Study on the Generating Mechanism of the Intermittency by Using the Feedback Linearization*

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Abstract
By using the feedback linearization, we studied the generating mechanism of intermittency. The well-known Lorenz model, Rössler model and BZ reaction model were selected as the subject of the present study. At first, we studied the basic properties of these models and it was found that the feedback linearization can apply to the analysis of these models. Subsequently, these models were modified into the linear equations with nonlinear inputs by the feedback linearization and we considered the generating mechanism of intermittency. The consideration results showed that the detailed information of the boundary has an important meaning to study the generating mechanism of intermittency. Moreover, it was shown that the generating mechanism of intermittency can be explained intuitively by the collision with the boundary.

Key words: Chaos and Fractal, Nonlinear Control, Nonlinear Vibration, Nonlinear Dynamics, Complex System

1. Introduction
Intermittency gives a strange impression seemingly, but this phenomenon is found easily in everyday life, i.e., the laminar-turbulent transitional flow. Intermittency has attracted the interest of many researchers for a long time because of its strangeness. A number of theoretical studies have been carried out to clarify the basic properties and generating mechanism of intermittency. Pomeau and Manneville(1) proposed the simple dynamical models and classified the type of intermittency in terms of the bifurcation. Moreover, the existence of an additional mechanism for generating intermittency was shown by several researchers and it was shown that the attractors confined to a manifold whose dimension is smaller than that of the full phase space is strongly related to the mechanism for the generation of the intermittency(2)-(3). In recent years, intermittency was observed in the neural systems(4) and the research range of intermittency has been expanding rapidly. However, we cannot find the intuitive explanations on the generating mechanism of the intermittency and there is much left to be studied hereafter.

It is needless to say that the excellent mathematical tool is effective to understand the generation mechanism of the intermittency. Of course, the mathematical tool that can analyze the nonlinear system is required to understand the intermittency. Up to now, several mathematical tools have been applied to the analysis of nonlinear systems or the control of nonlinear systems. Within these mathematical tools, the feedback linearization is one of the most famous and interesting ones(5)-(6). The idea of the feedback linearization is to transform a nonlinear system into a linear system with nonlinear control input by using the transformed state vector. This mathematical tool has an advantage that does not
include the approximation entirely. So, we can obtain the exact information on the subjective nonlinear systems if this tool can be applied to the analysis of the subjective nonlinear systems. Moreover, the nonlinear properties of the system emerge via the transformed state vector and the transformed control input\(^{(5)-(6)}\). For this reason, the nonlinear properties of the system can be clarified from the investigations on the transformed state vector and the transformed control input. We can understand the advantage of the feedback linearization easily from these considerations.

In the present study, we applied the feedback linearization to the analysis of the well-known Lorenz model\(^{(7)}\), Rössler model\(^{(8)}\) and BZ reaction model\(^{(9)}\) for the clarification of the generating mechanism of intermittency. The result of preliminary analysis showed that the feedback linearization is possible for these three models. Then, we investigated the behavior of the transformed state vectors and control inputs, and we found that the generating mechanism of the intermittency can be understood intuitively by the collision with the boundary.

2. Feedback Linearization

Consider the following single input nonlinear system

\[
\frac{dx}{dt} = f(x) + g(x)u,
\]

where, \(t\) is time, \(x \in \mathbb{R}^n\) is the state vector, \(u \in \mathbb{R}\) is the control input and \(f, g: \mathbb{R}^n \to \mathbb{R}^n\) are at least \(n-1\) times differentiable functions. According to the existing studies\(^{(5)-(6)}\), the feedback linearization is possible if the following two conditions are satisfied.

(a) Vectors \(ad^k_f g(x)\) with \(k = 0, \cdots, n-1\) are linearly independent.

(b) Distribution \(\text{span} \{ ad^0_f g(x), ad^1_f g(x), \cdots, ad^{n-2}_f g(x) \}\) is involutive.

In these conditions, vector \(ad^k_f g(x)\) is defined by

\[
ad^k_f g(x) = [f, ad^{k-1}_f g](x) \quad (k \neq 0),
\]

\[
ad^0_f g(x) = g(x),
\]

where, \([a, b](x) = (a(x) \in \mathbb{R}^n, b(x) \in \mathbb{R}^n)\) is called as the Lie bracket of vector fields and defined by \(\phi^{(4)-(6)}\) \([a, b](x) = \frac{\partial}{\partial x} \cdot a(x) - \frac{\partial}{\partial x} \cdot b(x)\). The above two conditions deduce the existence of the scalar function \(\phi(x)\), which satisfies the following conditions.

\[
L_{ad^i_f} \phi(x) = 0 \quad (i = 0, 1, 2 \cdots, n-2),
\]

\[
L_{ad^{n-1}_f} \phi(x) \neq 0.
\]

In these equations, \(L_a b(x) = \frac{\partial b}{\partial x} \cdot a(x)\) \((a(x) \in \mathbb{R}^n, b(x) \in \mathbb{R})\) is the Lie derivative. We now introduce the state vector \(\xi \in \mathbb{R}^n\) defined as

\[
\xi = (L^0_f \phi(x), L^1_f \phi(x), \cdots, L^{n-1}_f \phi(x))^T.
\]

Here, \(L^k_f \phi(x)\) with \(k = 0, \cdots, n-1\) is given by

\[
L^k_f \phi(x) = L_f L^{k-1}_f \phi(x) \quad (k = 0, 1, 2 \cdots, n-1),
\]

\[
L^0_f \phi(x) = \phi(x).
\]

The purpose of this derivation is to obtain the state vector \(\xi\), and we should note that this
vector satisfies the following equation (5)-(6). That is,
\[ \frac{d\xi}{dt} = \begin{cases} 0 & 1 & \cdots & 0 \cr \vdots & \ddots & \cdots & \vdots \cr 0 & \cdots & 1 \end{cases} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} + \xi \]
(7)

\[ v = L_g L_f^{-1}(x) \cdot u + L_f \phi(x). \]
(8)

This equation means that the nonlinear properties of the system emerge via the definition of state vector \( \xi \) (Eq.(4)) and control input \( v \) (Eq.(8)). So, the nonlinear properties of the system can be clarified from the investigations on Eqs.(4) and (8). Equation (7) is a linear system with nonlinear control input. The study on a linear system with nonlinear control input has been carried out for a long time, and it was shown that the chaotic behavior can emerge in such system(10).

3. Application to the Lorenz Model

The Lorenz model has been one of the most famous nonlinear systems(7) and a large number of studies on the intermittency in the Lorenz model have been carried out for a long time(1). The Lorenz model is written as
\[ \begin{align*}
\frac{dx_1}{dt} &= -\sigma x_1 + \sigma x_2 \\
\frac{dx_2}{dt} &= -x_2 - x_1 x_3 + r x_1 \\
\frac{dx_3}{dt} &= x_1 x_2 - b x_3
\end{align*} \]
(9)

Here, \( \sigma, r \) and \( b \) are constants. In the present study, we regard the constant \( r \) as the control input after Cresco and Sun(6). So, we convert the symbol \( r \) into \( u \) in order to clarify the correspondence to Eq.(1). From the comparison of Eq.(9) with Eq.(1), we obtain
\[ f(x) = (-\sigma x_1 + \sigma x_2, -x_2 - x_1 x_3, x_1 x_2 - b x_3)^T, \]
(10)

\[ g(x) = (0, x_1, 0)^T. \]
(11)

At first, we consider whether the Lorenz model can satisfy the above two conditions (Eq.(2) and Eq.(3)). In this case, the value of \( n \) is 3 and we can obtain the following three vectors after some manipulations. That is,
\[ \begin{align*}
ad_j^0 g(x) &= \begin{cases} 0 \\ x_1 \\ 0 \end{cases}, \\
ad_j^1 g(x) &= \begin{cases} -\sigma x_1 \\ (1-\sigma)x_1 + \sigma x_2 \\ -x_1^2 \end{cases}, \\
ad_j^2 g(x) &= \begin{cases} -\sigma (1-\sigma) x_1 - 2\sigma^2 x_2 \\ (1-\sigma)x_1^2 + \sigma(1-\sigma)x_2 - 2\sigma x_1 x_2 - x_3^3 \\ (3\sigma - b - 1)x_1^2 - 2\sigma x_1 x_2 \end{cases} \end{align*} \]

Here, we define a constant \( \delta \) as \( \delta = b - 2\sigma \). These numerical formulas mean that these three vectors are linearly independent for the case of \( \delta \neq 0 \). Usually, the values of \( \sigma \) and \( b \) are set at 8/3 and 10 respectively(7). These values deduce the condition of \( \delta \neq 0 \) and mean that the Lorenz model can satisfy the condition (a) mentioned above. Subsequently, let us consider the condition (b). After some manipulations, we obtain
\[ [ad_j^m g, ad_j^n g](x) \in \text{span}\{ad_j^m g(x), ad_j^n g(x)\} \quad (m = 0, 1 \quad \text{and} \quad n = 0, 1). \]
That is, one can see that the Lorenz model can satisfy the condition (b). These results show that the feedback linearization is possible for the Lorenz model under the condition of $\delta \neq 0$.

Next, let us consider the scalar function $\phi(x)$ which satisfies Eqs.(2) and (3). In the Lorenz model, Eq.(2) is written as

$$x_1 = x_2,$$

$$x_2 = x_3.$$

Solving these equations, we obtain

$$\phi(x) = \frac{x_1^2}{2} - \alpha x_3. \quad (12)$$

It is confirmed easily that this function satisfies the condition of Eq.(3). So, we can find that this function satisfies all the required conditions. By using this function, the state vector $\xi$ can be written as

$$\begin{pmatrix}
\xi_1 \\
\xi_2 \\
\xi_3
\end{pmatrix} = \begin{pmatrix}
\frac{1}{2} x_1^2 - \alpha x_3 \\
- \alpha x_1^2 + \sigma bx_3 \\
\sigma \alpha x_1 x_3 + 2\sigma^2 x_1^2 - \sigma b x_3
\end{pmatrix} \quad (13)$$

In addition, the control input $v$ is given by

$$v = p_1 \xi_1 + p_2 \xi_2 + p_3 \xi_3 + g_{NL}(\xi). \quad (14)$$

Here, $p_1$, $p_2$, and $p_3$ are constants. These constants are written as

$$p_1 = \frac{2\alpha b^2 - 8\alpha^2 b - 2\kappa b}{\delta} + 2\sigma^2 b + 2\sigma ub,$$

$$p_2 = \frac{\h^3 - 8\alpha^3 - k(b + 2\alpha)}{\delta} + 2\sigma(\sigma + b) + 2\sigma u,$$

$$p_3 = -\frac{k}{\delta} + 2\sigma \quad (k = \delta + b^2 + \sigma b - 6\alpha^2).$$

Moreover, $g_{NL}(\xi)$ is given by

$$g_{NL}(\xi) = \frac{1}{\delta} \left( b\xi_1 + \xi_2 \right) \left( 2\sigma \xi_1 + \xi_2 \right) + \frac{(b\xi_2 + \xi_3)^2}{2(b\xi_1 + \xi_2)}.$$

Here, we introduce the new state vector $z = (z_1, z_2, z_3)^T$ to simplify the numerical expression, which is defined as

$$\begin{pmatrix}
z_1 \\
z_2 \\
z_3
\end{pmatrix} = \begin{pmatrix}
b\xi_1 + \xi_2 \\
2\sigma \xi_1 + \xi_2 \\
h\xi_2 + \xi_3
\end{pmatrix} \quad (15)$$

By using the state vector $z$, the final expression of the Lorenz model can be written as follows.

$$\begin{pmatrix}
\frac{dz}{dt} = \begin{pmatrix}
0 & 0 & 1 \\
2\sigma & -b & 1 \\
q_1 & q_2 & q_3
\end{pmatrix} z + \begin{pmatrix} 0 \\
0 \\
1
\end{pmatrix} h_{NL}(z), \\
h_{NL}(z) = -\frac{2z_1 z_2}{\delta} + \frac{z_3^2}{2z_1}. \quad (17)$$
Here, \( q_1 = [p_1 - 2\sigma p_2 + 2b\sigma(b + p_2)]/\delta \), \( q_2 = [-p_1 + b p_2 - b^2 (b + p_2)]/\delta \) and \( q_3 = b + p_3 \).

Before discussing the intermittency of the Lorenz model, we show the relation between the state vector \( \mathbf{x} \) and the transformed state vector \( \mathbf{z} \). This relation plays an important role to consider the generating mechanism of the intermittency. From Eqs.(13) and (15), we have

\[
x_1^2 = \frac{2z_1}{\delta}.
\]

This equation means that the range of \( z_1 \) is confined to the region of \( z_1 \geq 0 \) or \( z_1 \leq 0 \) according to the sign of \( \delta \). That is, we can find that the range of \( z_1 \) is limited by the boundary at \( z_1 = 0 \).

The calculated trajectory of the Lorenz model on the plane of \( z_3 = 0 \) is shown in Fig.1. In this figure, the values of \( u, \sigma \) and \( b \) were set at 28, 10 and 8/3 respectively. The Runge-Kutta-Gill method was adopted to obtain the numerical results and the time step width was set at 0.005. In addition, the numerical calculation was carried out within the range of \( 0 \leq t \leq 50 \), and the initial values were set at \( x_1 = x_2 = x_3 = 1 \). This figure shows that we cannot reveal the calculated trajectory by using a closed curve, suggesting the chaotic nature of this system. Moreover, this figure shows that the geometry of the trajectory is almost an ellipse pruned along \( z_1 = 0 \). This geometry means the boundary at \( z_1 = 0 \) has a strong influence on the trajectory of the Lorenz model. To clarify the behavior near the boundary at \( z_1 = 0 \), we show the calculated trajectory within the range of...
16 \leq t \leq 20$ in Fig. 2. From this figure, we can find that the trajectory changes suddenly due to the collision with the boundary and the trajectory after the collision is strongly affected by the trajectory before the collision. These results mean that the generating mechanism of intermittency can be explained qualitatively by the collision with the boundary. As mentioned above, a number of theoretical studies on the generating mechanism of the intermittency have been carried out, and it was suggested that the generating mechanism of intermittency can be described by an aperiodic recurrence mechanism that turns trajectories back toward the unstable fixed point\(^{(11)}\). Of course, the collision with the boundary corresponds to the aperiodic recurrence mechanism. That is, we can find that the feedback linearization can concretize the aperiodic recurrence mechanism that turns trajectories back toward the unstable fixed point. We can understand the meaning of the present study easily from these considerations. Figure 3 shows the calculated trajectory within the range of $5 \leq t \leq 15$. In this figure, the trajectory does not collide with boundary at $z_1 = 0$ and we cannot find the chaotic nature. That is, a system spends a long time near a weakly unstable fixed point. These results suggest that the intermittency is produced by the collision with the boundary, too. It is well known that the Lorenz model has two unstable fixed points under the condition of $u > 1\(^{(11)}\). The positions of the unstable fixed points are given by\(^{(11)}\)

$$x_1 = x_2 = \pm \sqrt{b(u-1)} , \quad x_3 = u - 1 .$$

This result means that the number of the unstable fixed point is one in the case of the present coordinate system, and the position of the unstable fixed point is given by $(z_1, z_2, z_3) = (-6.24 \times 10^2, -4.68 \times 10^3, 2.50 \times 10^4)$ under the conditions of Figs. 1, 2 and 3. Of course, this position is compatible with the results of these figures. These results show that the present coordinate facilitates the understanding of the Lorenz system and we can understand the meaning of the present study easily. Subsequently, we show the calculated trajectory for situations where the intermittency does not appear. Figure 4 shows the calculated trajectory of the Lorenz model under the conditions of $u = 24$, $\sigma = 10$ and $b = 8/3$. From this figure, we can find that the calculated trajectory converges to a certain limit-cycle with the increase in time. Moreover, it should be noted that the collision with the boundary at $z_1 = 0$ does not occur after the lapse of sufficient time. These results suggest that the generating mechanism of intermittency can be explained by the collision with the boundary qualitatively, too.

4. Application to the Rössler Model

The Rössler model\(^{(8)}\) can be written as

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_2 - x_3 \\ x_1 + ax_2 \\ bx_1 - (c - x_1)x_3 \end{pmatrix} .$$

(19)

Here, $a$, $b$ and $c$ are constants. In the present study, we regard the constant $c$ as the control input. So, we convert the symbol $c$ into $u$ in order to clarify the correspondence to Eq.(1). From the comparison of Eq.(19) with Eq.(1), we obtain

$$f(x) = (-x_2 - x_3, x_1 + ax_2, bx_1 + x_1x_3)^T ,$$

(20)

$$g(x) = (0, 0, -x_3)^T .$$

(21)

Moreover, we can obtain the following three vectors.
These numerical formulas mean that these three vectors are linearly independent and mean that the Rössler model can satisfy the condition (a) mentioned above. Subsequently, let us consider the condition (b). After some manipulations, we obtain

$$[ad_f^m, ad_f^n]g(x) \in \text{span}[ad_f^1g(x), ad_f^0g(x)] \quad (m = 0, 1 \text{ and } n = 0, 1).$$

That is, one can see that the Rössler model can satisfy the condition (b). These results show that the feedback linearization is possible for the Rössler model. Next, let us determine the scalar function $\phi(x)$ which satisfies Eqs.(2) and (3). In the Rössler model, Eq.(2) can be written as

$$-x_3 \frac{\partial \phi}{\partial x_3} = 0, \quad -x_3 \frac{\partial \phi}{\partial x_1} - bx_3 \frac{\partial \phi}{\partial x_3} = 0.$$ 

Solving these equations, we obtain

$$\phi(x) = x_2.$$  \hspace{1cm} (22)

As this function satisfies the condition given by Eq.(3), we can obtain the transformed state vector $\xi$ from this function. That is,

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 + ax_3 \\ ax_1 + (a^2 - 1)x_2 - x_3 \end{pmatrix}.$$  \hspace{1cm} (23)

By using the state vector $\xi$, the final expression of the Rössler model can be written as follows.

$$\frac{d\xi}{dt} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xi + \begin{pmatrix} 0 \\ 0 \\ h_{NL}(\xi) \end{pmatrix},$$ \hspace{1cm} (24)

$$h_{NL}(\xi) = (a^2 - b - 1)(\xi_2 - a\xi_3) + a(a^2 - 2)\xi_3 + (a\xi_1 - \xi_2 + u - a)(-\xi_1 + a\xi_2 - \xi_3).$$ \hspace{1cm} (25)

Equation (23) shows the transformed state vector $\xi$ is described by a linear transformation of the state vector $x$. This result means the range of $\xi$ is not confined and the collision with the boundary does not occur as an inevitable result. So, it is easily expected that we cannot find the intermittency in the Rössler model. This result is compatible with the well-known facts. The Rössler model was primarily intended to have some similar properties to the Lorenz model, but also be easier to analyze. \(8\). We can find the many numerical and theoretical studies on the property of the Rössler model, and the difference between the Rössler model and Lorenz model became clear gradually. \(12\). In contrast, the feedback linearization can clarify the basic properties of the nonlinear systems by using the transformed state vector and we can understand the difference between the Rössler model and Lorenz model intuitively. The meaning of the feedback linearization can be easily understood from these considerations.
5. Application to the Analysis of BZ Reaction

The most widely accepted model of the BZ reaction consists of three variables “Oregonator” model\(^9\). The non-dimensional Oregonator model can be written as

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt} \\
\frac{dx_3}{dt}
\end{bmatrix} = \begin{bmatrix}
(s(x_2 - x_1x_2 + x_1 - qx_1^2)) \\
(-x_2 - x_1x_2 + fx_3) / s \\
w(x_1 - x_3)
\end{bmatrix}.
\]

(26)

Here, \(s, q, f\) and \(w\) are constants. In the present study, we regard the constant \(f\) as the control input. So, we convert the symbol \(f\) into \(u\) in order to clarify the correspondence to Eq.(1). From the comparison of Eq.(26) with Eq.(1), we obtain

\[
f(x) = \begin{bmatrix}
(s(x_2 - x_1x_2 + x_1 - qx_1^2)) \\
(-x_2 - x_1x_2 + fx_3) / s \\
w(x_1 - x_3)
\end{bmatrix},
\]

(27)

\[
g(x) = (0, x_3 / s, 0)^T.
\]

(28)

In addition, we can obtain the following three vectors easily.

\[
ad^0_j g(x) = \begin{bmatrix}
0 \\
x_3 / s \\
0
\end{bmatrix}, \quad ad^1_j g(x) = \begin{bmatrix}
-x_3(1-x_1) \\
w(x_1 - x_3) / s + x_3(1 + x_1) / s^2 \\
0
\end{bmatrix},
\]

\[
ad^2_j g(x) = (p_1, p_2, p_3)^T.
\]

Here,

\[
p_1 = sx_3(x_2 - x_1x_2 + x_1 - qx_1^2) + w(x_1 - 1)(x_1 - x_3) + s(-x_2 - 1 + 2qx_1)x_3(1-x_1) - (1-x_1){w(x_1 - x_3) + x_3(1 + x_1) / s},
\]

\[
p_2 = (w + x_3 / s)(x_2 - x_1x_2 + x_1 - qx_1^2) + \{-w / s + (1 + x_1) / s^2\} - x_2x_3(1-x_2) / s + (1 + x_1){w(x_1 - x_3) + x_3(1 + x_1) / s},
\]

\[
p_3 = wx_3(1-x_1).
\]

These numerical formulas show these three vectors are linearly independent and mean the Oregonator model can satisfy the condition (a) mentioned above. Subsequently, let us consider the condition (b). After some manipulations, we obtain

\[
[ad^0_j g, ad^0_j g](x) = [ad^1_j g, ad^1_j g](x) = [ad^2_j g, ad^2_j g](x) = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}.
\]

That is, one can see that the Oregonator model satisfies the condition (b) and the feedback linearization is possible. To obtain the transformed state vector, let us determine the scalar function \(\phi(x)\) which satisfies the Eqs.(2) and (3). In this case, Eq.(2) can be written as

\[
-x_3 \frac{\partial \phi}{\partial x_2} = 0,
\]

\[
-x_3(1-x_1) \frac{\partial \phi}{\partial x_3} + \frac{w}{s}(x_1 - x_3) + x_3(1 + x_1) \frac{\partial \phi}{\partial x_2} = 0.
\]

Solving these equations, we obtain
\[ \phi(x) = x_3. \]  

This function satisfies the condition of Eq.(3). So, we can obtain the transformed state vector \( \xi \) from this function. That is,

\[
\begin{pmatrix}
\xi_1 \\
\xi_2 \\
\xi_3
\end{pmatrix} =
\begin{pmatrix}
x_3 \\
w(x_1 - x_3) \\
sw(x_2 - x_1x_3 + x_1 - qx_1^2) - w^2(x_1 - x_3)
\end{pmatrix}.
\]  (30)

The inverse relation is given by

\[
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} =
\begin{pmatrix}
\xi_1 + \xi_2 / w \\
\xi_3 + w\xi_2 - sw[\xi_1 + \xi_2 / w - q(\xi_1 + \xi_2 / w)^2] \\
sw(1 - \xi_1 - \xi_2 / w)
\end{pmatrix}.
\]  (31)

The third relation in Eq.(30) is a quadratic equation with respect to \( x_1 \). So, the following relation is derived from the condition for the real roots.

\[ D = (-swx_2 + sw - w^2)^2 + 4swq(-\xi_1 + swx_2 + w^2x_1) \geq 0 \]  (32)

We can rewrite this equation as

\[ D / w^2 = (sx_2 - s + w)^2 + 4sxq(x_2 + qx_1 - s + w) \geq 0. \]  (33)

This condition shows the existence of the boundary and the collision with the boundary occurs at \( D = 0 \). As mentioned above, it is expected that the collision with the boundary has a strong relation to the generation mechanism of the intermittency. So, we can expect that the abrupt change of the trajectory occurs at \( D = 0 \). To confirm this expectation, one numerical result on the Oregonator model is shown in Fig.5. This figure shows the change of \( D \) and \( x_1 \) with time, and the following constants and initial values were used after Troy (9).

\[ f = 1.0, \quad q = 8.375 \times 10^{-6}, \quad w = 0.161, \quad s = 77.27, \quad x_1(0) = 1.0, \quad x_2(0) = 900.0 \]

\[ x_3(0) = 19573.0 \]

From the lower panel of this figure, we can find the abrupt switching from the periods of stasis to the large amplitude bursts. Moreover, we can find that the approach of \( D \) to zero corresponds to the abrupt increase in \( x_1 \). This result shows that we can predict the abrupt increase in \( x_1 \) by using the condition of \( D = 0 \), and confirms the validity of the above discussion.

![Fig.5 Change of \( x_1 \) and \( D \) with time](image-url)
6. Discussion and Conclusions

It was suggested that the generating mechanism of intermittency can be described by the aperiodic recurrence mechanism that turns back to the unstable fixed point\(^{(3)}\). So, the elucidation of the aperiodic recurrence mechanism plays an important role to understand the intermittency. In the present study, the feedback linearization was applied to clarify the detailed feature of the aperiodic recurrence mechanism. As a result, we found the collision with the boundary corresponds to the aperiodic recurrence mechanism. Moreover, the feedback linearization was effective to study the detailed features of the boundary. For these reasons, it is expected that the feedback linearization is one of the most excellent tools to elucidate the intermittency.

In connection with these results, we quote the study on the motion of a ball bouncing upon a vertically vibrated plate. This ball is expected to exhibit a simple vertical motion seemingly, but this ball exhibits a complex behavior under the certain conditions\(^{(13)-(14)}\). This fact shows the important role of the collision in the onset of the chaos or the intermittency, and we can find that the bouncing ball and the present study share the problem of the collision with the boundary. So, an analogy between the bouncing ball and the present study will be the interesting subject in future study.

The feedback linearization has been applied to the control of the nonlinear systems for a long time\(^{(5)-(6)}\), and excellent results have been produced by many researchers. However, the feedback linearization has a disadvantageous point that the application range is restricted by the structure of the objective nonlinear system. So, the studies which relax the restriction have been carried out for a long time\(^{(15)}\). Future studies are anticipated.

Chaos or intermittency is not so singular problem in the field of mechanical engineering. In addition, there is a possibility that these factors have a strong effect on the performance of the mechanical systems (ex. laminar-turbulent transition). For this reason, the detailed knowledge on the chaos or intermittency will play the important role to improve the efficiency of the mechanical systems. More detailed study should be performed in future.

References

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