Integrated Design of Structural and Semi-active Control Systems: Inverse Lyapunov Approach*

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Abstract

An integrated design method of structural and semi-active control systems for civil structures is presented. The Vibration Control Device (VCD) that has been developed by authors is used as the semi-active control device. A new semi-active control method, which is referred to as the inverse Lyapunov approach, is proposed. In the inverse Lyapunov approach we firstly assume a bang-bang type semi-active control law based on an unknown Lyapunov function. The Lyapunov matrix that defines the Lyapunov function is optimized so that quantitative performance measures on vibration suppression of civil structures are optimized. The control design problem of the semi-active control law in the inverse Lyapunov approach results in an optimization problem of the Lyapunov matrix to construct the Lyapunov function. In the present study, in addition to the design parameters to determine the Lyapunov matrix, structural design parameters, the stiffness between neighboring two floors of the structural system itself and the parameters of the VCD, are optimized simultaneously so that the structural responses subject to recorded and/or artificial earthquake waves are optimized in the sense of the specifications on vibration suppression of civil structures. We adopt the Genetic Algorithm (GA) to get the optimal Lyapunov matrix and the structural design parameters.

Key words: Semi-Active Control, Civil Structures, Inverse Lyapunov Approach, Integrated Design of Structural and Control Systems, Genetic Algorithm

1. Introduction

An integrated design method of structural and semi-active control systems for civil structures is presented. The Vibration Control Device (VCD) that has been developed by authors is used as the semi-active control device. The VCD is composed of a mechanism of a ball screw with a flywheel and an electric motor with an electric circuit. The VCD is installed between neighboring two floors like general dampers. The large inertial resistance force proportional to the relative acceleration between two floors is generated by the mechanism of the ball screw with the flywheel. The damping resistance force is generated by the electric motor with the electric circuit. The electric circuit is connected to both terminals of the electric motor and is able to change its electric resistance continuously by a command signal voltage. The variable damping property is realized with the electric circuit. A new semi-active control method, which is referred to as the inverse Lyapunov approach, is proposed. In the inverse Lyapunov approach we firstly assume a bang-bang type semi-active control law based on an unknown
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conventional dampers. In Fig. 1, qi integrated design methodology. sign parameters. A simulation study is presented to show the effectiveness of the proposed design methodology.

The structural responses subject to recorded and/or artificial earthquake waves are optimized in the sense of specifications on vibration suppression of civil structures, e.g., relative displacement between neighboring floors and the absolute acceleration of each floor, etc. We adopt the Genetic Algorithm (GA) to get the optimal Lyapunov matrix and the structural design parameters of the VCD. In the present study, in addition to the design parameters to determine the Lyapunov matrix, the structural stiffness between neighboring two floors of the structural system itself and the parameters of the VCD are optimized simultaneously so that the structural responses subject to recorded and/or artificial earthquake waves are optimized in the sense of specifications on vibration suppression of civil structures, e.g., relative displacement between neighboring floors and the absolute acceleration of each floor, etc. We adopt the Genetic Algorithm (GA) to get the optimal Lyapunov matrix and the structural design parameters. A simulation study is presented to show the effectiveness of the proposed design methodology.

Notations are as follows: \( \mathbb{R}^n \): the set of \( n \)-dimensional real vectors, \( \mathbb{R}^{m \times n} \): the set of \( m \times n \) real matrices, \( \mathbf{S}^n \): the set of \( n \)-dimensional, real and symmetric matrices, \( \mathbf{A}^T \): the transpose of a matrix \( \mathbf{A} \), \( t \): time, \( \mathbf{I}_n \): an \( n \)-dimensional identity matrix, \( \mathbf{0}_{m \times n} \): \( m \times n \) zero matrix, \( \delta_{ij} \): Kronecker delta function (\( \delta_{ij} = 1, i = j \), \( \delta_{ij} = 0, i \neq j \)), \( \text{RMS}(a(t)) \): RMS value of a scalar signal \( a(t) \), i.e., \( \text{RMS}(a(t)) = \sqrt{\frac{1}{T_f} \int_0^{T_f} a^2(t)dt} \).

2. Modeling of the Structural System

2.1. Principle of the Vibration Control Device (VCD)

The schematic diagram of the VCD is shown in Fig. 1. The VCD is assumed to be installed between neighboring two floors of a structure, e.g., the \( i \)-th and \((i - 1)\)-th floors, as conventional dampers. In Fig. 1, \( q_i(t) \), \( m_i^{\text{VCD}} \) and \( d_i^{\text{VCD}}(t) \) are the displacement of the \( i \)-th floor, an equivalent mass and a time-varying damping coefficient of the VCD, respectively. The resistance force produced by the relative motion of two floors is denoted by \( f_i^{\text{VCD}}(t) \). The VCD can be viewed as a dynamic system whose input signals are the relative acceleration and the velocity between two floors and the output is the force \( f_i^{\text{VCD}}(t) \). The mathematical model of the VCD is described as the following:

\[
\sum f_i^{\text{VCD}}(t) = m_i^{\text{VCD}}(\ddot{q}_i(t) - \dot{q}_{i-1}(t)) + d_i^{\text{VCD}}(t)(\dot{q}_i(t) - \dot{q}_{i-1}(t)) \quad (1)
\]

The VCD whose resistance force property is given as Eq. (1) has been developed by authors in recent years\(^{(1)}\). The assembly and the design parameters of the VCD in Ref. (1) are shown in Fig. 2 and Table 1 respectively. The VCD is a mechanism consists of a ball screw, a flywheel, an electric motor and an electric circuit to control the damping property of the VCD. The ball screw and the flywheel produce a large inertia force (the term \( m_i^{\text{VCD}}(\ddot{q}_i(t) - \dot{q}_{i-1}(t)) \) in Eq. (1)) which is proportional to the relative acceleration between two floors. The damping force denoted by \( d_i^{\text{VCD}}(t)(\dot{q}_i(t) - \dot{q}_{i-1}(t)) \) in Eq. (1) is generated by the electric motor (generator) and the electric circuit for the energy dissipation. In the designed VCD the coefficients \( m_i^{\text{VCD}} \)
and $d_i\text{VCD}(t)$ in Eq. (1) are given as follows:\(^{(1)}\):

$$m_i\text{VCD} = K_d(l_1 + r^2l_2), \quad d_i\text{VCD}(t) = K_d \frac{r^2}{R_a + R}$$ \hspace{1cm} (2)

where $K_d$, $r$, $l_1$ and $l_2$ are the constant related to the ball screw, the gear ratio, the moment of inertia of the flywheel and that of the electric motor respectively. In the damping coefficient $d_i\text{VCD}(t)$, $K_d$, $R_a$ and $R$ are the torque constant, the back-emf (back electromotive force) constant, the electric resistance of the motor and the resistance connected to the motor terminal respectively. The damping coefficient of the VCD can be controlled by changing the resistance $R$. We have developed an electronic circuit to change the electronic resistance according to a command voltage that is produced by an implemented semi-active control law. More detailed dynamical properties of the VCD are presented in Ref. (1) and we are currently developing larger scale of VCD that is able to produce a large resistance force.

### 2.2. Model of Civil Structures with VCD

Let us consider a structure which is installed $n_{\text{VCD}}$ VCDs, whose inertial and (variable) damping coefficients are denoted by $m_i\text{VCD}$ and $d_i\text{VCD}(t)$ $(i = 1, \ldots, n_{\text{VCD}})$ respectively. The equation of motion of the structure is given as the following:

$$M\ddot{\mathbf{q}}(t) + \mathbf{D}(t)\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{b}_2\ddot{\mathbf{w}}(t) + \mathbf{b}_1(t)\dot{\mathbf{w}}(t) + \mathbf{b}_0\mathbf{w}(t),$$ \hspace{1cm} (3)

$$\mathbf{M} = \mathbf{M}^0 + \mathbf{M}^\text{VCD}, \quad \mathbf{D}(t) = \mathbf{D}^0 + \mathbf{D}^\text{VCD}(t), \quad \mathbf{b}_1(t) = \mathbf{b}_1^0 + \mathbf{b}_1^\text{VCD}(t), \quad \mathbf{b}_2 = \mathbf{b}_2^0 + \mathbf{b}_2^\text{VCD}$$ \hspace{1cm} (4)

where $\mathbf{q}(t) \in \mathbb{R}^n$ and $\mathbf{w}(t) \in \mathbb{R}^n$ are the displacement and the disturbance vectors of the structure respectively. Matrices $\mathbf{M}, \mathbf{D}, \mathbf{K} \in \mathbb{S}^n$ and $\mathbf{b}_j \in \mathbb{R}^{n \times n}$ $(j = 0, 1, 2)$ are the mass, damping, stiffness and influence coefficient matrices. In Eq. (4) matrices $\mathbf{M}^0$, $\mathbf{D}^0$, $\mathbf{b}_1^0$ and $\mathbf{b}_2^0$ are the original mass, damping and influence coefficient matrices of the structure itself without VCDs respectively. On the other hand matrices $\mathbf{M}^\text{VCD}$, $\mathbf{D}^\text{VCD}(t)$, $\mathbf{b}_1^\text{VCD}(t)$ and $\mathbf{b}_2^\text{VCD}$ are those related to the installed VCDs respectively. Note that matrices $\mathbf{D}$ and $\mathbf{b}_1$ accordingly become time varying because they contain the variable damping coefficient of the VCD. The state-space form of Eq. (3) is given as the following:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t),$$ \hspace{1cm} (5)

where

$$\mathbf{x}(t) := \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}, \quad \mathbf{u}(t) := \begin{bmatrix} \mathbf{w}(t) \\ \dot{\mathbf{w}}(t) \end{bmatrix}, \quad \mathbf{A} := \begin{bmatrix} 0_{n \times n} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D}(t) \end{bmatrix}.$$
\[ B := \begin{bmatrix} M^{-1} & 0_{n \times n_{\text{cd}}} \\ b_0 & b_1(t) & b_2 & \cdots \end{bmatrix} \]

All coefficient matrices in Eq. (5) can become time varying because possibly they contain the variable damping coefficient of the VCDs, i.e., matrices \( A \) and \( B \) are functions on the variable damping coefficient \( d_i^{\text{VCD}}(t), i = 1, \ldots, n_{\text{VCD}} \) of the VCDs and can be written as follows:

\[
A = A_0 + \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) A_i, \quad (6)
\]

\[
B = B_0 + \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) B_i, \quad (7)
\]

where

\[
A_0 = \begin{bmatrix} 0_{n \times n_{\text{cd}}} & I_n \\ -M^{-1}K & -M^{-1}D_0 \end{bmatrix}, \quad A_i = \begin{bmatrix} 0_{n \times n_{\text{cd}}} & I_n \\ 0_{n \times n_{\text{cd}}} & -M^{-1}D_i \end{bmatrix},
\]

\[
B_0 = \begin{bmatrix} 0_{n \times n_{\text{cd}}} \\ -M^{-1} \begin{bmatrix} b_0 & b_1 & b_2^T & \cdots \end{bmatrix} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0_{n \times n_{\text{cd}}} & b_1^T & b_2^T & \cdots \end{bmatrix}.
\]

\[
D^{\text{VCD}}(t) = \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) D_i, \quad B_i^{\text{VCD}}(t) = \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) b_i^T, \quad i = 1, \ldots, n_{\text{VCD}}.
\]

Note that all coefficient matrices in Eq. (5) can become time varying because possibly they contain the variable damping coefficient of the VCDs, i.e., matrices \( A \) and \( B \), are functions on the variable damping coefficient \( d_i^{\text{VCD}}, i = 1, \ldots, n_{\text{VCD}} \) of the VCDs.

In this paper we assume each variable damping coefficient of the VCD can be controlled in a following range:

\[
d_i^{\text{VCD}} \leq d_i^{\text{VCD}}(t) \leq \bar{d}_i^{\text{VCD}}, \quad i = 1, \ldots, n_{\text{VCD}}
\]

where \( d_i^{\text{VCD}} \geq 0 \) and \( \bar{d}_i^{\text{VCD}} \geq 0 \) are the maximum and minimum damping coefficients of the \( i \)-th VCD respectively.

3. Semi-active Control: Inverse Lyapunov Approach

For the structural system given in Eq. (5) define a Lyapunov function \( V(t) \) given as the following:

\[
V(t) = x^T(t)Px(t)
\]

where \( P \in S^m, \ P \geq 0 \) is the Lyapunov matrix. Note that the Lyapunov matrix \( P \) is time invariant, i.e., not a function on \( t \). Then with Eqs. (5)-(7) the time derivative of the Lyapunov function is given as the following:

\[
\dot{V}(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)
\]

\[
= (Ax(t) + Bu(t))^T Px(t) + x^T(t)P(Ax(t) + Bu(t))
\]

\[
= x^T(t)(A^T_0 P + PA_0)x(t) + 2u^T(t)B^T_0Px(t)
\]

\[
+ \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) \left[ x^T(t)(A^T_i P + PA_i)x(t) + 2u^T(t)B^T_iPx(t) \right] + \sum_{i=1}^{n_{\text{VCD}}} d_i^{\text{VCD}}(t) \left[ x^T(t)(A^T_i P + PA_i)x(t) + 2u^T(t)B^T_iPx(t) \right] \]

Then the variable damping coefficient \( d_i^{\text{VCD}}(t), i = 1, \ldots, n_{\text{VCD}} \) minimizing \( \dot{V}(t) \) for every time instant is obtained as the following:

\[
d_i^{\text{VCD}}(t) = \begin{cases} \inf \{ d_i^{\text{VCD}}(t) : \dot{V}(t) \leq 0 \} & i = 1, \ldots, n_{\text{VCD}} \\ \sup \{ d_i^{\text{VCD}}(t) : \dot{V}(t) \geq 0 \} & i = 1, \ldots, n_{\text{VCD}} \end{cases}
\]

\[
(11)
\]
Note that the above bang-bang control nature always retains for any Lyapunov matrices \( \mathbf{P} \in \mathbf{S}^{2n} \), \( \mathbf{P} \succeq 0 \), i.e., we always have the above bang-bang control if we would like to minimize \( \dot{V}(t) \), the time derivative of the Lyapunov function \( V(t) \), although timing for switching \( q^{\text{VCD}}(t) \), \( i = 1, \ldots, n_{\text{VCD}} \) can be altered if we take another Lyapunov matrix. In other words we can choose any Lyapunov matrices \( \mathbf{P} \in \mathbf{S}^{2n}, \mathbf{P} \succeq 0 \) and the bang-bang control minimizing \( \dot{V}(t) \) at every time instant \( t \) is always available for the selected \( \mathbf{P} \).

In conventional studies about the semi-active control based on the Lyapunov function, e.g., Ref. (2), the Lyapunov matrix \( \mathbf{P} \) is selected as the following:

\[
\mathbf{P} = \frac{1}{2} \begin{bmatrix}
\mathbf{K} & \mathbf{0}_{n\times n} \\
\mathbf{0}_{n\times n} & \mathbf{M}
\end{bmatrix}
\]  

(12)

For the Lyapunov matrix \( \mathbf{P} \) in Eq. (12) the Lyapunov function \( V(t) \) becomes the sum of the kinetic and the potential energies at the time instant \( t \) given by

\[
V(t) = \frac{1}{2} \left\{ \mathbf{q}^T(t)\mathbf{M}\mathbf{q}(t) + \mathbf{q}^T(t)\mathbf{K}\mathbf{q}(t) \right\}.
\]  

(13)

In this case the energy dissipation rate that is defined as \(-\dot{V}(t)\) is maximized with the bang-bang control law in Eq. (11). The control law is physically reasonable in a qualitative sense because we can easily imagine that there is a strong link between the swift energy dissipation and the good vibration suppression in the structural system.

However, although the above estimation may be qualitatively valid, it is difficult to show a concrete proof that selecting the Lyapunov matrix as Eq. (12), i.e., selecting the Lyapunov function \( V(t) \) as the sum of the kinetic and the potential energies as Eq. (13), can directly lead to the quantitative performance improvement on vibration suppression of civil structures, e.g., the relative displacement between two neighboring stories and the absolute acceleration of each storey etc..

In the present paper, to address the above problem of the Lyapunov-based semi-active control we propose a new approach that is referred to as the inverse Lyapunov approach by using the fact about the invariant bang-bang control structure of the Lyapunov-based semi-active control scheme. In this approach we firstly assume the bang-bang type control for an unknown Lyapunov matrix \( \mathbf{P} \) so that the time derivative of the Lyapunov function corresponding to the matrix \( \mathbf{P} \) is minimized. Under the assumption we search the matrix \( \mathbf{P} \in \mathbf{S}^{2n}, \mathbf{P} \succeq 0 \) so that the quantitative performance on vibration suppression that can be obtained by the simulated response of the structural system subject to recorded or artificial earthquake disturbances is optimized under some parameterizations of the matrix \( \mathbf{P} \). The approach is a reversed version of the conventional Lyapunov-based bang-bang type semi-active control that firstly defines the Lyapunov function \( V(t) \) like Eq. (13) and the bang-bang control law is obtained to minimize \( \dot{V}(t) \) (maximize the dissipation rate \(-V(t)\)). In the present inverse Lyapunov approach we search the Lyapunov matrix \( \mathbf{P} \) so that the simulated response of the semi-active control system subject to earthquake waves becomes good in a sense of quantitative criteria on vibration control of civil structures. The proposed inverse Lyapunov approach, the optimization of the Lyapunov function \( V(t) \) (the Lyapunov matrix \( \mathbf{P} \)) with the premise of the bang-bang control law, is expected to achieve the better performance than that of the standard Lyapunov-based semi-active control.

However if we consider all elements of the Lyapunov matrix \( \mathbf{P} \) as the design parameters in the optimization problem directly the number of the parameters in the optimization may become prohibitive. For the \( n \)-dof structural system the Lyapunov matrix \( \mathbf{P} \) becomes \( 2n \times 2n \) real symmetric positive semi-definite matrix. Thanks to the symmetric property of the matrix \( \mathbf{P} \), the number \( n_p \) that we need to optimize becomes

\[
n_p = \frac{2n(2n+1)}{2}.
\]

For relatively small \( n \), say \( n = 3 \) (three storey building), \( n_p \) becomes \( n_p = \frac{(2\times3)(2\times3+1)}{2} = 21 \). Maybe it is acceptable but for larger \( n \), e.g., \( n = 20, n_p = \frac{(2\times20)(2\times20+1)}{2} = 820 \).
To address the problem we parameterize the Lyapunov function as the following:

\[ V(t) = g'V'(t) + \sum_{i=1}^{n-1} g_j^i V_j'(t) + \sum_{j=1}^{m} g_j^j V_j'(t) + \sum_{k=1}^{m} g_k^k V_k'(t), \quad (14) \]

\[ g', g_j^i, g_j^j, g_k^k \geq 0, \quad i = 1, \ldots, n - 1, \quad j = 1, \ldots, m, \quad k = 1, \ldots, m \]

where \( V'(t), V_j'(t), V_j^j(t) \) and \( V_k'(t), j = 1, \ldots, n - 1, \quad k = 1, \ldots, m \) are defined as follows respectively:

\[ V'(t) = \frac{1}{2} \{ q^T(t) M_0 q(t) + q^T(t) K q(t) \}, \quad (15) \]

\[ V_j'(t) = c_j^2(t), \quad i = 1, \ldots, n - 1, \quad (16) \]

\[ V_j^j(t) = s_j^2(t), \quad j = 1, \ldots, m, \quad V_k'(t) = s_k^2(t), \quad k = 1, \ldots, m \quad (17) \]

where \( r_{i+1}(t), \quad i = 1, \ldots, n - 1 \) is the relative displacement between neighboring two floors given as \( r_{i+1}(t) = q_{i+1}(t) - q_i(t), i = 1, \ldots, n - 1 \). Components \( V_j^j(t) \) and \( V_k'(t) \) are the squared \( j \)-th modal displacement \( s_j(t) \) and the \( k \)-th modal velocity \( \dot{s}_k(t) \) respectively. The modal displacement is obtained by following a procedure from the displacement of each floor \( q_i(t), \quad i = 1, \ldots, n \). The equation of motion of the structural system given in Eq. (3) can be rewritten as the following modal form:

\[ \ddot{s}(t) + \Gamma(t) \dot{s}(t) + \Omega^2 s(t) = u_m \dot{w}(t) + b_m \ddot{w}(t) + c_m w(t), \quad (18) \]

\[ s(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_n(t) \end{bmatrix}^T, \quad q(t) = T s(t), \quad T \in \mathbb{R}^{2n \times 2n}, \quad T^T M T = I_{2n} \]

\[ \Omega = \text{diag}(\omega_1, \ldots, \omega_n), \quad 0 < \omega_1 < \omega_2 < \cdots < \omega_n, \quad \Omega^2 = T^T K T, \]

\[ \Gamma(t) = \Gamma_0 + \sum_{i=1}^{n \times CP} \alpha_i(t) I_i, \quad \Gamma_0 = \text{diag}(\gamma_1^0, \ldots, \gamma_m^0) = T^T D_0 T, \quad \Gamma_i = \text{diag}(\gamma_1^i, \ldots, \gamma_m^i) = T^T D_i T, \]

\[ h_0 = T^T b_0, \quad h_1(t) = T^T b_1(t), \quad h_2 = T^T \theta, \]

where \( \omega_i \), \( i = 1, \ldots, n \) is the \( i \)-th natural frequency that is obtained as the solution to the following generalized eigenvalue problem:

\[ \omega_i^2 M t_i = K t_i, \quad t_i, \quad i = 1, \ldots, n \quad (19) \]

where \( t_i \in \mathbb{R}^{n} \), \( i = 1, \ldots, n \) is the eigenvector corresponding to the eigenvalue \( \omega_i^2 \) and satisfies \( t_i^T M t_j = \delta_{ij}, \quad i, \quad j = 1, \ldots, n \). The matrix \( T \) in Eq. (18) is defined as the following:

\[ T = \begin{bmatrix} t_1 & \cdots & t_n \end{bmatrix} \quad (20) \]

With the matrix \( T \) in Eq. (20) we have the modal displacement vector \( s(t) \in \mathbb{R}^n \) from the absolute displacement vector \( q(t) \) as

\[ s(t) = U q(t), \quad U = \begin{bmatrix} u_1 & \vdots & u_n \end{bmatrix} = T^{-1}, \quad u_i \in \mathbb{R}^{1 \times n}, \quad i = 1, \ldots, n. \quad (21) \]

and each element of the vector \( s(t) \) denoted by \( s_i(t), \quad i = 1, \ldots, n \) is the \( i \)-th modal displacement. The modal velocity vector \( \dot{s}(t) \in \mathbb{R}^n \) can be obtained with the almost same way described above by taking

\[ \dot{s}(t) = U \dot{q}(t), \quad (22) \]

instead of Eq. (21). Then, the Lyapunov function in Eq. (14) is rewritten as the following:

\[ V(t) = x^T(t) \left( g' P' + \sum_{i=1}^{n-1} g_j^i P_i' + \sum_{j=1}^{m} g_j^j P_j' + \sum_{k=1}^{m} g_k^k P_k' \right) x(t) \quad (23) \]
where

$$P^* = \begin{bmatrix} K & 0_{nxn} \\ 0_{nxn} & M_0 \end{bmatrix}, \quad P^*_i = \begin{bmatrix} (C_i^T C_i)^{-1} & 0_{gxn} \\ 0_{gxn} & 0_{gxn} \end{bmatrix},$$

$$C_i = \begin{bmatrix} 1 & 0_{1x(n-2)} \\ 0_{1x(n-2)} & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0_{1x(i-1)} & 1 & 0_{1x(n-i-1)} \end{bmatrix}, i = 2, \ldots, n - 2,$$

$$C_{i-1} = \begin{bmatrix} 0_{1x(n-3)} & 1 & 0_{1x(n-i-1)} \end{bmatrix}, \quad C_n = \begin{bmatrix} 0_{1x(n-2)} & 1 \end{bmatrix},$$

$$P^*_j = \begin{bmatrix} u_j^T u_j & 0_{gxn} \\ 0_{gxn} & 0_{gxn} \end{bmatrix}, \quad j = 1, \ldots, m, \quad P^*_k = \begin{bmatrix} 0_{gxn} & 0_{gxn} \\ 0_{gxn} & u_k^T u_k \end{bmatrix}, \quad k = 1, \ldots, m.$$

We optimize the weighting factors for each component of the Lyapunov function $V(t)$ in Eq. (14), denoted by $g_i \geq 0$, $g_i^f \geq 0$, $i = 1, \ldots, n - 1$, $g_j^d \geq 0$, $j = 1, \ldots, m$ and $g_k^d \geq 0$, $k = 1, \ldots, m$ to improve the control performance on vibration suppression.

Under the parameterization as Eq. (14) the number of design parameters $n_p$ becomes $n_p = 1 + (n - 1) + 2m$ where $n$ and $m$ are the number of dof of the structural systems and the number of the employed modes of vibration respectively. In the above $n_p$, 1 is for $g_i$, $n - 1$ is for $g_i^f$, $i = 1, \ldots, n - 1$ and $2m$ is for $g_j^d$, $j = 1, \ldots, m$ and $g_k^d$, $k = 1, \ldots, m$ in Eq. (23). For example if we set $n = 20$ ($n$ is the number of storey) and $m = 3$, i.e., lower three modes of vibration are employed for the performance evaluation, $n_p = 1 + (20 - 1) + 2 \cdot 3 = 26$ that is much smaller than the case when each element of the Lyapunov matrix is taken directly as the design parameter ($n_p = 820$ that is shown in the above). This fact implies the current parameterization of the Lyapunov function makes the proposed inverse Lyapunov approach to be tractable for realistic civil structures with relatively high degrees of freedom.

4. Integrated Design of the Structural System and the Semi-active Control Law

Under the semi-active control law in § 3 the integrated design of the structural and control systems is conducted. The integrated design of the structural and control system is the simultaneous optimal design of design parameters existing both in the control object and controller. The integrated design problem was firstly considered in control system design for the actively controlled large space structures in 1980’s to achieve the small structural weight and the good vibration suppression simultaneously(6). In contrast to the standard structural or control system design problem no analytical methods for the global optimal solution to the integrated design problem have been found and some iterative procedure to obtain a locally optimal solution have been proposed so far(7)–(9). In fact, even in the simplest case, the integrated design problem becomes a BMI (Bilinear matrix inequality) optimization problem(9) that is NP hard problem, i.e., the global optimal solution cannot be obtained with the acceptable amount of computation.

In the present study an integrated design problem of civil structural systems with the semi-active control based on the inverse Lyapunov approach is considered. As structural design parameters we employ the stiffness $k_i$, $i = 1, \ldots, n$ between neighboring two floors, the equivalent mass of the VCDs $m_{i}^{VCD}$, $i = 1, \ldots, n_{VCD}$ and the maximum damping coefficient of the VCD $d_j^{VCD}$, $j = 1, \ldots, n_{VCD}$. Those structural design parameters can be adjusted in the following range:

$$k_i \leq k_i \leq (k_i)_u, \quad i = 1, \ldots, n$$

(24)

$$m_j^{VCD} \leq m_j^{VCD} \leq (m_j^{VCD})_u, \quad j = 1, \ldots, n_{VCD},$$

(25)

$$d_j^{VCD} \leq d_j^{VCD} \leq (d_j^{VCD})_u, \quad j = 1, \ldots, n_{VCD}$$

(26)

where the symbols with the subscripts _u and _i are the upper and the lower values of design parameters respectively.
The control design parameters are the weighting factors $q_c \geq 0$, $q_d^i \geq 0$, $i = 1, \ldots, n - 1$, $q_d^f \geq 0$, $j = 1, \ldots, m$ and $q_d^e \geq 0$, $k = 1, \ldots, m$ (Eq. (23)) in the semi-active control based on the inverse Lyapunov approach. By changing those weighting factors the performance of the semi-active control can be optimized.

We define the integrated design problem as a minimization problem of a performance index defined as the following:

$$J = \sum_{i=1}^{n_e} J_i, \quad J_i = \sum_{j=1}^{4} w_j^{i} J_j, \quad w_j^{i} \geq 0, \quad i = 1, \ldots, n_e, \quad j = 1, \ldots, 4. \quad (27)$$

where $n_e$ is the number of earthquake waves employed to obtain the simulated structural responses in the optimization. The objective function $J$ is the weighted sum of $J_i$, $i = 1, \ldots, n_e$, $j = 1, \ldots, 4$ with the weighting factor $w_j^{i} \geq 0$, $i = 1, \ldots, n_e$, $j = 1, \ldots, 4$. Components $J_j$, $i = 1, \ldots, n_e$, $j = 1, \ldots, 4$ are defined as follows respectively:

$$J_1^i = \sum_{k=1}^{n} \frac{\text{RMS}(r_k^i(t))}{\text{RMS}(0)} \quad (28)$$

$$J_2^i = \sum_{k=1}^{n} \frac{\text{RMS}(a_k^i(t))}{\text{RMS}(0)} \quad (29)$$

$$J_3^i = \sum_{k=1}^{n} \frac{\max_{0 \leq t \leq T_j^{i}} |r_k^i(t)|}{\max_{0 \leq t \leq T_j^{i}} |r_0^i(t)|} \quad (30)$$

$$J_4^i = \sum_{k=1}^{n} \frac{\max_{0 \leq t \leq T_j^{i}} |a_k^i(t)|}{\max_{0 \leq t \leq T_j^{i}} |a_0^i(t)|} \quad (31)$$

$i = 1, \ldots, n_e$, $k = 1, \ldots, n - 1$, $l = 1, \ldots, n$

where $r_k^i$, $a_k^i$ and $T_j^{i}$, $i = 1, \ldots, n_e$, $k = 1, \ldots, n$ are the relative displacement of all neighboring two floors including the one between the 1st floor and the ground ($r_1(t) = q_1(t) - w(t)$, $r_k(t) = q_k(t) - q_k(t-1), k = 2, \ldots, n$), the absolute acceleration of k-th floor and the duration of the i-th earthquake disturbance respectively. Note that each RMS value of the structural response is calculated over the earthquake duration only. The free structural vibration after the ending time of the earthquake disturbance is not included in evaluating the RMS value. The superscripts $^\ast$ and $^0$ represent two cases, the case of the semi-active control with VCD and that without control, i.e., each response with the superscript $^0$ is the structural response without VCDs. Note that components in Eqs. (28)-(31) are similar to the performance indices that are widely used as quantitative indices for evaluating the control performance on vibration suppression of civil structures$^{(4),(5)}$.

In the formulated optimization problem we cannot describe the relationship between the performance index and the design parameters in the structural and the control systems because of the nonlinear nature of the control law and the performance index. Hence, we cannot apply gradient methods that are generally efficient on the convergence property to the formulated integrated design problem. In the present study the Genetic Algorithm (GA) is adopted as the optimization method without gradient information. It is well known that GA is used in various types of optimization problems that are difficult to obtain the gradient of the objective function on the design parameter and shows nice results in many applications although we cannot guarantee any convergence properties in a deterministic sense. Note also that, however, thanks to the non-gradient-based property and the population search strategy of GA the possibility of a convergence to bad locally optimal solutions, which is the most serious problem in traditional gradient-based optimization methods, can be reduced.

5. Simulation Example

We consider a fifteen story building with three VCDs as an example of the present semi-active control. The VCDs are installed in the lowest three inter-stories, i.e., between the
ground and the 1st floor, 1st and 2nd, and the 2nd and 3rd respectively. The schematic model and the physical parameters of the building are shown in Fig. 3. Coefficient matrices in Eqs. (3) and (4), when the displacement vector $\mathbf{q}(t)$ is defined as $\mathbf{q}(t) = [q_1(t), q_2(t), \ldots, q_{15}(t)]^T$, are given as follows:

$$
\mathbf{M}_0 = \begin{bmatrix}
m_1 & 0 & \cdots & 0 \\
0 & m_2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & m_{15}
\end{bmatrix}, \\
\mathbf{M}^{\text{VCD}} = \begin{bmatrix}
m_{1\text{VCD}} & -m_2^{\text{VCD}} & \cdots & 0 \\
-m_2^{\text{VCD}} & m_2^{\text{VCD}} + m_3^{\text{VCD}} & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & m_3^{\text{VCD}} + m_4^{\text{VCD}}
\end{bmatrix},
$$

$$
\mathbf{M}_e = \begin{bmatrix}
k_1 + k_2 & -k_2 & \cdots & 0 \\
-k_2 & k_2 + k_3 & \cdots & -k_3 \\
\vdots & \ddots & \ddots & \vdots \\
0 & -k_3 & k_3 + k_4 & \cdots
\end{bmatrix},
$$

$$
\mathbf{K} = \begin{bmatrix}
\vdots & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & m_{15}^{\text{VCD}}
\end{bmatrix},
$$

$$
\mathbf{D}^p = \beta \mathbf{K}, \quad \beta = 10^{-2}, \\
\mathbf{D}^{\text{VCD}}(t) = \begin{bmatrix}
\mathbf{D}_0(t) & \mathbf{0}_{3\times 12} \\
\mathbf{0}_{12\times 3} & \mathbf{0}_{12\times 12}
\end{bmatrix}.
$$

Values of the nominal structural parameters $m_i$ and $k_i^{n}$ where $k_i^{n}$, $i = 1, \ldots, 15$ are nominal structural stiffness are shown in Table 2. The nominal parameters of VCDs are shown in the 2nd column of Table 3. The maximum and the minimum values of the stiffness and parameters of VCDs in Eqs. (24)-(26) are defined as $(k_i)^n = 0.5 k_i^{n}, (k_i)^m = 2(k_i)^n, i = 1, \ldots, 15, (m_j^{\text{VCD}})^n = 0.5(m_j^{\text{VCD}})^m, (m_j^{\text{VCD}})^m = 2(m_j^{\text{VCD}})^n, (d_j^{\text{VCD}})^n = 0.5(d_j^{\text{VCD}})^m$, and $(d_j^{\text{VCD}})^m = 2(d_j^{\text{VCD}})^n, j = 1, \ldots, 3$ where the parameters with the superscript $n$ are the nominal values of the corresponding design parameters shown in Tables 2 and the 2nd column of Table 3, respectively. For the optimization of the structural and the control design parameters (the weighting factors in Eq. (23)) we take four recorded and artificial earthquake waves, i.e., $n_e = 4, i = 1$: El Centro NS (1940), $i = 2$: BCJL1, $i = 3$: Hachinohe NS (1968) and $i = 4$: JMA Kobe NS (1995).
Table 3 Nominal and optimal design parameters of VCDs

<table>
<thead>
<tr>
<th>Design parameter [Unit]</th>
<th>Nominal</th>
<th>Optimal</th>
<th>Optimal (2-step)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_{\text{VCD}}) [kg]</td>
<td>(1.20 \times 10^7)</td>
<td>(6.03 \times 10^6)</td>
<td>(2.34 \times 10^7)</td>
</tr>
<tr>
<td>(m_{\text{VCD}}) [kg]</td>
<td>(1.20 \times 10^7)</td>
<td>(6.66 \times 10^6)</td>
<td>(2.19 \times 10^7)</td>
</tr>
<tr>
<td>(m_{\text{VCD}}) [kg]</td>
<td>(1.20 \times 10^7)</td>
<td>(6.50 \times 10^6)</td>
<td>(7.56 \times 10^6)</td>
</tr>
<tr>
<td>(d_{\text{VCD}}) [Ns/m]</td>
<td>(4.80 \times 10^7)</td>
<td>(9.32 \times 10^6)</td>
<td>(9.26 \times 10^6)</td>
</tr>
<tr>
<td>(d'_{\text{VCD}}) [Ns/m]</td>
<td>(4.80 \times 10^7)</td>
<td>(8.55 \times 10^6)</td>
<td>(2.96 \times 10^7)</td>
</tr>
<tr>
<td>(d''_{\text{VCD}}) [Ns/m]</td>
<td>(4.80 \times 10^7)</td>
<td>(9.53 \times 10^6)</td>
<td>(8.07 \times 10^7)</td>
</tr>
</tbody>
</table>

Note that all earthquake waves are scaled so that the peak ground acceleration (PGA) becomes 4.0 [m/s²]. The weighting factors in the objective function \(J\) in Eq. (27) are determined as \(w_j = 1, i = 1, \ldots, n, j = 1, \ldots, 4\) respectively.

To include the effect of the delay of the VCDs (such delays are unavoidable in general semi-active control devices) the variable damping coefficient of each VCD is assumed to be governed by the following first order differential equation:

\[
\frac{d(d_j^{\text{VCD}}(t))}{dt} = \frac{1}{T_d} d_j^{\text{VCD}}(t) + \frac{1}{T_d} d_j^f(t), \quad j = 1, 2, 3
\]

where \(d_j^f(t), j = 1, 2, 3\) and \(T_d > 0\) are the command signal for the \(j\)-th VCD and the time constant of the VCD respectively. Note that the command signal \(d_j^f(t)\) takes two values, i.e., \(d_j^f(t) = d_j^{VCD}\) or \(d_j^{\text{VCD}}\), \(j = 1, 2, 3\). The time constant is defined as \(T_d = 0.02\) [s] in the present simulation.

In the proposed integrated design approach all design parameters, i.e., the structural stiffness \((k_i, i = 1, \ldots, n, n = 15)\), the parameters of VCDs \((m_j^{\text{VCD}}\) and \(d_j^{\text{VCD}}, j = 1, 2, 3)\) and the factors in the inverse Lyapunov approach in Eq. (14) are simultaneously optimized with GA as shown in the previous section. In the present example, the 2-step design approach, which is the conventional control system design methodology, is also applied to the present optimal design problem for the comparison purposes. The result of the proposed integrated design method is compared with that of the 2-step design method. The outline of the 2-step design approach is summarized as the following:

**Step 1:** The structural stiffness parameters \(k_i, i = 1, \ldots, n, n = 15\) are firstly optimized in the range given by Eq. (24) so that the following performance index is minimized:

\[
J' = \sum_{i=1}^{n} w_j J_j', \quad w_j = 1, i = 1, \ldots, n, j = 1, \ldots, 4
\]

where

\[
(J_j')_i = \sum_{k=1}^{n} \text{RMS} \left(0 r_k^j(t)\right), \quad (J_j')_2 = \sum_{k=1}^{n} \text{RMS} \left(0 a_k^j(t)\right),
\]

\[
(J_j')_3 = \sum_{k=1}^{n} \max_{0 \leq t_j \leq T_j} \|r_k^j(t)\|, \quad (J_j')_4 = \sum_{k=1}^{n} \max_{0 \leq t_j \leq T_j} \|a_k^j(t)\|.
\]

**Step 2:** For the optimized structural stiffness obtained in Step 1 the design parameters of VCD and the factors in the inverse Lyapunov approach are optimized in the ranges in Eqs. (25) and (26) so that the performance index \(J\) in Eq. (27) is minimized.

Note that GA is also employed to obtain the optimal solution to each optimization problem in the above 2-step design and the same earthquake waves as those in the integrated design approach are employed to obtain the structural responses.

In the above 2-step design approach the structural response without VCDs are optimized in the sense of the RMS and the peak values firstly. Then the optimal design parameters of the VCDs and the semi-active control law are obtained for the optimal structure obtained with the structural optimization in Step 1 of the above 2-step design. In the present simulation example the performance of the semi-active control system is compared between the proposed integrated design case and the conventional 2-step design case.
The optimal structural stiffness values $k_i$, $i = 1, \ldots, 15$ in the integrated design case and the 2-step design case are shown in Fig. 4 with their maximum and minimum values respectively. The nominal structural stiffness is also shown in the figure for comparison. The values of structural stiffness in lower floors, especially from the 1st to 3rd ones that are VCDs are installed, become smaller than nominal stiffness values both in the cases of the integrated design and the 2-step design. Moreover it is also found that the values of the structural stiffness in middle (upper) floors in the integrated design case are smaller (larger) than those of the 2-step design case.

The optimized design parameters of the VCDs are shown in the 3rd and 4th columns of Table 3 where the 3rd column shows the result of the integrated design and the 4th column shows that of the 2-step design respectively. In the case of the integrated design all the values of the equivalent mass of the VCDs ($m_{VCD}^j$, $j = 1, 2, 3$) become smaller and the maximum damping coefficients ($d_{VCD}^j$, $j = 1, 2, 3$) become larger. On the other hand values of $m_{VCD}^1$, $m_{VCD}^2$, $d_{VCD}^1$ and $d_{VCD}^3$ become larger and the values of $m_{VCD}^3$ and $d_{VCD}^2$ become smaller in the case of the 2-step design method shown in the above.

The result for El Centro NS earthquake is shown in Fig. 5 and Table 4. The RMS and peak values of the relative displacement between neighboring two floors and the absolute acceleration of each floor are depicted in four cases, i.e., NC (Integrated design), SA (Integrated design) and Pon (Integrated design) are the result without VCDs, the result of the proposed semi-active control and the case where all variable damping coefficients $d_{VCD}^j(t)$, $j = 1, 2, 3$ are kept at their maximum respectively in the case of the integrated design, and SA (2-step design) is the case of the semi-active control in the case of the 2-step design. We can see that the best control performance is achieved in the case SA (integrated design) compared to other two cases, NC (Integrated design) and Pon (Integrated design), with a small increase of
the relative displacement in lower floors. Especially the absolute acceleration of each floor is reduced by more than 25% values compared to the case Pon both in the RMS and the peak values. Furthermore from 4th to 15th floor, values of relative displacement between neighboring two floors are reduced by more than 18%. Moreover the control performance of the case SA (Integrated design) is better than that of the case SA (2-step design) with a small performance degradation of the relative displacement in some middle floors. The result strongly supports the advantage of the proposed integrated design approach over the conventional 2-step design methodology because the performance of the integrated design approach cannot be achieved with the 2-step design method (Note: Because of the non-convex and the (possibly) non-differentiable characteristics of the optimization problem and the employed optimization method (GA) we cannot guarantee that we can always get the better control performance with the integrated design method than that when we take the 2-step design method. However, at least we can say that the better performance is achieved with the integrated design method than that when we take the 2-step design method.

Table 4  Result for El Centro NS (1940) earthquake

<table>
<thead>
<tr>
<th>Quantity</th>
<th>NC_{RMS}</th>
<th>SA_{RMS}</th>
<th>Pon_{RMS}</th>
<th>NC_{peak}</th>
<th>SA_{peak}</th>
<th>Pon_{peak}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$ [mm]</td>
<td>3.68</td>
<td>3.10</td>
<td>1.83</td>
<td>0.84</td>
<td>16.96</td>
<td>12.21</td>
</tr>
<tr>
<td>$r_2$ [mm]</td>
<td>5.52</td>
<td>1.48</td>
<td>1.29</td>
<td>0.27</td>
<td>1.15</td>
<td>18.36</td>
</tr>
<tr>
<td>$r_3$ [mm]</td>
<td>4.79</td>
<td>1.24</td>
<td>1.14</td>
<td>0.26</td>
<td>1.08</td>
<td>15.45</td>
</tr>
<tr>
<td>$r_4$ [mm]</td>
<td>1.55</td>
<td>0.44</td>
<td>0.60</td>
<td>0.28</td>
<td>0.74</td>
<td>4.67</td>
</tr>
<tr>
<td>$r_5$ [mm]</td>
<td>1.44</td>
<td>0.45</td>
<td>0.61</td>
<td>0.31</td>
<td>0.75</td>
<td>3.76</td>
</tr>
<tr>
<td>$r_6$ [mm]</td>
<td>3.50</td>
<td>1.18</td>
<td>1.57</td>
<td>0.34</td>
<td>0.75</td>
<td>9.37</td>
</tr>
<tr>
<td>$r_7$ [mm]</td>
<td>4.67</td>
<td>1.56</td>
<td>2.10</td>
<td>0.33</td>
<td>0.75</td>
<td>15.12</td>
</tr>
<tr>
<td>$r_8$ [mm]</td>
<td>3.99</td>
<td>1.26</td>
<td>1.71</td>
<td>0.32</td>
<td>0.74</td>
<td>14.80</td>
</tr>
<tr>
<td>$r_9$ [mm]</td>
<td>1.96</td>
<td>0.58</td>
<td>0.80</td>
<td>0.29</td>
<td>0.72</td>
<td>7.78</td>
</tr>
<tr>
<td>$r_{10}$ [mm]</td>
<td>5.36</td>
<td>1.46</td>
<td>2.11</td>
<td>0.27</td>
<td>0.69</td>
<td>21.84</td>
</tr>
<tr>
<td>$r_{11}$ [mm]</td>
<td>2.34</td>
<td>0.61</td>
<td>0.91</td>
<td>0.26</td>
<td>0.66</td>
<td>9.67</td>
</tr>
<tr>
<td>$r_{12}$ [mm]</td>
<td>2.44</td>
<td>0.61</td>
<td>0.96</td>
<td>0.25</td>
<td>0.64</td>
<td>10.23</td>
</tr>
<tr>
<td>$r_{13}$ [mm]</td>
<td>2.28</td>
<td>0.56</td>
<td>0.91</td>
<td>0.24</td>
<td>0.62</td>
<td>9.63</td>
</tr>
<tr>
<td>$r_{14}$ [mm]</td>
<td>2.40</td>
<td>0.58</td>
<td>0.97</td>
<td>0.24</td>
<td>0.60</td>
<td>6.27</td>
</tr>
<tr>
<td>$r_{15}$ [mm]</td>
<td>1.46</td>
<td>0.36</td>
<td>0.60</td>
<td>0.24</td>
<td>0.59</td>
<td>6.27</td>
</tr>
<tr>
<td>$q_1$ [m/s²]</td>
<td>0.68</td>
<td>0.30</td>
<td>0.41</td>
<td>0.45</td>
<td>0.75</td>
<td>4.70</td>
</tr>
<tr>
<td>$q_2$ [m/s²]</td>
<td>0.67</td>
<td>0.25</td>
<td>0.36</td>
<td>0.37</td>
<td>0.69</td>
<td>3.13</td>
</tr>
<tr>
<td>$q_3$ [m/s²]</td>
<td>0.80</td>
<td>0.22</td>
<td>0.33</td>
<td>0.27</td>
<td>0.66</td>
<td>3.56</td>
</tr>
<tr>
<td>$q_4$ [m/s²]</td>
<td>0.86</td>
<td>0.20</td>
<td>0.31</td>
<td>0.23</td>
<td>0.65</td>
<td>3.72</td>
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<tr>
<td>$q_5$ [m/s²]</td>
<td>0.89</td>
<td>0.21</td>
<td>0.32</td>
<td>0.23</td>
<td>0.65</td>
<td>3.70</td>
</tr>
<tr>
<td>$q_6$ [m/s²]</td>
<td>0.88</td>
<td>0.19</td>
<td>0.32</td>
<td>0.22</td>
<td>0.61</td>
<td>3.37</td>
</tr>
<tr>
<td>$q_7$ [m/s²]</td>
<td>0.77</td>
<td>0.16</td>
<td>0.30</td>
<td>0.21</td>
<td>0.54</td>
<td>3.46</td>
</tr>
<tr>
<td>$q_8$ [m/s²]</td>
<td>0.66</td>
<td>0.16</td>
<td>0.30</td>
<td>0.24</td>
<td>0.52</td>
<td>3.35</td>
</tr>
<tr>
<td>$q_9$ [m/s²]</td>
<td>0.58</td>
<td>0.15</td>
<td>0.28</td>
<td>0.26</td>
<td>0.54</td>
<td>3.03</td>
</tr>
<tr>
<td>$q_{10}$ [m/s²]</td>
<td>0.38</td>
<td>0.13</td>
<td>0.21</td>
<td>0.34</td>
<td>0.61</td>
<td>1.37</td>
</tr>
<tr>
<td>$q_{11}$ [m/s²]</td>
<td>0.37</td>
<td>0.13</td>
<td>0.19</td>
<td>0.35</td>
<td>0.67</td>
<td>1.47</td>
</tr>
<tr>
<td>$q_{12}$ [m/s²]</td>
<td>0.43</td>
<td>0.13</td>
<td>0.19</td>
<td>0.32</td>
<td>0.70</td>
<td>1.93</td>
</tr>
<tr>
<td>$q_{13}$ [m/s²]</td>
<td>0.54</td>
<td>0.15</td>
<td>0.21</td>
<td>0.27</td>
<td>0.68</td>
<td>2.08</td>
</tr>
<tr>
<td>$q_{14}$ [m/s²]</td>
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<td>0.28</td>
<td>0.25</td>
<td>0.62</td>
<td>2.95</td>
</tr>
<tr>
<td>$q_{15}$ [m/s²]</td>
<td>0.83</td>
<td>0.20</td>
<td>0.34</td>
<td>0.24</td>
<td>0.59</td>
<td>3.53</td>
</tr>
</tbody>
</table>

The time histories of the variable damping coefficient of $j$-th VCD ((VCD)$_j$), $j = 1, 2, 3$ and the peak values of the resistance forces are shown in Fig. 6 and Table 5. The variable damping coefficients $d_v^{(CD)}(t)$, $j = 1, 2, 3$ are switched between their maximum and minimum values according to the proposed semi-active control based on the inverse Lyapunov approach. Intermediate values between the maximum and minimum values can be seen because of the delay of the VCDs modeled as Eq. (32). The peak values are shown also in the case Pon for comparison. The peak values of the resistance forces in the case SA take similar values to those of the case Pon. The above results show the effectiveness of the proposed semi-active control because the better control performance without excessive resistance forces compared to those of the passive control, i.e., the case Pon, is achieved in the semi-active control. The result for JMA Kobe NS earthquake is also shown in Figs.
Table 5  Peak values of the resistance forces of VCD$_j$, $j = 1, 2, 3$ for El Centro NS (1940) earthquake

<table>
<thead>
<tr>
<th>VCD$_j$</th>
<th>SA [N]</th>
<th>Pon [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCD$_1$</td>
<td>$1.51 \times 10^7$</td>
<td>$2.69 \times 10^6$</td>
</tr>
<tr>
<td>VCD$_2$</td>
<td>$1.42 \times 10^7$</td>
<td>$9.23 \times 10^6$</td>
</tr>
<tr>
<td>VCD$_3$</td>
<td>$2.54 \times 10^6$</td>
<td>$2.91 \times 10^6$</td>
</tr>
</tbody>
</table>

Fig. 6  Variable damping coefficients of the VCDs for El Centro NS (1940) earthquake

Fig. 7  Result for JMA Kobe NS (1995) earthquake

Fig. 8  Variable damping coefficients of the VCDs for JMA Kobe NS (1995) earthquake

7, 8 and Tables 6, 7. Similar results to the case of El Centro NS earthquake is achieved and the control performance achieved by the integrated design is better than that achieved by the 2-step design.

Furthermore the structural response of the semi-active control system for Taft NS (1952) earthquake wave (PGA=4.0 [m/s$^2$]) that is not employed in the GA-based optimization process is obtained. The objective of the present simulation is to evaluate if the proposed semi-active control system shows good control performance for the forthcoming unknown earthquake or not, in other words, if the present semi-active control system has a certain amount of performance robustness or not. The results show in Figs. 9, 10 and Tables 8, 9. We can see that the present semi-active control system still achieves good performance on vibration suppression compared with the cases NC and Pon. Furthermore the control system obtained with the integrated design shows the higher performance on vibration suppression compared with that obtained with the 2-step design.

With those simulation results the present integrated design method is a reasonable way to obtain a good structural system and a semi-active control law for the structural system in a simultaneous manner.
Table 6  Result for JMA Kobe NS (1995) earthquake

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\text{NC}_{\text{RMS}}$</th>
<th>$\text{SA}_{\text{RMS}}$</th>
<th>Peak values</th>
<th>$\text{NC}_{\text{Peak}}$</th>
<th>$\text{SA}_{\text{Peak}}$</th>
<th>Peak values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$ [mm]</td>
<td>4.59</td>
<td>3.35</td>
<td>3.59</td>
<td>0.73</td>
<td>0.93</td>
<td>21.53</td>
</tr>
<tr>
<td>$r_2$ [mm]</td>
<td>6.57</td>
<td>3.15</td>
<td>1.98</td>
<td>0.48</td>
<td>1.59</td>
<td>31.75</td>
</tr>
<tr>
<td>$r_3$ [mm]</td>
<td>5.40</td>
<td>1.92</td>
<td>1.76</td>
<td>0.36</td>
<td>1.09</td>
<td>26.27</td>
</tr>
<tr>
<td>$r_4$ [mm]</td>
<td>1.60</td>
<td>0.64</td>
<td>0.92</td>
<td>0.40</td>
<td>0.69</td>
<td>7.48</td>
</tr>
<tr>
<td>$r_5$ [mm]</td>
<td>1.36</td>
<td>0.66</td>
<td>0.95</td>
<td>0.49</td>
<td>0.69</td>
<td>5.35</td>
</tr>
<tr>
<td>$r_6$ [mm]</td>
<td>3.30</td>
<td>1.76</td>
<td>2.53</td>
<td>0.53</td>
<td>0.69</td>
<td>15.42</td>
</tr>
<tr>
<td>$r_7$ [mm]</td>
<td>4.88</td>
<td>2.41</td>
<td>3.46</td>
<td>0.49</td>
<td>0.70</td>
<td>25.03</td>
</tr>
<tr>
<td>$r_8$ [mm]</td>
<td>4.65</td>
<td>2.00</td>
<td>2.87</td>
<td>0.43</td>
<td>0.70</td>
<td>22.63</td>
</tr>
<tr>
<td>$r_9$ [mm]</td>
<td>2.46</td>
<td>0.94</td>
<td>1.35</td>
<td>0.38</td>
<td>0.69</td>
<td>10.99</td>
</tr>
<tr>
<td>$r_{10}$ [mm]</td>
<td>7.07</td>
<td>2.44</td>
<td>3.54</td>
<td>0.35</td>
<td>0.69</td>
<td>31.82</td>
</tr>
<tr>
<td>$r_{11}$ [mm]</td>
<td>3.15</td>
<td>1.04</td>
<td>1.51</td>
<td>0.33</td>
<td>0.69</td>
<td>13.80</td>
</tr>
<tr>
<td>$r_{12}$ [mm]</td>
<td>3.33</td>
<td>1.07</td>
<td>1.57</td>
<td>0.32</td>
<td>0.68</td>
<td>14.07</td>
</tr>
<tr>
<td>$r_{13}$ [mm]</td>
<td>3.13</td>
<td>1.00</td>
<td>1.47</td>
<td>0.32</td>
<td>0.68</td>
<td>13.22</td>
</tr>
<tr>
<td>$r_{14}$ [mm]</td>
<td>3.31</td>
<td>1.06</td>
<td>1.57</td>
<td>0.32</td>
<td>0.68</td>
<td>14.29</td>
</tr>
<tr>
<td>$r_{15}$ [mm]</td>
<td>2.02</td>
<td>0.65</td>
<td>0.96</td>
<td>0.32</td>
<td>0.68</td>
<td>8.88</td>
</tr>
</tbody>
</table>

$q_1$ [m/s²] 0.76 0.58 0.61 0.76 0.95 5.81 4.87 4.73 0.84 1.03
$q_2$ [m/s²] 0.90 0.43 0.54 0.47 0.78 5.40 3.74 4.22 0.69 0.88
$q_3$ [m/s²] 1.18 0.36 0.50 0.30 0.71 5.19 2.87 3.88 0.55 0.74
$q_4$ [m/s²] 1.26 0.34 0.49 0.27 0.70 5.61 2.50 3.55 0.45 0.71
$q_5$ [m/s²] 1.29 0.35 0.50 0.27 0.70 5.75 2.43 3.43 0.42 0.71
$q_6$ [m/s²] 1.25 0.33 0.48 0.26 0.68 5.87 2.19 3.06 0.37 0.72
$q_7$ [m/s²] 1.07 0.29 0.44 0.27 0.66 5.09 1.77 2.94 0.35 0.60
$q_8$ [m/s²] 0.86 0.28 0.43 0.33 0.66 4.07 1.78 2.59 0.44 0.69
$q_9$ [m/s²] 0.73 0.27 0.41 0.37 0.66 3.81 1.64 2.22 0.43 0.74
$q_{10}$ [m/s²] 0.42 0.22 0.33 0.54 0.68 2.53 1.36 1.94 0.54 0.70
$q_{11}$ [m/s²] 0.41 0.22 0.31 0.52 0.69 2.73 1.35 2.02 0.49 0.67
$q_{12}$ [m/s²] 0.53 0.22 0.31 0.41 0.69 2.78 1.26 1.90 0.45 0.66
$q_{13}$ [m/s²] 0.72 0.24 0.35 0.34 0.69 3.44 1.37 2.32 0.40 0.59
$q_{14}$ [m/s²] 0.97 0.31 0.45 0.32 0.68 4.05 2.16 3.40 0.53 0.63
$q_{15}$ [m/s²] 1.14 0.37 0.54 0.32 0.68 5.01 2.68 4.06 0.53 0.66

Table 7  Peak values of the resistance forces of VCD, $j=1,2,3$ for JMA Kobe NS (1995) earthquake

<table>
<thead>
<tr>
<th>VCD</th>
<th>SA [N]</th>
<th>Pon [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCD₁</td>
<td>$3.64 \times 10^5$</td>
<td>$3.45 \times 10^5$</td>
</tr>
<tr>
<td>VCD₂</td>
<td>$8.60 \times 10^6$</td>
<td>$7.97 \times 10^6$</td>
</tr>
<tr>
<td>VCD₃</td>
<td>$6.23 \times 10^6$</td>
<td>$3.60 \times 10^6$</td>
</tr>
</tbody>
</table>

Fig. 9  Result for Taft NS (1952) earthquake

6. Conclusion

An integrated design method of structural and semi-active control systems for civil structural systems has been presented. The VCD (Vibration Control Device) that has been developed by authors is adopted for the semi-active control. A new semi-active control law referred to as the inverse Lyapunov approach is proposed. In the inverse Lyapunov approach the Lyapunov matrix that is used for the bang-bang type semi-active control based on Lyapunov function is searched so that the control performance of the semi-active control is optimized. The integrated optimal design problem is formulated as the simultaneous optimization prob-
Fig. 10 Variable damping coefficients of the VCDs for Taft NS (1952) earthquake

Table 8 Result for Taft NS (1952) earthquake

<table>
<thead>
<tr>
<th>Quantity</th>
<th>NC RMS</th>
<th>SA RMS</th>
<th>Pon RMS</th>
<th>NC peak</th>
<th>SA peak</th>
<th>Pon peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>r1 [mm]</td>
<td>8.82</td>
<td>11.71</td>
<td>14.64</td>
<td>1.33</td>
<td>0.80</td>
<td>1.16</td>
</tr>
<tr>
<td>r2 [mm]</td>
<td>13.60</td>
<td>6.50</td>
<td>6.10</td>
<td>0.48</td>
<td>1.07</td>
<td>0.70</td>
</tr>
<tr>
<td>r3 [mm]</td>
<td>12.32</td>
<td>5.85</td>
<td>5.46</td>
<td>0.47</td>
<td>1.07</td>
<td>0.70</td>
</tr>
<tr>
<td>r4 [mm]</td>
<td>4.30</td>
<td>1.93</td>
<td>2.33</td>
<td>0.45</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>r5 [mm]</td>
<td>4.23</td>
<td>1.94</td>
<td>2.35</td>
<td>0.46</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>r6 [mm]</td>
<td>10.43</td>
<td>4.86</td>
<td>5.90</td>
<td>0.47</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>r7 [mm]</td>
<td>13.24</td>
<td>6.20</td>
<td>7.54</td>
<td>0.47</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>r8 [mm]</td>
<td>10.37</td>
<td>4.82</td>
<td>5.88</td>
<td>0.47</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>r9 [mm]</td>
<td>4.62</td>
<td>2.12</td>
<td>2.59</td>
<td>0.46</td>
<td>0.82</td>
<td>0.53</td>
</tr>
<tr>
<td>r10 [mm]</td>
<td>11.40</td>
<td>5.13</td>
<td>6.30</td>
<td>0.45</td>
<td>0.81</td>
<td>0.53</td>
</tr>
<tr>
<td>r11 [mm]</td>
<td>4.63</td>
<td>2.06</td>
<td>2.53</td>
<td>0.44</td>
<td>0.81</td>
<td>0.53</td>
</tr>
<tr>
<td>r12 [mm]</td>
<td>4.56</td>
<td>2.00</td>
<td>2.47</td>
<td>0.44</td>
<td>0.81</td>
<td>0.53</td>
</tr>
<tr>
<td>r13 [mm]</td>
<td>4.05</td>
<td>1.77</td>
<td>2.19</td>
<td>0.44</td>
<td>0.81</td>
<td>0.53</td>
</tr>
<tr>
<td>r14 [mm]</td>
<td>4.10</td>
<td>1.79</td>
<td>2.22</td>
<td>0.44</td>
<td>0.81</td>
<td>0.53</td>
</tr>
<tr>
<td>r15 [mm]</td>
<td>2.45</td>
<td>1.07</td>
<td>1.33</td>
<td>0.44</td>
<td>0.80</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 9 Peak values of the resistance forces of VCD<sub>j</sub>, j = 1, 2, 3 for Taft NS (1952) earthquake

<table>
<thead>
<tr>
<th>VCD&lt;sub&gt;j&lt;/sub&gt;</th>
<th>SA [N]</th>
<th>Pon [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCD&lt;sub&gt;1&lt;/sub&gt;</td>
<td>4.11 x 10&lt;sup&gt;7&lt;/sup&gt;</td>
<td>3.85 x 10&lt;sup&gt;7&lt;/sup&gt;</td>
</tr>
<tr>
<td>VCD&lt;sub&gt;2&lt;/sub&gt;</td>
<td>1.99 x 10&lt;sup&gt;7&lt;/sup&gt;</td>
<td>2.06 x 10&lt;sup&gt;7&lt;/sup&gt;</td>
</tr>
<tr>
<td>VCD&lt;sub&gt;3&lt;/sub&gt;</td>
<td>7.89 x 10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>3.84 x 10&lt;sup&gt;6&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

The problem of the structural stiffness, the design parameters of the VCD and the Lyapunov matrix for the inverse Lyapunov approach. The performance index related to the structural response for the recorded and the artificial earthquake disturbance is optimized with Genetic Algorithm (GA). The simulation results for the 15-storey building show the effectiveness of the proposed design approach.
References


