Nonlinear Pressure Wave Analysis by Concentrated Mass Model*

(2nd Report, Modeling and Validity Verification of Branch Element)

Satoshi ISHIKAWA**, Takahiro KONDOU** and Kenichiro MATSUZAKI**

** Department of Mechanical Engineering, Faculty of Engineering, Kyushu University,
744 Motoooka, Nishi-ku, Fukuoka-shi, Fukuoka 819-0395, Japan
E-mail: ishikawa@mvib.mech.kyushu-u.ac.jp

Abstract

A pressure wave propagating in a tube often changes to a shock wave because of the nonlinear effect of the fluid it is traveling through. The purpose of this study is to establish a practical analytical model to analyze this phenomenon. In previous reports, a concentrated mass model was proposed to analyze nonlinear pressure wave phenomena in a straight cylindrical tube. In this paper, the modeling of a branched junction is proposed. The model of the branched junction consists of masses, nonlinear pressure elements, base support dampers, and nonlinear dampers. The nonlinear damper is derived from the pressure loss at the branched junction. To confirm the validity of the proposed branch model, the numerical results obtained from the concentrated mass model are compared with the theoretical value of the transmission loss of a branch pipe. Additionally, an experiment on a sound tube with a branch pipe is performed and the experimental results are compared with the numerical result. All computational results agree very well with the theoretical value and the experimental results. Therefore, it is concluded that the proposed branch model is valid for the numerical analysis of nonlinear pressure wave problem in a tube with a branch pipe.

Key words: Method of Vibration Analysis, Simulation, Modeling, Vibration Coupled with Fluid Motion, Nonlinear Vibration, Shock Wave, Nonlinear Wave, Branch Element

1. Introduction

A pressure wave propagating in a tube often changes to a shock wave because of the nonlinear effect of fluid^{(1),(2)}. In previous reports^{(3),(4)}, we examined one-dimensional nonlinear pressure waves, and proposed a concentrated mass model to analyze this phenomenon. Pressure waves generated in a sound tube and in a plane-wave tube were analyzed numerically by the proposed model to confirm the validity of the model.

A branch pipe is also often considered because it is a simple structure and considerably attenuates pressure waves at a particular frequency. Ishihara et al.⁽⁵⁾ studied experimentally the effect of a branch pipe on a pressure wave in a hydraulic oil tube. A transfer matrix method is often used to perform a linear pressure wave analysis in a pipe system which has branch pipe. Abe et al.⁽⁶⁾ proposed a transfer matrix treatment of a branch element which considers linear damping by the shear stress from the wall. Mohri et al.⁽⁷⁾ replaced the pressure loss at a branched junction with a linear damper, and proposed a transfer matrix

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method which considers the pressure loss by using an iterative calculation. Chae et al.⁽⁸⁾ proposed a treatment of a branch element in a difference method to perform a nonlinear pressure wave analysis.

In the present paper, we propose a treatment of a branch element using the concentrated mass model. First, we propose a model of a branch element in which the nonlinear dampers are derived from the pressure loss at a branched junction. Next, we compare the numerical results from the concentrated mass model with the transmission loss⁽⁹⁾ to confirm the validity of the branch model in a linear region. Then, we consider an experiment on a sound tube with a branch pipe to confirm the validity of the branch model in a nonlinear region. Furthermore, we investigate the reduction effect of a branch pipe on a standing pressure wave. This effect has not been studied before.

2. Concentrated Mass Model

2.1 Straight tube element^{(3),(4)}

In previous reports, we considered one-dimensional nonlinear pressure wave phenomena in a cylindrical tube with a uniform cross-section. The fluid is modeled as a concentrated mass model that consists of masses, connecting nonlinear springs, connecting dampers, and base support dampers, as shown in Fig. 1. In this section, we summarize this straight tube model. We neglect a steady flow. The fluid is divided into N uniform elements such that the radius is R and the cross-sectional area is A . The mass is concentrated on the nodal points. The mass at each nodal point and the length of each element in the equilibrium state are uniform. The displacement of nodal point i is x_i , the pressure in element i is p_i , the variation of the pressure is dp_i , and the pressure in the equilibrium state is p_0 . The pressure in element i is

$$p_i = p_0 + dp_i. \quad (1)$$

Then, the mass m , the damping coefficient of the base support damper c^b , the variation of the nonlinear restoring force acting on the spring f_i^k , and the damping coefficient of connecting damper c^c are

$$m = \rho_0 A l + 2\mu_b A l \frac{1}{\omega} \text{Im}[F(R)], \quad (2)$$

$$c^b = 2\mu_b A l \text{Re}[F(R)], \quad (3)$$

$$f_i^k = p_0 A \left\{ 1 - \frac{l^\gamma}{(l + x_i - x_{i-1})^\gamma} \right\}, \quad (4)$$

$$c^c = \frac{4\mu_c A}{3l}, \quad (5)$$

where ρ_0 is the density in equilibrium, ω is the angular frequency, μ_b is the viscosity

Fig. 1 Concentrated mass model

for shear stress, and γ is the ratio of specific heats. The function Γ is given by

$$\Gamma(R) = \frac{\lambda^2 J_1(j\lambda R)}{j\lambda R J_0(j\lambda R) - 2J_1(j\lambda R)}, \quad (6)$$

where $j = \sqrt{-1}$, $\lambda = \sqrt{j\omega\rho_0/\mu_b}$, and $J_n(z)$ is a Bessel function of the first kind and order n .

The second term of the equivalent mass in Eq. (2) and the damping coefficient of connecting damper in Eq. (3) describe the influence of the shear stress from the wall. The nonlinear restoring force acting on the spring in Eq. (4) is derived from adiabatic changes of the fluid, and the connecting damper in Eq. (5) is derived from a normal stress of the fluid. The forces acting on each mass point are calculated using Eqs. (2)~(5), and the motion of the mass is simulated by numerical integration. Then, the variation of the pressure in element i is given by

$$dp_i = -\frac{f_i^k}{A}. \quad (7)$$

2.2 Branch element

In this paper, we apply the concentrated mass model to a system with a branch pipe, as shown in Fig. 2. The branch element is defined as the area in which the main pipe intersects with the branch pipe (shaded area). The length of the element is equal to the diameter of the branch pipe, and the three masses are arranged on the boundaries of the element. We assume that the number of the left nodal point is $i-1$, the number of the right nodal point is i , and the number of the nodal point on branch pipe side is $b0$. The force from the variation of the pressure dp_i in the branch element acts on each nodal point. A connecting damper in the branch element is neglected. Furthermore, we consider a nonlinear damper derived from the pressure loss in branched junction; we discuss this damper in Section 2.3.

Assuming the mass in the branch element is distributed to the mass points $i-1$ and i , and the mass of the nodal point $b0$ is given from the adjoining element $b1$ on branch pipe side, the masses are

Fig. 2 Branch element model

$$\left. \begin{aligned} m_{i-1} &= \frac{\rho_0 A_m (l_{i-1} + D_b)}{2} + \frac{\mu_b A_m (l_{i-1} + D_b)}{\omega} \text{Im}[\Gamma(R_m)] \\ m_i &= \frac{\rho_0 A_m (D_b + l_{i+1})}{2} + \frac{\mu_b A_m (D_b + l_{i+1})}{\omega} \text{Im}[\Gamma(R_m)] \\ m_{b0} &= \frac{\rho_0 A_b l_{b1}}{2} + \frac{\mu_b A_b l_{b1}}{\omega} \text{Im}[\Gamma(R_b)] \end{aligned} \right\}, \quad (8)$$

where the index m indicates a variable for the main pipe, and the index b indicates a variable for the branch pipe. The second term in each mass describes the influence of the shear stress from the wall as well as the straight tube element. Considering the influence of the shear stress as in Eq. (3), the base support dampers are represented as

$$\left. \begin{aligned} c_{i-1} &= \mu_b A_m (l_{i-1} + D_b) \text{Re}[\Gamma(R_m)] \\ c_i &= \mu_b A_m (D_b + l_{i+1}) \text{Re}[\Gamma(R_m)] \\ c_{b0} &= \mu_b A_b l_{b1} \text{Re}[\Gamma(R_b)] \end{aligned} \right\}. \quad (9)$$

Assuming an adiabatic change, the variation of the pressure dp_i in the branch element is

$$\left. \begin{aligned} dp_i &= \frac{p_0 V_i^\gamma}{\{V_i + A_m (x_i - x_{i-1}) + A_b x_{b0}\}^\gamma} - p_0 \\ V_i &= A_m D_b \end{aligned} \right\}. \quad (10)$$

There are two forces due to this variation of the pressure, as shown in Fig. 2: $dp_i A_m$ acts on mass points $i-1$ and i ; $dp_i A_b$ acts on mass point $b0$. The elements other than the branch element are the straight tube elements described in Section 2.1.

2.3 Nonlinear damper in the branch element

When the fluid flows in a branched junction, there is a pressure drop caused by a dividing flow and a combining flow. In this section, the nonlinear damping representing this pressure loss is derived.

Ito et al.⁽¹⁰⁾ considered the flow patterns of ① ~ ③ in Fig. 3, and determined experimentally the head loss between points 1 and 3, Δh_{13} for a steady flow. For the dividing flow, with speed of flow v_1 at point 1, the head loss between points 1 and 3 is given by

$$\Delta h_{13} = \zeta_{13} \frac{v_1^2}{2g}. \quad (11)$$

For of the combining flow, with speed of flow v_3 at point 3, the head loss is given by

$$\Delta h_{13} = \zeta_{13} \frac{v_3^2}{2g}. \quad (12)$$

In the case of $D_b = D_m$, the experimental formulae for the loss coefficient are given in Table 1 for all patterns in Fig. 3. The variable X in the formulae is given in the X column

Fig. 3 Flow pattern for energy loss

in Table 1 using the flow rates Q_1 , Q_2 , and Q_3 in Fig. 3.

Now we assume that the pressure drop also occurs in the case of an unsteady flow, such as a pressure wave, and we derive the force required for the nonlinear damper representing this pressure drop. Eight kinds of flow patterns exist in the concentrated mass model for unsteady flow, as shown in Fig. 4. In the case of the patterns I~VI, the force of the nonlinear damper can be derived from ζ_{13} in Table 1. The flow rate Q is related to $\dot{x}_{i-1}$, $\dot{x}_i$, and $\dot{x}_{b0}$. Table 2 shows the route of loss, the pattern of ζ_{13} in Fig 3, and the fraction of velocity used for X in the case of the patterns I~VI. In each of the patterns I~VI, there are two routes of loss, these loss coefficients $(\zeta_{13})_1$ and $(\zeta_{13})_2$ are calculated using the formulae in Table 1. For example, in the case of the pattern I, there are loss routes from $i-1$ to i and from $i-1$ to $b0$. First route corresponds to the dividing flow of ① in Fig. 3, and second route corresponds to the dividing flow of ③. The loss coefficients, $(\zeta_{13})_1$ and $(\zeta_{13})_2$, are calculated using the formulae in Table 1 in which X is $|\dot{x}_{b0} / \dot{x}_{i-1}|$. The total head loss, Δh , becomes

$$\Delta h = \{(\zeta_{13})_1 + (\zeta_{13})_2\} \frac{\dot{x}_\alpha^2}{2g}, \quad (13)$$

where α is the index of the mass point to which the damper is attached (see Fig. 4). Assuming the energy of this head loss is absorbed by the damper, the damping force f_α^c acting on the mass point α is given by

$$f_\alpha^c = -\rho_0 A_\beta \{(\zeta_{13})_1 + (\zeta_{13})_2\} \frac{\dot{x}_\alpha |\dot{x}_\alpha|}{2}, \quad (14)$$

where $A_\beta = A_m$ if α is $i-1$ or i , and $A_\beta = A_b$ if α is $b0$. Meanwhile, in the case of the pattern VII or VIII, the energy loss is neglected. Some of the patterns I~VIII exit for

Table 1: Loss coefficients for branch flows ⁽¹⁰⁾

Pattern		ζ_{13}	X
①	combining	$0.045 + 1.38X - 0.90X^2$	Q_2 / Q_3
	dividing	if $0 \leq X \leq 0.22$ then $1.55(0.22 - X)^2 - 0.03$ if $0.22 \leq X \leq 1$ then $0.65(X - 0.22)^2 - 0.03$	Q_2 / Q_1
②	combining	$1.09 - 0.53X - 1.48X^2$	Q_2 / Q_3
	dividing	$0.59 + 1.18X - 0.68X^2$	Q_3 / Q_1
③	combining	$1.19 - 1.73X + 1.34X^2$	Q_2 / Q_3
	dividing	$0.99 - 0.82X + 1.02X^2$	Q_3 / Q_1

Fig. 4 Flow pattern of the concentrated mass model

Table 2: Loss pattern and variable X

	Route of loss		Pattern of ζ_{13}	X
I	1	$i-1 \rightarrow i$	① dividing	$ \dot{x}_{b0} / \dot{x}_{i-1} $
	2	$i-1 \rightarrow b0$	③ dividing	$ \dot{x}_{b0} / \dot{x}_{i-1} $
II	1	$i-1 \rightarrow i$	① combining	$ \dot{x}_{b0} / \dot{x}_i $
	2	$b0 \rightarrow i$	② combining	$ \dot{x}_{i-1} / \dot{x}_i $
III	1	$i \rightarrow i-1$	① dividing	$ \dot{x}_{b0} / \dot{x}_i $
	2	$i \rightarrow b0$	③ dividing	$ \dot{x}_{b0} / \dot{x}_i $
IV	1	$i \rightarrow i-1$	① combining	$ \dot{x}_{b0} / \dot{x}_{i-1} $
	2	$b0 \rightarrow i-1$	② combining	$ \dot{x}_i / \dot{x}_{i-1} $
V	1	$i-1 \rightarrow b0$	③ combining	$ \dot{x}_i / \dot{x}_{b0} $
	2	$i \rightarrow b0$	③ combining	$ \dot{x}_{i-1} / \dot{x}_{b0} $
VI	1	$b0 \rightarrow i-1$	② dividing	$ \dot{x}_{i-1} / \dot{x}_{b0} $
	2	$b0 \rightarrow i$	② dividing	$ \dot{x}_i / \dot{x}_{b0} $

Fig. 5 Transmission loss model

one cycle, so we alter the position of the damper at every time step during numerical integration.

3. Comparison with transmission loss and nonlinear analysis

3.1 Transmission loss

To confirm the validity of the branch element model in a linear region, we compare our analysis based on the concentrated mass model with the transmission loss (TL) which has been established as the attenuation performance of a side-branch muffler. Transmission loss is given by

$$TL = L_1 - L_2, \quad (15)$$

where L_1 is the sound pressure level of a wave incident on a branch pipe, L_2 is the sound pressure level of the wave transmitted in a main pipe direction, as shown in Fig. 5. As the value of TL grows, the decrease of pressure amplitude becomes larger. Non-reflecting boundary conditions are assumed at the right end and at the left end in Fig. 5; thus, the transmission loss represents the attenuation of a progressive wave. Assuming that the pressure wave is linear and that a damping is neglected, by the continuity of pressure and the balance of flow at the branched junction, the transmission loss can be written as follows⁽⁹⁾

$$TL = 10 \log \left\{ 1 + \frac{1}{4} \left(\frac{A_b}{A_m} \right)^2 \tan^2 \left(\frac{\omega}{a} l_b \right) \right\}, \quad (16)$$

where l_b is the length of the branch pipe, a is the speed of sound, and ω is the angular frequency of the pressure wave.

Fig. 6 Model for Analysis

Table 3: Parameter values for the analyzed model

L (m)	20	l_0 (m)	10
l_b (m)	0.85	D_m (mm)	48
D_b (mm)	48	p_0 (MPa)	0.1065
ρ_0 (kg/m ³)	1.29	γ	1.4
Δp_0 (Pa)	10 (8000)	c (m/s)	340
μ_b (Pa·s)	0 (1.82×10^{-5})	μ_c (Pa·s)	0 (1.0)

3.2 TL by concentrated mass model

To evaluate TL derived from the concentrated mass model, we perform a numerical analysis of a model of a pipe system with a branch pipe, as shown in Fig. 6. The boundary condition at the left end is the pressure boundary condition; at the right end non-reflecting boundary condition is assumed. The analysis parameters are listed in Table 3. The force f_0 acting from the left on nodal point 0 is given by

$$f_0 = \Delta p_0 A_m \sin \omega t, \quad (17)$$

where Δp_0 is the pressure amplitude at the left end. At the right end, the pressure p_{n+1} acting from the right on nodal point n is given by

$$p_{n+1} = \rho_0 a \dot{x}_n. \quad (18)$$

The displacement of the end of the branch pipe is 0.

$$x_{bn} = 0. \quad (19)$$

Numerical integration is performed using the Runge-Kutta method with a time interval of $1/26000$ s. The partition number of the main pipe is $n = 2000$, and the partition number of the branch pipe is $bn = 85$. Using the pressure amplitude Δp_B at point B after the branch pipe in Fig. 6 and the pressure amplitude Δp_{B0} at point B when there is no branch pipe, the transmission loss can be written as follows

$$TL = 20 \log \left(\frac{\Delta p_{B0}}{\Delta p_B} \right). \quad (20)$$

Because there are assumptions of linearity and non-damping when Eq. (16) is derived, we analyze the concentrated mass model in a linear region with a small pressure amplitude ($\Delta p_0 = 10$ Pa), and without the connecting damper ($\mu_c = 0$), without the base support damper ($\mu_b = 0$), and without the nonlinear damper.

3.3 Comparison between analytical results and theoretical values

Figure 7 shows the comparison between the theoretical values of TL from Eq.(16) and the results from the concentrated mass model. The horizontal axis shows the frequency, the vertical axis shows TL . The length of the branch pipe was fixed so that the first-order resonance frequency of the branch pipe is 100Hz. From Fig. 7 we can see that the decrease of pressure amplitude at 100Hz and 300Hz is large, while the change at 200Hz and 400Hz is zero. This reason is that the reflected wave from the fixed end of the branch pipe is 180 degrees out of phase with the incident wave for frequencies that are odd multiples of

Fig. 7 Transmission loss in the linear region

Fig. 8 Convex element

Fig. 9 Transmission loss in the nonlinear region

100Hz, while the reflected wave is in phase with the incident wave for even multiples of 100Hz. The model values (circles) agree well with the theoretical values (solid line). Therefore, the model of the branch element proposed in Section 2, that is, the array of nodal points, the distribution of the mass, and the treatment of the variation of pressure, is valid. The analytical values do not agree with the theoretical values if the mass is distributed equally to the three mass points, or if we take the convex element shown in Fig. 8.

3.4 Attenuation of nonlinear pressure wave

When the amplitude of the pressure boundary condition in Fig. 6 is large, a shock wave is generated because of nonlinear effects^{(3),(4)}. We investigate the decrease of pressure amplitude due to the branch pipe when the shock wave appears. We use the model shown in Fig.6 and the parameters in Table 3, as in Section 3.2. Note that the values in parentheses are used for Δp_0 , μ_b , and μ_c . Δp_0 is set to a large value to generate the shock wave. In addition, the viscosity μ_b for the base support damper is the real value of air, but the viscosity μ_c for the connecting damper is larger in order to suppress the generation of the numerical vibrations^{(3),(4)}. The nonlinear damper in Section 2.3 is neglected to clarify the effect of the branch pipe on the pressure amplitude.

Figure 9 shows the TL under these conditions. In the nonlinear region, TL from the concentrated mass model (dashed line) is smaller than the theoretical values (solid line) at 100Hz and 300Hz. Figure 10 shows the time-series of the pressure wave at point B with and

Fig. 10 Pressure wave at point B

Fig. 11 Power spectrum at point B

without the branch pipe when the frequency of the pressure boundary condition is 100Hz. While the amplitude of the shock wave with the branch pipe is smaller than the amplitude without branch pipe, the shrinking is only by a factor of 0.38. Figure 11 shows the power spectrum of the waveform in Fig. 10 when the 1st order is 100Hz. The spectra with the branch pipe are smaller than spectra without the branch pipe for odd orders, but the spectra are almost equal for even order. When the shock wave appears and higher harmonic components are generated, the even order components remain because the branch pipe can only reduce the odd components. For this reason, the TL in the nonlinear region at 100Hz and 300Hz is small (see Fig.9). If a second branch pipe for second order components (200Hz) is attached after the branch pipe for 100Hz, the components of 2nd and 6th and 10th orders can be reduced, but the components of 4th and 8th orders remain.

4. Application to the sound tube with branch pipe problem

4.1 Experimental apparatus

To confirm the validity of the branch element model in the nonlinear region, we performed experiments on a sound tube with a branch pipe, as shown in Fig. 12. The branch pipe is fixed to the main pipe. At the left end, the air is excited by a piston which is fixed to a vibration exciter. The acceleration amplitude of the piston, δ_a , is constant at all frequencies. The right end of the main pipe and the end of the branch pipe are closed and Kistler 603B pressure sensors are installed there. The pipe is a stainless pipe with outside diameter 38mm and thickness 2mm. The T-joint at the branched junction is a 'Byte type tube fitting (T-38)' made by Fujitoku that we processed to an inside diameter of 34mm. Table 4 shows the dimensions of the experimental apparatus. We performed experiments in six configurations: in five of these the distance from the piston to the branch pipe, l_0 , was 0.8m, 0.9m, 0.991m, 1.1m, and 1.2m; the final configurations was without the branch pipe. The length of the branch pipe, l_b , was fixed so that the first-order resonance frequency of the branch pipe was equal to the first-order resonance frequency of the main pipe.

4.2 Analysis method

We simulate the pressure wave phenomena with the concentrated mass model. The numerical integration is performed using the Runge-Kutta method with a time interval of $1/20000$ s, 80 partitions in the main pipe and 40 partitions in the branch pipe. The parameters are listed in Table 4. We use the base support damper, the connecting damper,

Fig. 12 Experimental apparatus

Table 4: Parameter values

L (m)	1.982	l_b (m)	1.0
D_m (mm)	34	D_b (mm)	34
p_0 (MPa)	0.1013	γ	1.4
δ_a (m/s ²)	350	ρ_0 (kg/m ³)	1.21
μ_b (Pa·s)	1.82×10^{-5}	μ_c (Pa·s)	0.9

Fig. 13 Frequency response without branch pipe

and the nonlinear damper. The boundary condition at the piston is the displacement

$$x_0 = \frac{\delta_a}{\Omega^2} \sin \Omega t. \quad (21)$$

where Ω is the exciting angular frequency. At the closed end, the displacement is

$$x_n = 0. \quad (22)$$

4.3 Experimental results and numerical results

Figure 13 and 14 are comparisons of the numerical results from the concentrated mass model and the experimental results when there is no branch pipe. Figure 13 shows the total amplitude of the pressure at the closed end as a function of the frequency of the piston. The model results (solid line) agree well with the experimental results (circles) in this case. Figure 14 shows time series of the waveform of dp at the closed end of the main pipe at 85.3 Hz, which is the resonance frequency. A shock wave is generated in the experimental result, and this phenomenon is simulated by the concentrated mass model, as we have discussed in previous reports^{(3),(4)}.

Figure 15 shows the frequency responses at the closed ends of the main pipe and the branch pipe as the distance of the piston to the branch pipe, l_0 , is varied. The numerical results without the nonlinear damper (dashed lines) are larger than the experimental results (circles), but the numerical results with the nonlinear damper (solid lines) agree well with the experimental result for each l_0 . So the nonlinear damper derived from the pressure loss

Fig. 14 Pressure wave without branch pipe (85.3Hz)

○ : Experimental result
 — : Numerical result with nonlinear damping
 --- : Numerical result without nonlinear damping

Fig. 15 Frequency response with branch pipe

Fig. 16 Pressure wave at resonance frequency

at the branched junction is necessary. When the branch pipe is at the center of the main pipe ($l_0 = 0.991\text{m}$), there is only one resonance and the amplitude of the resonance peak at main pipe end is larger than the corresponding amplitudes at other values of l_0 . We discuss this point later.

Figure 16 shows the time series waveform at the closed end of the main pipe at the resonance frequency for $l_0 = 0.9\text{m}$, 0.991m , 1.1m . For $l_0 = 0.9\text{m}$, 80.8Hz and $l_0 = 1.1\text{m}$, 91.5Hz , the waveforms are sinusoidal. But for $l_0 = 0.9\text{m}$, 90.3Hz and $l_0 = 1.1\text{m}$, 80Hz , the waveforms are distorted, and for $l_0 = 0.991\text{m}$, the shock wave is generated. So the nonlinear effect appears. The numerical results agree well with the experimental results, and the nonlinear phenomena are simulated. We conclude that the branch element model proposed in this paper is valid for nonlinear wave problems, such as a sound tube with a branch pipe.

In Fig. 15, there are two resonance peaks when the branch pipe is not at the center of the main pipe, but there is one resonance peak for $l_0 = 0.991\text{m}$ when the branch pipe is at the center of the main pipe. The amplitude of the resonance peak at the main pipe end when $l_0 = 0.991\text{m}$ is the largest. In addition, the shock wave is generated only when $l_0 = 0.991\text{m}$ (see Fig. 16). So the center of the main pipe is a significant position. A shock wave is also generated when the branch pipe is attached near the piston or near the right end of the main pipe. Figure 17 shows the pressure response to the variation of l_0 and the piston frequency. When the position of the branch pipe is not at the center, there are two resonance lines. These lines meet when the branch pipe is at the center of the main pipe. The amplitude of the pressure is large when the position of the branch pipe is at the center, near the piston, and near the right end. Figure 18 shows the first and second order natural frequencies f_p of paths I and II in Fig. 19 that are given by

$$f_p = \frac{n_r a}{2l_p}, \quad (23)$$

where n_r is the order of the natural frequency, l_p is the length of the path. The sound speed a is 342 m/s , which is the value for the parameters in Table 4. The resonance lines in Fig. 17 correspond to the natural frequency lines in Fig. 18. So the resonances of the pipe system in Fig. 12 consist of the resonance of path I and path II in Fig.19. If the branch

Fig. 17 Pressure response to the variation of piston frequency and l_0

Fig. 18 Natural frequency of paths I and II

Fig. 19 Paths of resonance

pipe is not at the center of the main pipe, the resonance divides into paths I and II. But, if the branch pipe is at the center of the main pipe, there is one resonance because the length of path I is the same as the length of path II, and the amplitude of the resonance grows because of the overlap of the two resonances. Because the amplitude of the resonance grows the shock wave is generated.

Considering the effect of the attenuation of the amplitude of pressure in the main pipe by the branch pipe, the branch pipe should be fixed a short distance from the center of the main pipe ($l_0 = 0.9\text{m}$ and $l_0 = 1.26\text{m}$ are the best) to reduce the amplitude of the resonance. If the frequency of an excitation is near the resonance frequency of the main pipe, the branch pipe should be fixed as far as possible from the center. That is, the branch pipe should be positioned at the pressure antinodes of a standing wave in the main pipe. The effect of the branch pipe is the largest when the branch pipe is positioned not at the velocity antinodes in a standing wave but at the pressure antinodes. To suppress the generation of the shock wave, the branch pipe should be fixed at a position other than the center or ends of the main pipe.

5. Conclusion

To analyze one-dimensional nonlinear pressure wave phenomena in a cylindrical tube

with a branch pipe, we propose a treatment of a branch element in the concentrated mass model. The model at the branch element consists of masses, nonlinear pressure elements, base support dampers, and nonlinear dampers. The nonlinear damper is derived from the pressure loss at the branched junction. To confirm the validity of the proposed branch model in a linear region, numerical results obtained from the concentrated mass model are compared with the theoretical values of the transmission loss in a branch pipe. The numerical results agree well with the theoretical values. Additionally, an experiment on a sound tube with a branch pipe is performed. The results of our model agree well with the experimental results qualitatively and quantitatively: for example a shock wave occurs at resonance frequency when the position the branch pipe is at the center of the main pipe. We also confirm the necessity and the validity of the nonlinear damper. Furthermore, we confirm that the effect of the branch pipe on the attenuation of the amplitude of pressure in the main pipe is highly dependent on the position of the branch pipe.

In summary, we conclude that the proposed branch model is suitable for the numerical analysis of nonlinear pressure wave problems in a tube with a branch pipe.

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