'_b(t) = r'^{-1}(\tau = t)b^T(\tau = t)P'(\tau = t) \quad (6)$$

where $f'_b(t)$ are time-varying feedback gains. The combination of Eqs. (1) and (5) requires progressive time calculations only. The responses in this method become similar to those of NOR. The theory of this method is quite different from that of NOR, which is based on the criterion function (2). However, in actual applications, it is important for controller designers to easily obtain useful trajectories that are similar to the optimal control results.

2.2 Use of Discrete-Time Riccati Equations

In order to reduce load of computers, this subsection introduces the use of following discrete-time Riccati equation instead of Eq. (5):

$$P'_d(n+1) = A_d^T(n)P'_d(n)A_d(n) - A_d^T(n)P'_d(n)b_d(n) \cdot [r'_d(n) + b_d^T(n)P'_d(n)b_d(n)]^{-1} b_d^T(n)P'_d(n)A_d(n) + Q'_d(n) \quad (7)$$

where $A_d(n)$ and $b_d(n)$ are discrete-time matrices of $A(t)$ and $b(t)$, respectively, obtained by the sampling period of actual implementation. n ($n = 0, 1, 2, \dots$) denotes the sample number of discrete-time. For convenience, in this paper, there are equations including both continuous-time t and the discrete-time sample n . Let us consider that each

of these n values corresponds to an appropriate t . The control input is obtained as follows:

$$u(t) = -f'_{bd}(n)x(t), \quad f'_{bd}(n) = [r'_d(n) + b_d^T(n)P'_d(n)b_d(n)]^{-1} b_d^T(n)P'_d(n)A_d(n). \quad (8)$$

2.3 Adaptive Nonstationary Control (ANSC) Method

The use of Eqs. (7) and (8) implies that the time-varying feedback gains can be obtained by real-time computations in actual implementations. This feature is convenient for the realization of some kind of adaptive control and realizes the applications of the nonstationary control to control object models which are not definite at starting time. Reference (5) assumes that the parameters possessing some information of the controlled object can be detected online/real-time. The real-time identified controlled object model is described as follows:

$$\dot{x}_m(t) = A_m(t)x_m(t) + b_m(t)u_m(t). \quad (9)$$

The computational load for the identification in this paper is the same as that of the conventional method in Ref. (5). The Riccati equation corresponding to model (9) is

$$P'_{md}(n+1) = A_{md}^T(n)P'_{md}(n)A_{md}(n) - A_{md}^T(n)P'_{md}(n)b_{md}(n) \cdot [r'_{md}(n) + b_{md}^T(n)P'_{md}(n)b_{md}(n)]^{-1} b_{md}^T(n)P'_{md}(n)A_{md}(n) + Q'_{md}(n) \quad (10)$$

where $A_{md}(n)$ and $b_{md}(n)$ are discrete-time matrices of $A_m(t)$ and $b_m(t)$, respectively obtained by the sampling period of actual implementation. The control input is obtained by

$$u_m(t) = -f'_{bmd}(n)x_m(t) \\ f'_{bmd}(n) = [r'_{md}(n) + b_{md}^T(n)P'_{md}(n)b_{md}(n)]^{-1} b_{md}^T(n)P'_{md}(n)A_{md}(n). \quad (11)$$

The method utilizing Eqs. (9)–(11) is termed “Adaptive Nonstationary Control method (ANSC method) for the state feedback control” in Ref. (5).

3. Realization of Output Feedback Control

3.1 Output Feedback Control Based on a Two-Degree-of-Freedom Control System

Applications of the ANSC method to output feedback control problems have not been

Fig. 1 Two-degree-of-freedom control system

discussed. Reference (5) deals with the positioning control problem of a flexible structure on an X-Y table. However, it is difficult to detect the relative displacement between the vibrating tip and the table as shown in Ref. (5) in many actual structures. Generally, the vibration status is detected by the absolute acceleration measurement by an acceleration pick-up. In the same manner as in general optimal control problems, an observer based on the Kalman filter and the acceleration measurement may be utilized to estimate the relative variables⁽⁸⁾. However, the use of Kalman filter in ANSC requires the online/real-time computations of another time-varying Riccati equation in addition to Eq. (10). When we consider the computational load, this method is not practical in many applications. Therefore, in this paper we propose another practical output feedback ANSC method.

The method proposed in this paper corresponds to a two-degree-of-freedom (2-DOF) control system that consists of an online simulator and a sliding mode controller as shown in Fig. 1. We assume that Eq. (9) represents the nominal system including the estimated parameters. Equations (9)–(11) are computed online as shown in the left part of Fig. 1. This is termed the “online simulator” in this paper. In actual implementations, the following discrete-time state equation is used instead of Eq. (9):

$$\mathbf{x}_{md}(n+1) = \mathbf{A}_{md}(n)\mathbf{x}_{md}(n) + \mathbf{b}_{md}(n)u_{md}(n). \quad (12)$$

The control input can be written as:

$$u_{md}(n) = -\mathbf{f}'_{bmd}(n)\mathbf{x}_{md}(n) \quad (13)$$

instead of Eq. (11). Some state variables in Eq. (12) which are of the same kind as actual detectable variables are adopted as the references for the 2-DOF system. At the same time, simulator input (13) is utilized as the feedforward control input. Here, the references and the feedforward control input are designed by taking characteristics such as vibration properties into account. Then, a simple feedback controller, namely, a sliding mode controller (SMC) is used only for reducing the influence of disturbances such as Coulomb friction in the driving system. SMC is one of the most popular selections for control problems with nonlinear disturbances. The design method for SMC is described in the following subsection. The SMC supplies the feedback control input based on the error between the references and the corresponding actual detected variables. The real control input is the sum of the feedforward control input and the feedback control input. If the detected variables follow the references satisfactorily, the output feedback ANSC is realized without the observer (Kalman filter). The principal idea of the structure in Fig. 1 is similar to that in the first author's previous study in Ref. (9). However, the references and the feedforward control input are generated online in Fig. 1 different from the case in Ref. (9).

3.2 Design of Sliding Mode Controller

In the same manner as in Ref. (9), we obtain the following simple model of the controlled object for the SMC design:

$$\dot{\mathbf{x}}_{dim}(t) = \mathbf{A}_{dim}\mathbf{x}_{dim}(t) + \mathbf{b}_{dim}u_{dim}(t) + \mathbf{d}_{dim}f_r(t). \quad (14)$$

It is assumed that the disturbance $f_r(t)$ is treated as a continuous function and the matching condition⁽¹⁰⁾ is satisfied. The linear model for designing SMC becomes

$$\dot{\mathbf{x}}_{dim}(t) = \mathbf{A}_{dim}\mathbf{x}_{dim}(t) + \mathbf{b}_{dim}u_{dim}(t) \quad (15)$$

$$\sigma(t) = \mathbf{S}\mathbf{x}_{dim}(t) \quad (16)$$

where $\sigma(t)$ is a switching function. This study utilizes the eventual sliding mode control method⁽¹⁰⁾. The control input is described by both the linear input term and the nonlinear input term as follows:

$$u_{smc}(t) = -[Sb_{dim}]^{-1} SA_{dim}x_{dim}(t) - k(x_{dim}, t) \frac{\sigma(t)}{\|\sigma(t)\| + \delta} \quad (17)$$

where $\delta > 0$ is a small value used for eliminating chattering. S is determined by the following Riccati equation:

$$P_{smc}A_{\varepsilon} + A_{\varepsilon}^T P_{smc} - P_{smc}b_{dim}b_{dim}^T P_{smc} + Q_{smc} = 0 \quad (18)$$

$$S = b_{dim}^T P_{smc} \quad (19)$$

where $A_{\varepsilon} = A_{dim} + \varepsilon I$. $\varepsilon > 0$ is selected such that the sliding mode control system guarantees sufficient stability.

The actual nonlinear control input in Eq. (17) varies its sign frequently during periods of high acceleration and deceleration periods. Therefore, in this study, we apply time-varying $\delta(t)$ to Eq. (17) and enlarge it during such a period. Figure 2 shows $\delta(t)$ in the case of the following application. Equation (17) is rewritten as follows:

$$u_{smc}(t) = -[Sb_{dim}]^{-1} SA_{dim}x_{dim}(t) - k(x_{dim}, t) \frac{\sigma(t)}{\|\sigma(t)\| + \delta(t)}, \quad \delta(t) > 0. \quad (20)$$

In the following application, the elements of Eqs. (15), (16), (18), (19) and (20) are not discretized. These equations as the continuous-time system are used for the numerical calculations and experiments.

Fig. 2 Time-varying parameter

3.3 Controlled Object Example

In this study, we select the controlled object in Ref. (5) (Fig. 3) as an example. This object consists of an X-Y table with a flexible structure installed on it. The table is equipped with a driving system consisting of DC servomotors, reduction gears and timing belts. The motion along only a single axis (X-axis in Fig. 3) is considered in this study. When the table is settled at some point, the natural frequency of the flexible structure is 4.55 Hz. The parameters of the controlled object are summarized in Table 1. In the experiments, the angular displacement of the motor is detected by an encoder. The vibration status of the flexible structure can be detected by both an acceleration pick-up installed on the structure and a gap sensor installed on the table. However, the information obtained by the gap sensor is not used for control in this study. Control computations for the experiments are carried

out by a digital signal processor (DSP, TMS320C31). The sampling period is 1.0 ms.

If the flexible structure can be modeled by a one-degree-of-freedom vibration system, we can obtain the controlled object model consisting of the vibrating part and the base part as shown in Fig. 4. The state equation for this model derived in the same manner as in Ref. (11), is as follows:

$$\begin{aligned}\dot{\mathbf{x}}_p(t) &= \mathbf{A}_p \mathbf{x}_p(t) + \mathbf{b}_p u_p(t) + \mathbf{d}_p f_r \\ \mathbf{x}_p(t) &= [x_s(t) \quad \dot{x}_s(t) \quad \theta_m(t) \quad \dot{\theta}_m(t) \quad i(t)]^T\end{aligned}\quad (21)$$

Fig. 3 Controlled object

Table 1 Parameters of the controlled object model

m_s	Equivalent mass of the vibrating part	4.50 kg
k_s	Equivalent spring constant of the vibrating part	3.68×10^3 N/m
c_s	Equivalent damping coefficient of the vibrating pt.	1.28 N·s/m
m_b	Equivalent mass of the base part	22.1 kg
J_b	Moment of inertia (Base side gear and the pulley)	3.57×10^{-5} kg·m ²
J_m	Moment of inertia (Motor and the motor side gear)	8.33×10^{-4} kg·m ²
r	Radius of the pulley	0.0159 m
α	Gear ratio ($= r_b/r_m$)	11.0
L	Inductance of the motor	2.0 mH
R	Resistance of the motor	1.0 Ω
K_T	Torque constant of the motor	0.215 N·m/A
K_E	Back electromotive force constant of the motor	0.215 V·s/rad
B	Braking constant of the motor	1.69×10^{-4} N·m·s/rad
f_r	Coulomb friction force of the driving system	131.4 N

Fig. 4 Controlled object model

where $x_s(t)$, $\theta_m(t)$ and $i(t)$ are the absolute displacement of the tip of the vibrating part [m], the angular displacement of the motor [rad] and the current in the motor [A], respectively. The control input $u_p(t)$ is the voltage applied to the motor $e(t)$ [V]. In Eq. (21), the influence of Coulomb friction in the driving system is taken into account by using sign functions in the same manner as in Ref. (11). The details of the modeling are given in the original paper in Japanese⁽¹²⁾.

If the influence of the motor's inductance is sufficiently small, the current of the motor $i(t)$ can be removed from $x_p(t)$. Generally, the influence of damping is very small in controlled objects like the structure in Fig. 3. Moreover, if we neglect the force from the vibrating part imparted to the base part and the Coulomb friction, a simple model is obtained as follows⁽⁵⁾:

$$\begin{aligned} \dot{x}_{pc}(t) &= A_{pc} x_{pc}(t) + b_{pc} u_{pc}(t) \\ x_{pc}(t) &= [x_s(t) \quad \dot{x}_s(t) \quad \theta_m(t) \quad \dot{\theta}_m(t)]^T, \quad u_{pc}(t) = e(t) \end{aligned} \quad (22)$$

$$A_{pc} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_s^2 & 0 & \omega_s^2 \frac{r}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{\alpha K_T K_E}{r I_l R} \end{bmatrix}, \quad b_{pc} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\alpha K_T}{r I_l R} \end{bmatrix}$$

where ω_s , α and I_l are $\omega_s = \sqrt{\frac{k_s}{m_s}}$, $\alpha \equiv \frac{r_b}{r_m}$ and $I_l \equiv m_b \frac{r}{\alpha} + 2J_m \frac{\alpha}{r} + J_b \frac{1}{\alpha r}$, respectively. The model (22) is utilized for the controller design in the following section.

4. Numerical Calculations and Experiments

4.1 The Control Problem Presented in This Paper

In this paper, we discuss the positioning and residual vibration reduction of the flexible structure in Fig. 3. The problem setting is almost the same as that in Ref. (5). The vibration during access motion is permitted in our study. Then, we can estimate the parameters of the vibration characteristics during access motion, and use the estimated data to reduce the residual vibration. From the viewpoint of applications, such a problem setting is similar to a positioning and residual vibration reduction problem with unknown mass. In this section, a very simple method for natural frequency estimation is applied. As mentioned in the previous section, the influence of damping of the flexible structure can be neglected in this study. Then, the natural frequency is estimated by the “peak-to-peak” period measurement

Fig. 5 Estimation of the natural frequency

of the tip acceleration as shown in Fig. 5. Other online estimation methods (e.g., in Ref. (13)), as well as this simple estimation technique, can be applied to this control problem. If the ANSC method is applied to the model (22) when the weights on vibration control are zeros, the controller does not reduce vibration. When the weights increase, the vibration reduction begins⁽⁵⁾. Therefore, we can exactly estimate the natural frequency when the weights are zeros.

4.2 Numerical Calculations

This subsection presents examples of numerical calculations. Let the access distance be set at 0.1 m under the access period of about 3 s. The models (22) and (21) are applied to the controller design and the response calculations, respectively. We prepare two kinds of weights – the time-varying weights on the angular displacement of the motor (for positioning) $[q_{pos}(t)]^2$ and the relative velocity of the tip of the structure (for residual vibration reduction) $[q_{vib}(t)]^2$. These weights are tuned by taking account of the above problem setting and the constraints of the actual experimental system. As a result, $\mathbf{Q}'_{md}(t)$, $q_{pos}(t)$ and $q_{vib}(t)$ are set as follows:

$$\mathbf{Q}'_{md}(t) = \mathbf{C}_{obj}^T(t) \mathbf{C}_{obj}(t) \quad (23)$$

$$\mathbf{C}_{obj}(t) = \begin{bmatrix} 0 & 0 & q_{pos}(t) & 0 \\ 0 & q_{vib}(t) & 0 & -(r/\alpha)q_{vib}(t) \end{bmatrix}$$

$$q_{pos}(t) = (r/\alpha) \left[1.0e5 + 2.0e7 / \{1.0 + \exp(-10.0(t-3.0))\} \right]^{\frac{1}{2}}$$

$$q_{vib}(t) = 0.5 \left[5.0e3 / \{1.0 + \exp(-10.0(t-2.8))\} \right]^{\frac{1}{2}}.$$

The weight on the control input $r'_{md}(t)$ is set to 1.0 [$r'_{md}(t)=1.0$] in this section. $q_{pos}(t)$ and $q_{vib}(t)$ are shown in Fig. 6 (a) and (b), respectively. The time-varying feedback gains obtained by these weights are shown in Figs. 6 (c)–(f). Here, we calculate Eq. (10) under $\mathbf{P}'_{md}(0) = \mathbf{0}$. $\mathbf{A}_{md}(n)$ and $\mathbf{b}_{md}(n)$ in Eq. (10) are determined by the result of the natural frequency estimation in the following response calculation (Fig. 8). The gains for the motor variables (Figs. 6 (e) and (f)) increase from zeros by following the weight $q_{pos}(t)$ (Fig. 6 (a)). On the other hand, the gains for vibration reduction (Figs. 6 (c) and (d)) are almost zero until about 2.0 to 2.3 s. Therefore, vibration reduction is not carried out during the access motion.

For the SMC design, a simple double-integrator system model is applied for Eq. (14)⁽⁹⁾. The detecting variables for the feedback control are only the angular displacement and velocity of the motor. Different from the previous study⁽⁵⁾, the relative displacement and velocity of the tip are not utilized for feedback control. The elements of Eq. (14) become:

$$\mathbf{x}_{dim}(t) = \begin{bmatrix} \theta_m(t) - \theta_{mref}(t) \\ \dot{\theta}_m(t) - \dot{\theta}_{mref}(t) \end{bmatrix}, \quad \mathbf{A}_{dim} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{b}_{dim} = \begin{bmatrix} 0 \\ (\alpha K_T)/(r I_{ld} R) \end{bmatrix}, \quad \mathbf{d}_{dim} = \begin{bmatrix} 0 & -\frac{1}{I_{ld}} \end{bmatrix}^T,$$

$$I_{ld} \equiv (m_s + m_b) \frac{r}{\alpha} + 2J_m \frac{\alpha}{r} + J_b \frac{1}{\alpha r}.$$

$\theta_{mref}(t)$ and $\dot{\theta}_{mref}(t)$ are the reference angular displacement and velocity, respectively. The design parameters of the SMC design are tuned as follows:

Fig. 6 Time-varying weights and feedback gains

Fig. 7 Numerical calculation result (proposed method)

Fig. 8 Natural frequency estimation

Fig. 9 Numerical calculation result (the controller based on a time-invariant natural frequency [2.0 Hz])

$$k(\mathbf{x}_{dim}, t) = 4.3(\mathbf{S}\mathbf{b}_{dim} > 0) \text{ or } -4.3(\mathbf{S}\mathbf{b}_{dim} < 0), \quad \mathbf{Q}_{smc} = \text{diag}[1.0 \quad 1.0 \times 10^{-4}], \quad \varepsilon = 0.001.$$

$\delta(t)$ in Fig. 2 is also utilized.

The resulting responses are shown in Fig. 7. The result of the estimation of the natural frequency for this case is shown in Fig. 8. The estimation stops if the differences in the estimated frequency are smaller than 0.05 Hz during three vibration periods. As a reference, Fig. 9 shows the result from the case in which the nonstationary controller, based on the time-invariant model whose natural frequency is 2.0 Hz, is applied. From the comparison of Figs. 7 and 9, we can understand that the proposed method effectively reduces the residual vibration. These results are similar to those of the state feedback control case in Ref. (5). Thus, the effectiveness of the method is verified.

4.3 Experiments

Experimental results of the proposed method corresponding to the numerical calculation, are shown in Figs. 10 and 11. In the same manner as for the numerical calculations, Fig. 12 also shows the result of the case in which the nonstationary controller based on the 2.0 Hz time-invariant model is applied. In the experiments, taking the influence of noises on the acceleration pick-up into account, the natural frequency is estimated by the acceleration data sampled every 15.0 ms. Discrete-time matrices $\mathbf{A}_{md}(n)$ and $\mathbf{b}_{md}(n)$ are obtained by using the zero-order holder and Taylor approximation of matrix exponential functions. Terms whose orders are higher than the square of sampling period are truncated.

The experimental results are similar to the results of the numerical calculations. Therefore, the effectiveness of the proposed method has also been verified experimentally.

Fig. 10 Experimental result (proposed method)

Fig. 11 Natural frequency estimation (experiment, estimated result: 4.46Hz)

Fig. 12 Experimental result (the controller based on a time-invariant natural frequency [2.0 Hz])

5. Conclusions

In this paper we introduce the state feedback adaptive nonstationary control (ANSC) methods proposed by one of the authors and discuss a novel ANSC method for the output feedback control problems. Instead of using a Kalman filter based observer, in this paper we adopt a two-degree-of-freedom control system which consists of an online simulator and a sliding mode controller. This structure enables us to reduce the computational load in comparison with the use of the Kalman filter.

In this paper, we selected the same positioning control problem of a vibration system as in Ref. (5) for the application and verification of the proposed method. The effectiveness of the proposed method was verified by numerical calculations and experiments. The original paper (12) (in Japanese) shows an example of a computational load comparison between the proposed method and the Kalman-filter-based method. It indicates that the computational time of the proposed method is reduced to about 60 % of that for the other method. The proposed method corresponds to a practical output feedback ANSC method.

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