Hybrid Control of an Euler-Bernoulli Beam Using Direct Velocity Feedback and Wave-Filter-Based Active Wave Control*

Hiroyuki IWAMOTO**, Nobuo TANAKA** and Simon G. HILL**

**Department of Aerospace Engineering
Tokyo Metropolitan University,
6-6 Asahigaoka, Hino-city, Tokyo, 191-0065 Japan
E-mail: hiwamoto@sd.tmu.ac.jp

Abstract
Active wave control strategy enables the inactivation of vibration mode, which is valid for suppressing the vibration of a distributed parameter structure. However, when active wave control is applied, new vibration modes are produced in the uncontrolled region. To overcome this problem, this paper proposes a novel control strategy based on a hybrid combination of direct velocity feedback (DVFB) and active wave control. The two control methods have complementary qualities; DVFB is for improving the stability, and active wave control is for its unique control effects. First, a transfer matrix method in the Laplace domain is introduced to describe wave propagation phenomena of an Euler-Bernoulli beam. Then the wave filtering method which uses point sensors is presented. Based on the filtering method, the characteristic equation and control laws of the reflected wave absorbing control are derived. Next, the independence of the two control methods in the proposed hybrid control system is investigated by a numerical simulation. This is followed by the discussion of the stability problem of the hybrid control system via a Nyquist diagram method and three types of root loci. Finally, the control effects of the proposed control system are presented, demonstrating the validity of the proposed method.

Key words: Active Wave Control, Wave Filter, Direct Velocity Feedback, Hybrid Control

1. Introduction
Vibration minimization and suppression of flexible structures such as large space structures is a practical and thus important topic in mechanical engineering. Typically large space structures are constructed of materials with a minimum amount of damping and the overall structure is very flexible due to its thin and large components: as a result the structure is prone to vibration beyond safe limits. The vibration may have significant effects on missions in space including fatal damage to the structure itself and/or contamination of any measured data such that it is meaningless. Although there have been a number of papers on active control of flexible structures, the mainstream methods are based on modal expansion theorem1)(4). Such methods can explicitly deal with system eigenvalues, so that their damping ratios and modal frequencies can be assigned. Furthermore, since the mode shapes corresponding to the eigenvalues are dependent on the global properties, it can reduce the vibration of the whole structure. However, the conventional modal approaches cannot provide a sufficiently accurate model over a modally rich frequency range since

*Received 7 Jan., 2010 (No. 10-0005)
[DOI: 10.1299/jsdd.4.440]
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natural frequencies and mode shapes are sensitive to the inevitable modeling errors. Even if robust control methodologies such as H-infinity control\(^{(5)}\)(\(^{(6)}\)) are applied to optimally truncated models\(^{(7)}\), the controller will be of large order, even for the case where the number of target vibration modes is small. To overcome this difficulty, an active wave control method\(^{(8)}\)(\(^{(14)}\)) has been developed in the last two decades. Since active wave control is based on a hypothesis as regards how the vibration mode is to be excited, it enables the inactivation of all vibration modes, which is valid for suppressing the vibration of a distributed parameter structure. The excitation mechanism of vibration modes is hypothesized as follows:

1. A disturbance force excites a target structure.
2. Excitation energy propagates through a structure as travelling waves.
3. When the travelling waves reach the structural boundary, reflected waves as well as near-field decaying from the boundary are produced.
4. A standing wave is formed by the interference of the travelling and reflected waves.
5. If the shape of the standing wave is tuned to the mode shape, then its structural vibration mode is excited.

It follows from the hypothesis described above that the standing wave is the direct cause of modal excitation. Therefore, if the travelling or reflected waves that form the standing wave are eliminated with any control input, no standing wave is produced, and hence the vibration mode becomes inactive. This is the concept of an active wave control method. This method is especially suitable for the case where the controlled object is a distributed parameter structure unlike the modal control method since it aims to absorb or cancel only one incident wave out of the four wave components; positive and negative travelling waves and near-fields decaying from each boundary.

However, conventional feedback wave control has two drawbacks. First, wave controllers are expressed as functions of disturbance location. This property is undesirable since the advantage of feedback control over feedforward is that no information on disturbance is required for building a control strategy. This problem was, however, has been resolved in Iwamoto and Tanaka by introducing a wave filtering method\(^{(15)}\)(\(^{(17)}\)). It was also presented that the design of the wave-filter-based feedback wave controller is not affected by control location and boundary condition. The second drawback of conventional feedback wave control is that vibration modes are newly produced in the uncontrolled region, and hence the stability margin is close to zero. This is because the active wave control method aims not at augmenting the damping but at cancelling a wave propagating in a part of a structure. To overcome this problem, this paper proposes the hybrid combination of direct velocity feedback (DVFB)\(^{(18)}\) and active wave control. A schematic diagram of the hybrid control system using DVFB and wave-filter-based active wave control is shown in Fig. 1.

![Conceptual diagram of the hybrid control system using DVFB and wave-filter-based active wave control](image-url)
control system is illustrated in Fig. 1. The two control methods have complementary qualities, i.e., DVFB is for improving the stability and active wave control is for its unique control effects.

This paper is ordered as follows. Firstly, a transfer matrix method in the Laplace domain is introduced to describe wave propagation phenomena of an Euler-Bernoulli beam. Then, wave filtering method using point sensors is presented. Based on the filtering method, the characteristic equation and control law of the reflected wave absorbing control (RWAC) are derived. Next, the independence of the two control methods in the proposed hybrid control system is investigated by a numerical simulation. This is followed by the discussion of the stability problem of the hybrid control system via a Nyquist diagram method and three types of root loci. It is found that higher stability margin is realized compared to the conventional active wave control method. Finally, the control effects of the proposed control system are presented, demonstrating the validity of the proposed hybrid control method.

2. Design of the hybrid control system

2.1. Transfer matrix for a flexible beam in the Laplace domain

Since pure flexural wave propagation is seldom observed in a real structure, a solution of the equation of a distributed parameter system is generally described by superposition of vibration modes. This work, however, places an emphasis on wave propagation, employing a progressive wave solution. Based on the assumption that the effects of shear deformation and rotary inertia can be ignored, a displacement of an Euler-Bernoulli beam, \( \xi(x,t) \) satisfies the equation,

\[
E I \frac{\partial^4 \xi(x,t)}{\partial x^4} + \rho A \frac{\partial^2 \xi(x,t)}{\partial t^2} = f(x,t)
\]  

where \( E, I, A, \rho \) and \( f(x,t) \) are Young’s modulus, area moment of inertia, cross-sectional area, mass density and applied force per unit length, respectively. Assuming that initial conditions are zero, the Laplace transform of the homogeneous equation of Eq. (1) is given by

\[
\frac{\partial^4 \tilde{\xi}(x,s)}{\partial x^4} - \kappa^4 s^2 \tilde{\xi}(x,s) = 0
\]  

where \( s \) is the complex variable in the Laplace domain, and \( \kappa \) is defined as

\[
\kappa = -\frac{\rho A}{EI}.
\]  

A progressive solution to Eq. (2) is described by

\[
-\tilde{\xi}(x,s) = c_1 e^{-\kappa x} + c_2 e^{i\kappa x} + c_3 e^{-i\kappa x} + c_4 e^{i\kappa x}
\]

where \( j \) is the imaginary operator and \( c_1, c_2, c_3 \) and \( c_4 \) are the positive travelling wave amplitude, the near-field amplitude decaying from the left boundary, the negative travelling wave amplitude and the near-field amplitude decaying from the right boundary, respectively. Differentiating the bending displacement, the slope \( \theta(x,s) \), the internal bending moment \( M(x,s) \) and internal shear force \( Q(x,s) \) are obtained, and then the state vector \( z(x,s) \) is defined as

\[
z(x,s) = \begin{pmatrix} -\tilde{\xi}(x,s) & \tilde{\theta}(x,s) & \tilde{M}(x,s)/EI & \tilde{Q}(x,s)/EI \end{pmatrix}
\]

where \( ^T \) denotes the transpose of the expression.

Consider the beam element whose length is \( l \). Defining the left and right end of the element as node \( i-1 \) and \( i \), respectively, the relation between two state vectors at these nodes is described as

\[
z_i = T_{i,i-1}(l)z_{i-1}
\]

where \( T_{i,i-1} \) is the transfer matrix of the state vectors between the node \( i \) and \( i-1 \), and is
defined as

\[
T_{l-1}(l) = \begin{pmatrix}
    t_1 & t_4 & t_3 & t_2 \\
    K^4 s^4 t_2 & t_1 & t_4 & t_3 \\
    K^4 s^4 t_3 & K^4 s^4 t_2 & t_1 & t_4 \\
    K^4 s^4 t_4 & K^4 s^4 t_3 & K^4 s^4 t_2 & t_1
\end{pmatrix}
\]  (7)

where

\[
t_1 = \frac{e^{j \kappa x_1} + e^{j \kappa x_2} + e^{j \kappa x_3} + e^{j \kappa x_4}}{4},
\]  (8)

\[
t_2 = \frac{-j e^{j \kappa x_1} + e^{j \kappa x_2} + e^{j \kappa x_3} + e^{j \kappa x_4}}{4 \kappa^2},
\]  (9)

\[
t_3 = \frac{e^{j \kappa x_1} + e^{j \kappa x_2} - e^{j \kappa x_3} + e^{j \kappa x_4}}{4 \kappa^2},
\]  (10)

\[
t_4 = \frac{je^{-j \kappa x_1} - je^{j \kappa x_2} + e^{j \kappa x_3} + e^{-j \kappa x_4}}{4 \kappa}.
\]  (11)

2.2. Wave filtering method for a flexible beam using four point sensors

Although there are some types of wave filtering methods, the simplest filter consists of four sensors and four sub-filters. Schematic diagram of a wave filter is shown in Fig. 2. The required wave amplitude in this control is that of positive travelling wave at the rightmost sensor point. In this case, the outputs of each point sensor used for the wave filter are described by

\[
-\sqrt{\xi}(x_{sw1}, s) = c_1 e^{-j \kappa x_{sw1}} + c_2 e^{j \kappa x_{sw2}} + c_3 e^{j \kappa x_{sw3}} + c_4 e^{j \kappa x_{sw4}},
\]  (12)

\[
-\sqrt{\xi}(x_{sw2}, s) = c_1 e^{-j \kappa x_{sw1}} + c_2 e^{-j \kappa x_{sw2}} + c_3 e^{j \kappa x_{sw3}} + c_4 e^{j \kappa x_{sw4}},
\]  (13)

\[
-\sqrt{\xi}(x_{sw3}, s) = c_1 e^{-j \kappa x_{sw1}} + c_2 e^{-j \kappa x_{sw2}} + c_3 e^{-j \kappa x_{sw3}} + c_4 e^{j \kappa x_{sw4}},
\]  (14)

\[
-\sqrt{\xi}(x_{sw4}, s) = c_1 e^{-j \kappa x_{sw1}} + c_2 e^{-j \kappa x_{sw2}} + c_3 e^{-j \kappa x_{sw3}} + c_4 e^{-j \kappa x_{sw4}},
\]  (15)

where \(x_{sw1}, x_{sw2}, x_{sw3}\) and \(x_{sw4}\) are the location of each point sensor used for the wave filter, and the corresponding nodes are defined as \(sw1, sw2, sw3\) and \(sw4\), respectively. Equations (12) ~ (15) are combined into the matrix as

\[
s_{sw} = L_{sw} w(x_{sw4})
\]  (16)

where \(s_{sw}, L_{sw}\) and \(w(x_{sw4})\) are the sensor output vector, the sensor separation matrix and wave vector at \(x = x_{sw4}\), respectively, and defined as

\[
s_{sw} = \begin{pmatrix}
-\sqrt{\xi}(x_{sw1}, s) \\
-\sqrt{\xi}(x_{sw2}, s) \\
-\sqrt{\xi}(x_{sw3}, s) \\
-\sqrt{\xi}(x_{sw4}, s)
\end{pmatrix}^T
\]  (17)

\[
L_{sw} = \begin{bmatrix}
    1 & 1 & 1 & 1 \\
    c & d & a & b \\
    c^2 & d^2 & a^2 & b^2 \\
    c^3 & d^3 & a^3 & b^3
\end{bmatrix}
\]  (18)

Fig. 2 Schematic diagram of the wave filter using four point sensors
and the elements of the sensor separation matrix \( L_{RP} \) are defined as
\[
a = e^{-j \kappa l_x},
\]
(20)
\[
b = e^{j \kappa l_x},
\]
(21)
\[
c = \kappa l_x,
\]
(22)
\[
d = e^{-j \kappa l_x}.
\]
(23)

Therefore, the wave filtering equation given by
\[
w(x_{w4}) = L_{RP}^{-1} s_{RP}.
\]
(24)

From Eq. (24), the required signal is obtained by four sub-filters as follows
\[
c_i e^{-j \kappa l_x} = -F_{RPI} \overline{\xi}(x_{w4}, s) - F_{RPI2} \overline{\xi}(x_{w2}, s) - F_{RPI3} \overline{\xi}(x_{w3}, s) - F_{RPI4} \overline{\xi}(x_{w4}, s)
\]
(25)

where
\[
F_{RPI} = \frac{1}{(a-c)(b-c)(a-d)},
\]
(26)
\[
F_{RPI2} = -\frac{(a+b+d)(a-c)(b-c)(a-d)}{a},
\]
(27)
\[
F_{RPI3} = \frac{(ab + ad + 1)(a-c)(b-c)(a-d)}{a},
\]
(28)
\[
F_{RPI4} = -\frac{a(c-a)(b-c)(a-d)}{a}.
\]
(29)

It should be pointed out that \((a-c)\), which is the common part of denominator in Eqs. (26)–(29), becomes zero if the sensor separation \(L_s\) is equal to an integral multiple of half wavelength\(^{15)–(17)}\). Hence, the separation must be smaller than the shortest half wavelength in the frequency range of interest.

2.3. Initial state vector and control law

In this section, characteristic equations and control laws of the RWAC are derived. As shown in Fig. 3, only one DVFB is introduced in this paper. However, any number of DVFB can be installed in a hybrid control system in principle. Supposing that the \(i\)th and \(j\)th state variables at right end are zero, and the \(m\)th and \(n\)th state variables at left end, \(z_{m0}\) and \(z_{n0}\) are non-zero, non-zero state variables at the left end (node 0) are described by
\[
z_{m0} = -\frac{1}{EI} (\alpha_1 f_{w} + \alpha_2 f_{w2} + \alpha_3 f_{d})
\]
(30)
\[
z_{n0} = -\frac{1}{EI} (\alpha_1 f_{w} + \alpha_2 f_{w2} + \alpha_3 f_{d})
\]
(31)

where
\[
\Delta = 40 f_{w} 40 f_{m} - 40 f_{w} 40 f_{m},
\]
(32)
\[
\alpha_1 = 40 f_{w} a_{1} f_{w} - 40 f_{m} a_{1} f_{m},
\]
(33)
\[
\alpha_2 = 40 f_{w} a_{2} f_{w} - 40 f_{m} a_{2} f_{m},
\]
(34)
\[
\alpha_3 = 40 f_{w} a_{3} f_{w} - 40 f_{m} a_{3} f_{m},
\]
(35)
\[
\alpha_4 = 30 f_{w} f_{w} a_{4} f_{w} - 30 f_{m} f_{m} a_{4} f_{m},
\]
(36)

\[\text{Fig. 3 Beam model with disturbance, DVFB and active wave control forces}\]
\[ \begin{align*}
\alpha_{22} &= 30l_{jm}32t_{44} - 30l_{lm}32t_{44}, \\
\alpha_{33} &= 30l_{jm}32t_{44} - 30l_{lm}32t_{44}.
\end{align*} \] (37)

where \( g_{ij} \) denotes the \( k \)th row and \( l \)th column variable in the transfer matrix \( T_{ij} \). Then the wave control force is given by

\[ f_w = EIG_{ewBP}(F_{BP1}x_{w1} + F_{BP2}x_{w2} + F_{BP3}x_{w3} + F_{BP4}x_{w4}) - (F_{BP1}x_{w1} + F_{BP2}x_{w2} + F_{BP3}x_{w3} + F_{BP4}x_{w4})G_{swBP}x_{w1}. \] (39)

where \( G_{swBP} \) is the RWAC law using a wave filter. Based on the transfer matrix description, the outputs of each point sensor in the wave filter are described by

\[ \begin{align*}
-\bar{x}(x_{w1}, s) &= z_{w1}x_{w1} + z_{w2}x_{w2} + z_{w3}x_{w3} + z_{w4}x_{w4} \quad (40) \\
-\bar{x}(x_{w2}, s) &= z_{w1}x_{w1} + z_{w2}x_{w2} + z_{w3}x_{w3} + z_{w4}x_{w4} \quad (41) \\
-\bar{x}(x_{w3}, s) &= z_{w1}x_{w1} + z_{w2}x_{w2} + z_{w3}x_{w3} + z_{w4}x_{w4} \quad (42) \\
-\bar{x}(x_{w4}, s) &= z_{w1}x_{w1} + z_{w2}x_{w2} + z_{w3}x_{w3} + z_{w4}x_{w4} \quad (43)
\end{align*} \]

Here, the velocity feedback control force \( f_{cv} \) is given by

\[ f_{cv} = EIG_{cvBP}(\beta z_{w1} + \beta z_{w2}) \] (44)

where \( G_{cv} \) is the DVFB gain. Substituting Eqs. (40)–(44) into Eq. (39), the wave feedback control force is rewritten as

\[ f_w = EIG_{ewBP}(\beta z_{w1} + \beta z_{w2}) \] (45)

Furthermore, substituting Eqs. (44) and (45) into Eqs. (30) and (31), the non-zero variables of the state vector at the node 0 (initial state vector) are described in the matrix form as

\[ \begin{pmatrix} z_{w0} \\ z_{e0} \end{pmatrix} = -A_{BP}^T b / EI \] (48)

where

\[ A_{BP} = \begin{pmatrix} -G_{cv} \alpha_{11}t_{11} + G_{cvBP} \alpha_{12} \beta_1 & -G_{cv} \alpha_{11}t_{11} + G_{cvBP} \alpha_{12} \beta_2 \\ -G_{cv} \alpha_{21}t_{11} + G_{cvBP} \alpha_{22} \beta_1 & -G_{cv} \alpha_{21}t_{11} + G_{cvBP} \alpha_{22} \beta_2 \end{pmatrix} \] (49)

\[ b = \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \end{pmatrix}^T. \] (50)

Next, the RWAC law is derived. Applying the coordinate transformation to the state vector with the wave number matrix \( K \) in the Laplace domain, the wave vector is given by

\[ w(x_a) = K^{-1}z_a \] (51)

where

\[ K = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -\kappa_1 \sqrt{s} & j \kappa_1 \sqrt{s} & \kappa_1 \sqrt{s} & -j \kappa_1 \sqrt{s} \\ \kappa_2^2 S & -\kappa_2^2 S & \kappa_2^2 S & \kappa_2^2 S \\ -\kappa_3^2 \sqrt{s} & j \kappa_3^2 \sqrt{s} & \kappa_3^2 \sqrt{s} & j \kappa_3^2 \sqrt{s} \end{pmatrix} \] (52)

Hence, the reflected wave amplitude at an arbitrary position \( x_a \) (node a) in the element 3 is defined as

\[ c_{e}e^{-\phi_{e}} = \frac{e^{\sqrt{G_{cv}} \kappa_1}}{4k^3}(k^1 jk^2 - k - f) \begin{pmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \end{pmatrix} \begin{pmatrix} z_{w0} \\ z_{e0} \end{pmatrix} \] (53)

where

\[ h_{11} = 20l_{im} - G_{cv} \alpha_{11} t_{11} + G_{cvBP} \beta_1, \] (54)

\[ h_{21} = 20l_{im} - G_{cv} \alpha_{21} t_{11} + G_{cvBP} \beta_2. \] (55)
Moreover, substituting Eq. (48) into Eq. (55), and setting it to be zero, the RWAC law using a wave filter is given by

\[ G_{rwBP} = \frac{G_{rwBP2}}{G_{rwPD}} \]  

(62)

where

\[ G_{rwBP1} = (\gamma_1 \alpha_3 + \gamma_2 \alpha_2) \Delta - G_{rw}(\gamma_1 \alpha_3 + \gamma_2 \alpha_2)(\alpha_1 \alpha_21 - \alpha_2 \alpha_1) \]  

(63)

\[ G_{rwBP2} = (\gamma_1 \beta_2 - \gamma_2 \beta_1)(\alpha_1 \alpha_23 - \alpha_2 \alpha_2) + j \Delta(\alpha_3 \beta_2 + \alpha_2 \beta_2) \]  

- \[ G_{rw}(\beta_1 \alpha_2 - \beta_2 \alpha_2)(\alpha_3 \alpha_2 - \alpha_2 \alpha_3) \]  

(64)

\[ \gamma_1 = k^2 h_{11} + j k^2 h_{21} - kh_{31} - j h_{41}, \]  

(65)

\[ \gamma_2 = k^2 h_{12} + j k^2 h_{22} - kh_{32} - j h_{42}. \]  

(66)

### 3. Numerical example

In this section, the advantages of the proposed hybrid control are presented by numerical calculation. The nominal specification of a flexible beam is listed in Table 1. In all simulations, a disturbance, wave control and DVFB points are set at \( x_d = 1 \)m, \( x_{cw} = 0.3 \)m and \( x_{cv} = 0.1 \)m, respectively, and sensor separation is fixed to 0.03m. Although an optimization scheme(19) for the placement of DVFB can be applied to the hybrid control system, it is not to be considered here since the placement is limited within the narrow range from \( x = 0 \)m to \( x = 0.11 \)m in this paper. Moreover, in the numerical calculation except for root loci, a loss factor, \( \eta = 0.001 \) is introduced to avoid numerical overflow. Such a small loss factor does not significantly affect the simulation reliability(13).

#### 3.1. Independence of the wave-filter-based wave controller

Although there are some methodologies termed “Hybrid control” in control engineering field, one of the widely used methods in active vibration control for a flexible structure is a HAC/LAC method(20)~(23). HAC is generally introduced to suppress the global vibration of the target structure in a narrow frequency band. On the contrary, LAC is for improving the stability of the control system, and the DVFB(18) is frequently used since it can augment the damping of all vibration modes with simple control architecture. However, such a hybrid control method has the interaction between the two control schemes, and hence it is impossible to independently design the each control unit. In contrast, the proposed method has the independent control schemes since the wave-filter-based wave controller is

<table>
<thead>
<tr>
<th>Table 1 Nominal specification of a target beam</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total length</strong></td>
</tr>
<tr>
<td>1.105 m</td>
</tr>
<tr>
<td><strong>Boundary condition</strong></td>
</tr>
<tr>
<td><strong>Clamped-clamped</strong></td>
</tr>
</tbody>
</table>
dependent only on the distance between the measurement and control points as presented in Iwamoto and Tanaka (14).

Figure 4 shows the frequency characteristics of the wave-filter-based wave controller versus DVFB gains when the wave measurement point is set at \( x_{sw} = 0.3 \text{m} \) (collocated). As is apparent in the figure, wave control laws are not affected by the DVFB. This result remains unchanged even when collocation of wave-filter-based active wave control does not hold. Therefore, if the DVFB gain and/or control location of DVFB becomes different from the initial setup, the wave controller need not be changed. This property is an advantage over the conventional HAC/LAC method.

3.2. Stability evaluation of the hybrid control system

In this section, stability of the conventional and proposed control systems with and without the parameter fluctuation is evaluated and compared by two methods. Firstly the root locus method is used. In this paper, three types of root loci are used for the evaluation of the first five poles, and the detail will be explained in each case. Secondly, a Nyquist diagram method is used to examine the stability margin of the system (9)(10)(13). Although this method can deal with only a single-input single-output (SISO) control system, it can be applied to the hybrid control system by regarding the system as an active wave control system. Defining the transfer function of the target structure and the wave controller as \( G_{sys} \) and \( G_{cw} \), respectively, the characteristic equations derived from Eq. (48) is expressed in the form, \( 1 + G_{sys}G_{cw} = 0 \), based on the classical control theory. In this case, the DVFB gain \( G_{cv} \) is included in \( G_{sys} \). Then the open-loop transfer functions of each control system are given by

\[
G_{ro}G_{wave} = G_{ro Ey}(\alpha_{22} \beta_{2} + \alpha_{21} \beta_{1}) - G_{ro Ey}(\alpha_{22} \alpha_{21} - \alpha_{11} \alpha_{22}) \times \left( \alpha_{21} \nu \nu' \nu'' - \alpha_{22} \nu \nu'' \right) / (\nu \nu'' \nu'') \]

Putting the real part of the Laplace variable to be zero, plotting the equations described above in the complex plane gives a Nyquist diagram.

Since a distributed parameter structure has an infinite number of vibration modes, it is difficult to investigate the root loci of all poles and the stability margin in the frequency range up to infinity. However, recalling that wave control laws and frequency responses of sub-filters in a wave filter cannot be expressed as rational functions of the Laplace variable in general, these elements must be approximated in a certain frequency range (11)(13). Therefore, if it is certificated that the system is stable with some degree of the stability...
margin in a sufficiently wide frequency range, the proposed hybrid control system can be constructed in the frequency range of interest; however, this practical technique is beyond the scope of this paper.

We now consider the case where no parameter fluctuation is assumed. Under this condition, the characteristics of the control system are not affected by collocation problem. Figure 5 illustrates the root loci of the wave-filter-based active wave control system when multiplying the wave control law by the coefficient \( C (0 \leq C \leq 1) \) with a regular step of 0.1. In the completely controlled state \((C=1)\), all poles other than the fourth one converge to the vicinity of the real axis. It should be noted that there is a large difference between the states of \( C=0.9 \) and \( C=1 \). This result indicates that the optimally controlled state is sensitive to the inaccuracy of the controller in the sense of pole location. However, since these poles are sufficiently distant from the imaginary axis, the system is unlikely to become unstable because of them. In contrast, the fourth pole is positioned on the imaginary axis after the system is completely controlled \((C=1)\), and thus the control system is on the stability limit. This phenomenon indicates that a vibration mode is newly produced in the uncontrolled region.

Figure 6 shows the Nyquist diagram when the designated wave is perfectly suppressed \((C=1)\). In this work the frequency range of interest is up to 2000Hz. As seen in the figure, the locus never encircles the point \((-1, 0)\). The number of times that the locus passes around the point is equal to that of poles placing on the imaginary axis in the frequency range of interest. However, the stability margin slightly remains, so that it contradicts the result of the root loci shown in Fig. 5. The reason is clear; a small loss factor is introduced for a Nyquist diagram method to avoid a numerical overflow as described earlier, so that the minute margin can be considered as zero. Thus, the control system may easily become
unstable due to the parameter fluctuation. This is the drawback of the conventional active wave control method.

The problem described above is resolved by additionally introducing the DVFB method to an active wave control system. Figure 7 depicts the root loci of the proposed hybrid control system when the designated wave is completely suppressed \((C=1)\) and the DVFB gain \(G_{cv}\) increases from 0 to 400 kg/s. The pole location is calculated when the DVFB gain is set at 0, 6, 12, 18, 24, 30, 31, 32, 33, 34, 35, 40 and 400 kg/s. As shown in the figure, the fourth pole, which is the cause of vulnerability to the parameter fluctuation as previously mentioned, moves from the imaginary axis to the left side and to the vicinity of the real axis as the \(G_{cv}\) increases up to 31 kg/s. This result means that the damping of the mode associated with the fourth pole is augmented by the DVFB controller. Moreover, this pole moves toward the origin along the vicinity of the real axis while the \(G_{cv}\) increases from 31 to 400 kg/s. As a consequence, the locus draws a rough arc. This property is similar to that of a single degree of freedom (SDOF) system, so that the state of \(G_{cv}=31\) kg/s is around a kind of “critical damping”. When this state is achieved, the response resulting from the fourth pole is not vibratory any more. In the simulation after this, the DVFB gain in the hybrid control system is set to 31 kg/s.

Figure 8 shows the Nyquist diagram of the hybrid system in the frequency range up to 2000 Hz. Note that a dash line indicates the unit circle in the complex plane. As is clear from the figure, gain and phase margins are increased in comparison with the conventional active wave control described in Fig. 6. In this case, the phase margin is approximately 14 degrees for a lead and delay, and the gain margin is 1.13 for increment and infinity for decrement, and thus the stability is improved by additionally introduced DVFB as compared to the conventional active wave control.
Next, consider the case where the control target has a parameter fluctuation. In this paper, the parameter fluctuation is defined by the variation of $\kappa$ that is directly related to the wave number. The variation range is from 80 to 120 percent. This range is approximately equivalent to that from 40 to 200 percent of line density $\rho A$, or that from 50 to 250 percent of bending rigidity $EI$. Figure 9(a) depicts root loci of the wave-filter-based active wave control system when the parameter $\kappa$ is varied with a regular step of 4 percent and collocation holds. As is evident from the figure, there is a large difference between the nominal and the minimally fluctuated (by plus or minus 4 percent) states except for the fourth pole in each case. Hence, the optimally controlled state is sensitive to the parameter fluctuation. In contrast, the fourth pole is not affected significantly by the variation. However, this pole moves to the unstable region when the parameter $\kappa$ is negatively varied.

Figure 9(b) shows root loci of the proposed hybrid control systems under the same conditions as Fig. 9(a). As is apparent in the figure, the locus of the fourth pole draws a rough arc in each control system and is sufficiently distant from the imaginary axis even if the parameter $\kappa$ is varied by plus or minus 20 percent while the other poles are not affected significantly by additionally introduced DVFB. Hence, the proposed method is more robust against the parameter fluctuation of the target structure than active wave control. It should be noted that the damping of the fourth pole is reduced when the parameter is negatively varied. This implies that the stability of the proposed method is sensitive to the negative variation of $\kappa$ rather than the positive one.

Next, consider the case where collocation of the sensor/actuator does not hold. Figure
Fig. 10 Root loci of the hybrid control systems with and without collocation when the parameter \( \kappa \) is changed in the range from 80 to 120 percent. As is clear from the figures, when the parameter \( \kappa \) does not change, pole location is not affected by the collocation problem. In contrast, when the parameter \( \kappa \) is varied, the wave measurement point has an influence on the pole location. Especially, in the case of non-collocation, the fifth pole is close to the imaginary axis when the parameter variation is minus 20 percent. Hence, the hybrid control system without collocation is significantly affected by the negative variation of the parameter \( \kappa \) in this setup. However, it is remarkable that the control system is still stable in spite of the fact that the variation is approximately equivalent to that of minus 60 percent of line density or plus 150 percent of bending rigidity. If this is a problem, the hazardous pole location can be improved by modifying the DVFB gain from 31kg/s. If this is the case, the modification leads to the relatively large change of the fourth pole location since it is sensitive to the DVFB gain as shown in Fig. 7. Thus, considering that the DVFB gain of 31kg/s is the minimum one for the maximum damping of the fourth pole, the modification will be a trade-off in a practical case.

3.3. Control performance of the hybrid control system

In this section, the control performance of the hybrid control system with and without the parameter fluctuation is investigated using frequency response and displacement...
distribution criteria. Modal frequencies of the clamped-clamped beam with and without the parameter variation are listed in Table 2.

First of all, consider the case where the target structure does not have the parameter fluctuation. Figure 11 depicts the dynamic compliances at the wave control point \(x_{cw}=0.3\text{m}\) with and without control. As is apparent in the figure, when active wave control is applied, a new peak appears at 61.46Hz where the gain curve has a notch in the case of non-control, indicating that a new vibration mode is produced in the uncontrolled region. As discussed in the previous section, this mode results from the fourth pole of the target structure and is the cause that stability margin is zero. However, if the hybrid control method is applied, the new peak is adequately suppressed while the other part of gain curve changes little. Thus, the proposed hybrid control overcomes the drawback of the conventional method.

Figure 12 illustrates envelopes of displacement distribution with and without control at the first five modal frequencies and 61.46Hz. In the case of non-control, the displacement distribution forms the standing wave, and the maximum amplitudes are 4.51mm at the first modal frequency 6.53Hz, 1.89mm at the second modal frequency 17.99Hz, 0.84mm at the third modal frequency 35.27Hz, 0.44mm at the fourth modal frequency 58.3Hz, 4.11µm at 61.46Hz and 0.25mm at the fifth modal frequency 87.1Hz, respectively. In contrast, when active wave control is applied, it can be graphically confirmed at some frequencies that nodal points shift their positions in the controlled region. It means that the travelling wave

<table>
<thead>
<tr>
<th>Parameter variation</th>
<th>-20%</th>
<th>0% (nominal)</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mode</td>
<td>10.2Hz</td>
<td>6.53Hz</td>
<td>4.53Hz</td>
</tr>
<tr>
<td>Second mode</td>
<td>28.11Hz</td>
<td>17.99Hz</td>
<td>12.49Hz</td>
</tr>
<tr>
<td>Third mode</td>
<td>55.11Hz</td>
<td>35.27Hz</td>
<td>24.49Hz</td>
</tr>
<tr>
<td>Fourth mode</td>
<td>91.1Hz</td>
<td>58.3Hz</td>
<td>40.49Hz</td>
</tr>
<tr>
<td>Fifth mode</td>
<td>136.09Hz</td>
<td>87.1Hz</td>
<td>60.48Hz</td>
</tr>
</tbody>
</table>

Fig. 11 Dynamic compliances at the active wave control point \(x_{cw}=0.3\text{m}\) with and without control: (a) without control, (b) with active wave control, (c) with hybrid control method
largely contributes to the displacement in that region, and the maximum amplitude is reduced to 7.24µm at the first modal frequency, 4.07µm at the second modal frequency, 3.28µm at the third modal frequency, 16.3µm at the fourth modal frequency and 1.58µm at the fifth modal frequency, respectively. However, the maximum amplitude at 61.46Hz is increased to 1.58mm since a vibration mode is newly produced at this frequency in the uncontrolled region.

Next, consider the case of the hybrid control system. In this case, the maximum amplitudes at the first, second, third and fifth modal frequencies are not changed significantly when the DVFB is additionally applied in the uncontrolled region while those of the fourth modal frequency and 61.46Hz are reduced. Observe that the newly produced mode is sufficiently suppressed and maximum amplitude is reduced to 1.94µm that is 0.12% as compared to the case of the conventional active wave control method. It should be noted that the reflected wave is completely suppressed at all frequencies by the proposed method. Thus, the DVFB does not disturb the unique (not sure what you mean by unique) control effect of active wave control.

Next, consider the case where the target structure has the parameter fluctuation. In this case, the hybrid control method without collocation provides different results from the control with collocation. Figure 13 shows the gain curves of dynamic compliances at the active wave control point (xsw=0.3m) with the parameter variation by plus and minus 20

Fig. 12 Envelopes of displacement distribution of the beam at the first five modal frequencies and 61.46Hz with and without control
percent. When the variation is minus 20 percent, the frequency range of interest is up to 150Hz in order to cover the fifth modal frequency 136.09Hz. As illustrated in Fig. 13(a), in the case where the parameter variation is minus 20 percent, there is a relatively large gain peak in the hybrid control system characteristics when the wave measurement point is not collocated ($x_{sw}=0.2m$) while the peak does not appear in the case of collocation. In contrast, when the parameter is varied by plus 20 percent, there is no gain peak in each control system regardless of whether collocation holds or not.

Next, displacement distribution under the parameter variation is investigated. In this simulation, standing wave ratio (SWR) \( R \) is used for the evaluation that indicates the ratio of the amplitude of positive and negative travelling waves, and is defined as

\[
R = \frac{\xi_{\text{max}}}{\xi_{\text{min}}} \tag{69}
\]

where \( \xi_{\text{max}} \) and \( \xi_{\text{min}} \) are the maximum and minimum displacement amplitudes in far-field. If the reflected wave is completely suppressed (i.e., in the ideal case), SWR is equal to 1. In contrast, when an envelope forms a perfect standing wave, it becomes infinity. Figure 14 illustrates envelopes of displacement distribution of the hybrid control system at the fifth modal frequency when the parameter is changed by plus and minus 20 percent; Comparison with respect to a collocation problem.

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Fig. 13 Gain curves of dynamic compliances at an active wave control point with and without hybrid control when the parameter is changed by plus and minus 20 percent; Comparison with respect to a collocation problem

Fig. 14 Envelopes of displacement distribution with hybrid control at the fifth modal frequency when the parameter is changed by plus and minus 20 percent; Comparison with respect to a collocation problem
modal frequency with and without collocation when the parameter $\kappa$ is changed by plus and minus 20 percent. The corresponding SWRs for each case are also described in the figure. In the case where the parameter variation is minus 20 percent, the SWRs in each control system degrade when collocation does not hold. This is because the fifth modal frequency under this condition is close to the peak frequency as shown in Fig. 13; in other words, the resonance as well as instability phenomena cannot arise in a finite structure without a standing wave. Consider then the case where the parameter variation is plus 20 percent. As is clear from the figure, the degradation of the SWRs is less than the previous case, and hence the proposed method provides the robust performance against the positive parameter fluctuation.

4. Conclusion

This paper has presented hybrid control of a flexible beam using wave-filter-based active wave control and direct velocity feedback to improve the stability of the conventional active wave control system. The main results of this work are summarized as follows;

1. The wave control laws are not affected by the DVFB, and this property remains unchanged even when collocation of wave-filter-based active wave control does not hold. This property is an advantage over the conventional HAC/LAC method.

2. The stability is investigated by using Nyquist diagrams and root loci. When applying the DVFB, the damping of a new mode produced in the uncontrolled region by active wave control is reduced. Therefore, the proposed hybrid control system is more stable than the conventional active wave control system.

3. When the parameter $\kappa$ is varied, the collocation problem affects the stability of the hybrid control system. However, even if the variation is plus or minus 20 percent, the fifth pole is still in the stable region.

4. Control performances of the proposed hybrid control system are investigated from a viewpoint of frequency characteristics and displacement distribution. In the nominal case, the DVFB reduces a new mode produced in the uncontrolled region while it does not disturb the unique control effect of active wave control.

5. If the parameter is varied by minus 20 percent and collocation does not hold, a new peak appears in the gain curve of the dynamic compliance at the wave control point. The degradation for the negative parameter variation can be also observed in the displacement distribution by using SWR. In contrast, when the parameter variation is plus 20 percent, any peaks appear in the compliance, and the degradation of SWRs is small. Hence, the proposed method provides the robust performance against the positive parameter fluctuation.

References


(6) Kar, I. N., Miyakura, T., Seto, K., Bending and torsional vibration control of a flexible


