Active Control of Vibrations in a Rolling Process by Nonlinear Optimal Controller*

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Abstract

In this paper a method of suppressing vibrations in an industrial rolling process with varying rotational frequency is presented. The vibrations in a rolling process are problematic as they do not only cause structural fatigue on the machinery, but also deteriorate the quality of the end product. The traditional approach for this type of a problem would be to avoid the critical frequencies of the process by changing the rotation speed of the reel, thus decreasing the vibrations. However, in practice this is hard to achieve as the radial velocity of the reel should be constant, while rotation speed of the reel varies depending on the diameter of the reel. The changes in radial velocity are typically not allowed as the rolling process is usually part of a larger process, where change in rotation speed affects the whole process. This paper introduces a generic modified LQ-control law to tackle the problem. This control design was designed in a previous ACRVEM-project (Active Control of Radial Rotor Vibrations in Electric Machines) and has been successfully used in suppression of radial rotor vibrations in electric drives, resulting in 90% damping of the vibrations. The major drawback of the controller has been the limitations due to its linear nature; the control is applicable only at a certain predefined rotational frequency, and outside this frequency the controller becomes unstable. In order to resolve this problem, a nonlinear optimal state-feedback controller based on continuous gain scheduling is introduced. This modification of the original controller is capable of suppressing the vibrations over the whole operational frequency range. In this paper the modelling and identification of the rolling process is first discussed. After the model has been obtained the control design procedures for both linear and nonlinear controllers are presented in detail. The performances of both controllers are analyzed in extensive simulations. Finally the simulation results are validated by implementing the designed controllers in the actual rolling process. It will be shown that this control methodology is highly effective for this type of vibration damping problems resulting in over 90% decrease in the vibrations over the whole frequency range of rotation.

Key words: Active Control of Vibrations, Continuous Gain Scheduling, LQ-Control, Nonlinear Optimal Control, Rolling Process

1. Introduction

Cost effectiveness and quality of the end product are some of the key issues in industry today. Both of these quantities have an impact on the earnings of a company, and therefore efforts to increase them are constantly made. The overall cost effectiveness is a sum of many factors, like the efficiency of the production process and the related maintenance costs. The quality of the end product is closely related to the production process. There exist
several factors that may deteriorate the process performance. One of these is the unwanted vibrations occurring in the process. The vibration may cause severe wear and tear of the process components. This structural fatigue results in increased maintenance costs and down times as some of the components may need to be replaced. In the worst case the wear and tear may cause process break downs and even pose a severe threat for the safety. Beside the structural fatigue, the impact of vibrations may be seen in the end product as a variation of quality in the produced material. The overall efficiency of the process may be limited by some design restrictions set by the possible vibrations, which is the case in electric motor design for example. It is thus evident that just by damping the unwanted vibrations the process performance can be enhanced(1).

Vibrations are a characteristic feature of mechanical systems and cannot be completely avoided; however the question remains whether the process can be designed such that these vibrations are minimal during the normal operation. The forces exciting the vibrations can arise from several different sources. Rotating components within a process are one of these sources and most of the industrial processes have at least one of these rotating components, be it the rotor of an electric motor or a reel of a rolling process.

The vibration forces in rotating components are typically excited by some rotor imbalances or misalignments of the shafts, and therefore the excitation frequency is usually closely related to the rotation speed of the rotor and its harmonic multiples. In a sense this makes the damping problem both easier and more difficult to solve as the excitation frequency is tied to the rotation speed, which results in a periodic disturbance making the problem well defined. On the other hand, the change of operation speed also changes the excitation frequency making the design problem significantly harder. The process structure should be designed such that none of its natural frequencies gets excited at any normal operation mode. This is however seldom possible as these components are typically part of a bigger production process that sets its own design restrictions. One way to overcome the problem is to implement a damper into the process that suppresses the vibrations.

In rotating processes the frequency of vibration excitation can be very broad, thus the passive damping methods become ineffective in the sense of overall damping. This problem can be tackled by the means of active vibration control, in which a counter force is produced by an actuator to compensate the vibration force. This results in very effective damping of vibrations at any desired frequency within the operational range of the actuator. The problem of vibration damping now transforms into a control problem, allowing the use of all traditional and well proven design and analysis methods. The major drawbacks of active vibration control are its relatively high price, required computing capacity and the possibility of creating an unstable closed-loop system. During the past decades the computing capacity has significantly increased while it has become less expensive. This development has made active vibration control a practical and useful approach. Although it is a relatively new field of research, numerous extensive results have already been obtained. Some of the application fields among many other include helicopters(2),(3), magnetic bearings(4), electric motors(5),(6) and automobiles(7).

In this paper the active control of vibrations in an industrial rolling process is considered. The vibrations are induced by a reel with a varying rotation frequency. The required control forces are generated by a hydraulic actuator acting also as the support for the reel. The focus of this paper is in control design for such processes, including both the linear and nonlinear control approaches, as well as in the extensive simulations and tests with the actual process. The control design methods presented in this paper, namely the LQ-design and gain-scheduling are well known and in wide use. However, the gain-scheduling is usually related to control schemes in which the control parameters are picked from some predefined look-up table. The control strategy presented in this paper applies gain scheduling of the controller parameters over a continuous set of optimal
controllers, effectively forming an autonomous and continuous nonlinear optimal control law with the scheduling variable as an additional exogenous input; hence the authors prefer to refer the controller as nonlinear instead of gain-scheduled – although in a sense it is both. The linear control methods presented in this paper have been tested with success in a previous project related to active control of radial rotor vibrations in an electric machine (ACRVEM). In that process the active vibration control resulted in 97% damping\(^8\).

This paper is structured as follows. The test process and the problem formulation are presented in §2. Section 3 describes the control design in detail. The simulations and test runs with the actual process are presented in §4. The conclusions and discussion are made in §5.

2. Problem formulation

Vibration control in a rolling process is quite problematic as the rollers are usually part of a larger system that sets tight limits on the rotation speed of the rotor. The surface speed of the rolled material is usually required to be static and thus the actual rotation speed of the reel varies as a function of its diameter. This poses several problems for the control design. In this paper the reel is assumed to have a constant diameter and varying rotation frequency. This assumption simplifies and linearizes the process model, while preserving the original problem related to the varying rotation frequency. The goal of the control effort is to minimize the reel vibrations. The studied test-bed process and the related modelling aspects are presented in the following subsections.

2.1. Test process

The test process is an industrial rolling process consisting of a reel, hydraulic actuator and a force sensor. The natural frequency of the process is 39Hz. The hydraulic actuator acts both as the source of the control forces as well as a support for the reel. The actuator is connected on the support structures through a force sensor, providing the information on the forces acting on the reel. A diagram of the process is given in Fig. 1.

![Diagram of the process](image)

Figure 1 An industrial rolling process

The pressure of the hydraulic actuator is controlled by a valve. The stem position in the valve is controller with a voltage signal ranging from -10V to +10V corresponding to ±100% opening of the two-way valve. The force sensor produces a voltage signal proportional to the sensed force. The only measurements available from the process are the rotation speed of the reel and the force sensor output. A common practice in vibration control is to measure acceleration, speed or position of the vibrating component\(^9\). Although setting up one of these measurements is easy, the sole force measurement was found a better choice as it was already available in the process and it provides the same information as the acceleration. So instead of minimizing the acceleration of the reel, the force measured by the sensor is minimized. One should note though that this leads to some further problems as the vibration sensed as force is not necessarily zero mean.
The output of the force sensor is used as the control parameter. It is sampled at 1kHz rate and fed into a dSpace system in which the control law is implemented. After computing the required control action the dSpace system converts it into a 16 bit digital output. This is converted into an analogue voltage signal that is fed through a current amplifier. The amplified signal is then fed to the hydraulic valve producing the required control forces. The preceding control scheme is shown in Fig. 2.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{setup.png}
\caption{The test setup and control scheme}
\end{figure}

2.2. Modelling

The process can be described with two partial models, one corresponding to the dynamics of the process and one describing the dynamics of the disturbance. The vibrations can then be interpreted as a force disturbance at the process output that is sensed by the force sensor. The process can be considered as a representation of the dynamics related to the actuator and the reel. The interaction of the partial models is illustrated in Fig. 3.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{models.png}
\caption{Interaction of the partial models}
\end{figure}

The studied process is a very complex one that has numerous natural frequencies. However most of these frequencies are conducted from the support structures and can thus be neglected without the loss of generality as far as the damping of the reel vibrations is considered. The resulting model describes the conversion of the control signal through the dynamics of the actuator and the reel into a force output. The input of the process is the actuator voltage and the output is the sensed force. The process can be described with the following state-space representation:

\begin{equation}
\begin{aligned}
\dot{x}_{\text{pro}}(t) &= A_{\text{pro}} x_{\text{pro}}(t) + B_{\text{pro}} u(t) \\
y_{\text{pro}}(t) &= C_{\text{pro}} x_{\text{pro}}(t)
\end{aligned}
\end{equation}

where $u(t)$ is the control voltage and $y_{\text{pro}}(t)$ is the produced force.
The preceding model is obtained through black-box identification of time-domain data. The identification procedure is described in the following subsection.

A common way of expressing the disturbance is sinusoidal signal acting at some desired frequency (3). This choice for vibration model can easily be justified because after the initial transient a harmonic disturbance in a linear stable process converges into a sinusoidal signal regardless of the point of excitation. The traditional approach assumes the disturbance to be zero mean. This assumption is not applicable for the given process and therefore the disturbance model has to be slightly modified to take the bias factor into account. This is obtained by introducing a new static state. A self-excited biased sinusoidal signal can be expressed as:

\[
\begin{align*}
    \dot{x}_s(t) &= \begin{pmatrix}
    -\omega_s^2 & -\varepsilon & 0 \\
    0 & 1 & 0 \\
    0 & 0 & -\varepsilon
    \end{pmatrix}
    \begin{pmatrix}
    x_s(t) \\
    x(t)
    \end{pmatrix}, \\
    d(t) &= \begin{pmatrix}
    1 & 0
    \end{pmatrix}
    \begin{pmatrix}
    x_s(t)
    \end{pmatrix}
\end{align*}
\]

where \(\omega_s\) is the disturbance frequency and \(\varepsilon > 0\) is a design parameter that is zero unless stated otherwise. The corresponding initial values are \(x_s(0) = [A \quad 0 \quad b]^T\), where \(A\) is the vibration amplitude and \(b\) is the value of the bias.

2.3. Identification

The process model is obtained through black-box identification of time-domain data obtained from the process measurements. The input of the process is the control voltage fed into the actuator and the output is the measured force. The input signal was chosen as a sinusoidal sweep signal due to the stiffness of the hydraulic actuator that prohibits the use of white-noise or impulse signals. The sweep is made from 5Hz to 150Hz as it is the feasible operation range of the actuator and the process operation frequency is assumed to stay well within these limits. The amplitude of the signal is chosen to be \(\pm 2\) V as it is high enough to provide the full control force, yet low enough to stay within the linear operation area of the valve. The sampling rate for the measurements is chosen as 1kHz, which is high enough to capture all the desired phenomena and low enough to preserve the process dynamics. The data is pre-processed before the identification is carried out. In pre-processing the data is filtered with an ideal frequency domain band-pass filter with lower cut-off frequency set at 5Hz and higher at 150Hz. This corresponds to the frequency band on which the information of the input signal is contained. The identification is made by a traditional prediction error method (PEM), which is an iterative method minimizing the error between a simulated model output and the actual measurements (10). A 6th order state-space representation is chosen as the model structure used in the identification. The appropriate model order can be approximated from the frequency spectrum of the measured data. After the model has been identified it has to be validated. The validation is carried out in both frequency and time domain by comparing the measured process output against the simulated model output. The validation results are shown in Figs. 4 and 5.
The fit of the model in time domain is ~76% which can be considered adequate for a high order process approximated with a low order model. The fit in frequency domain is clearly better with only slight variation from the measured data. The validation results suggest that the model is applicable for the given process.

2.4. Model composition

Before any control design can be done, the partial models have to be combined into a single plant. The plant describes the dynamics of a rolling process subject to an output force disturbance. The model is combined according to the layout in Fig. 3. The state-space representation can be given as:

\[
\begin{align*}
\dot{x}(t) &= A_{\text{pro}} x(t) + B_{\text{pro}} u(t) \\
y(t) &= C_{\text{pro}} x(t)
\end{align*}
\]

where \( u(t) \) is the control voltage and \( y(t) \) is the measured net force.

3. Control design

After the model describing the process has been obtained, a controller is designed to tackle the vibration problem. Two different control laws are designed for the process. First a linear quadratic optimal controller is designed. It has however a very limited range of operation in the sense of the rotation frequency. Thus it is not effective enough to yield adequate performance for the given problem. Therefore a nonlinear extension of the linear control law capable of damping the vibrations over the whole frequency range is presented. The design procedures for both controllers are described in the following subsections.
3.1. Linear controller

The linear controller is implemented as an optimal state feedback controller that minimizes a quadratic cost function (LQ-control). The controller consists of two parts that can be designed separately, namely the optimal feedback gain $L$ and a state-estimator. Some of the intermediate phases of the design are omitted here and can be found in(8),(11).

The optimal feedback is such that it minimizes a quadratic cost function:

$$J = \int_0^\infty \left( z(\tau)^T Q z(\tau) + u(\tau)^T R u(\tau) \right) d\tau,$$

where $z(t)$ is the minimized performance variable, $Q$ is the associated weighting matrix chosen as unity matrix and $R$ is the weighting for the applied control. The performance variable $z(t) = C_s x(t)$ is chosen as the net forces acting at the point of disturbance excitation. For an output error process this corresponds to $z(t) = y(t)$. In the given problem the bias term of the disturbance is not to be controlled, thus

$$z(t) = [C_{pre} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}] y(t) .$$

Substitution of this into Eq. (4) yields the traditional LQ-problem:

$$J = \int_0^\infty \left( x(\tau)^T C_s^T Q C_s x(\tau) + u(\tau)^T R u(\tau) \right) d\tau,$$

and the associated optimal control effort becomes:

$$u^*(t) = -R^{-1}B^T S(t)x^*(t),$$

where $^*$ denotes the optimality and $S(t)$ is the solution of the Riccati Equation ($S(t_0) = 0$):

$$-\dot{S}(t) = A^T S(t) + S(t)A - S(t)BR^{-1}B^T S(t) + Q.$$

This equation has to be solved numerically, which poses a few problems for the given problem. Even as the chosen model structure fulfils all the theoretical demands set for the Riccati equation to have a solution, the numerical solution may not exist. The numerical methods require the process to have no uncontrollable poles on the imaginary axis$^{(12)}$ in order for a unique solution to exist. The model for sinusoidal disturbance has two of such poles. In order to find a solution for the problem, the design parameter $\varepsilon$ in Eq. (2) is given a small value. This corresponds to adding a slight damping to the disturbance and perturbing its poles from the imaginary axis. It can be shown that by choosing $10^{-10} \leq \varepsilon \leq 10^{-8}$ the difference between the damped and undamped ($\varepsilon = 0$) sinusoidal disturbance signal is minimal within a well-defined time horizon. This choice of $\varepsilon$ also guarantees that all of submatrices of $S$ are finite.

The ratio of the disturbance signal with and without damping can be given by (for $\varepsilon << 1$):

$$J_{rel} = \frac{y_s(t)}{y_1(t)} = e^{-\frac{\varepsilon \tau}{2}},$$
where \( y_1(t) \) is the undamped sinusoidal signal, \( y_2(t) \) is the damped sinusoidal signal and \( \varepsilon = 2\zeta \omega \) is the design parameter.

A criterion for a feasible maximal optimization horizon can be derived from Eq. (8) as:

\[
t_f \leq -\frac{2 \ln(J_{\text{out}})}{\varepsilon},
\]

where \( J_{\text{out}} \) is the lowest allowable ratio between the damped and undamped disturbance signals and \( t_f \) is the corresponding maximal optimization horizon.

**Remark:** In practice the optimization horizon can be set infinite, corresponding to the stationary solution of Eq. (7) where \( \dot{S}(t) = 0 \). This assumption is possible as the original servo problem (creating a control signal that follows the disturbance signal in opposite phase) has been converted into a regulator problem, hence \( \dot{S}(t) \) converges very close to its final value well within feasible optimization horizon \( t_f \) given by Eq. (9) when the numerical solution of Eq. (7) is used.

The second part of the controller design process is the state-estimator design. As the process is fully observable the design can be carried out in a straightforward fashion. The common approach is to use Kalman-filter based estimation\(^{13}\). For the given process a traditional state-observer is chosen for the task as it is computationally lighter, the impact of possible measurement noise is negligible and it simplifies the design of the nonlinear controller. The traditional state-observer can be given as\(^{14}\):

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K \{ y(t) - \hat{y}(t) \},
\]

where \( \hat{x}(t) \) are the estimated states, \( y(t) \) is the measured process output and \( \hat{y}(t) \) is the estimated output.

The dynamics of the relative estimation error can be given as\(^{9}\):

\[
\dot{x}_n(t) = A_n x_n(t) + B_n u_n(t),
\]

where \( A_n = A^T, \ B_n = C^T, \ x_n(t) \) is an auxiliary variable and \( u_n(t) = -K_n x_n(t) \).

This corresponds to the duality problem, where the observability of the pair \((A,C)\) (the original system) can be studied as the controllability of the pair \((A^T,C^T)\) in Eq. (11).

The state-weighting matrix \( K \) can be solved as an optimal solution of a linear quadratic problem that minimizes a cost function in terms of estimation error convergence:

\[
J_{\text{obs}} = \int_0^\infty \left( x_n(\tau)^T Q_{\text{obs}} x_n(\tau) + u_n(\tau)^T R_{\text{obs}} u_n(\tau) \right) d\tau,
\]

where \( Q_{\text{obs}} \) and \( R_{\text{obs}} \) are the weighting matrices for the state estimation error and the sensitivity to measurements.

The final phase in controller design is to combine the estimator dynamics and the optimal feedback law. The dynamics of the resulting linear time invariant optimal controller can be written as:

\[
\begin{align*}
\dot{x}_{\text{cont}}(t) &= (A - BL - KC)x_{\text{cont}}(t) + Ky(t) \\
 u(t) &= -Lx_{\text{cont}}(t)
\end{align*}
\]

where \( x_{\text{cont}}(t) \) is the state vector of the optimal controller, \( y(t) \) is the measured output and \( u(t) \) is the optimal control effort.
In the implementation phase this controller can be discretized with any desired discretization method to correspond to the used sampling rate.

3.2. Nonlinear controller

In practice a process with a varying disturbance frequency cannot be controlled with a linear controller because the disturbance dynamics changes as the function of the frequency resulting in the change of state-matrix in Eq. (3). This may render both the state-estimator and state-feedback law unstable. In order to solve the problem, the controller has to be modified in such way that the change of the process parameters is taken into account. To that end next a nonlinear controller with a continuous gain scheduling in terms of rotation frequency is introduced. The state-estimator in Eq. (10) can be converted into its nonlinear counterpart given as:

$$\dot{x}(t, \omega) = (A(\omega) - K(\omega)C) \dot{x}(t, \omega) + Bu(t) + K(\omega)y(t),$$

where $\omega$ is the disturbance frequency in Hertz, $K(\omega)$ is the frequency dependent state-estimator gain, $A(\omega)$ is the frequency dependent system matrix, $u(t)$ is the applied control and $y(t)$ is the measured process output.

Now the state-feedback matrix $K$ changes as a function of the disturbance frequency and can be expressed as:

$$K(\omega) = \left[ f_1(\omega) \ f_2(\omega) \ \cdots \ f_n(\omega) \right]^T,$$

where $f_1-\omega_n$ are functions of frequency.

The optimal solution of $K$ as a function of disturbance frequency forms a smooth hypersurface as shown in Fig. 6. The functions in Eq. (15) are chosen to correspond the elements of $K$ as a function of disturbance frequency in the given hypersurface. In practice the hypersurface can be formed by solving the optimal linear control problem for each disturbance frequency within the desired frequency band. Naturally the mesh cannot be infinitely tight so the solutions are taken in discrete points; one choice for the resolution is the maximal resolution of the device measuring the disturbance or rotation frequency.

Figure 6 Projections of the hypersurface spanned by the elements of $K$ as a function of frequency
For the given problem the functions in Eq. (15) are chosen as polynomials. Now the optimal state-feedback $K$ can be given as:

$$K(\omega_n) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} u_n(\omega_n),$$

where $a_{ij}$ are the coefficients of the polynomials and $u_n(\omega_n) = [\omega_n^{m-1} \cdots \omega_n^2 \omega_n 1]^T$.

By substituting Eq. (16) into Eq. (14) yields:

$$\dot{x}(t, \omega_n) = \left( A(\omega_n) - K(\omega_n)C \right) x(t, \omega_n) + Bu(t) + K u_n(\omega_n) y(t).$$

The optimal control gain $L(\omega_n)$ can be solved in a similar way as $K(\omega_n)$. Finally the nonlinear optimal controller can be written as:

$$\begin{cases} \dot{x}_{\text{cont}}(t, \omega_n) = (A_{\text{cont}}(\omega_n) - BL(\omega_n)) x_{\text{cont}}(t, \omega_n) + B(\omega_n) y(t) \\ u(t, \omega_n) = -L(\omega_n) x_{\text{cont}}(t, \omega_n) \end{cases}.$$ (18)

The offline computation requirements related to the control design are relatively high. However, after the design procedure has been carried out the computational burden of the resulting control law is light. In practice the controller is easy to implement as a discrete parameter varying transfer function. What is notable is the fact that the control law is optimal in any point within its design range, if the frequency variation is frozen. Hence, the nonlinear control law can be used as it is for the process even if the frequency is not varying.

4. Simulations and test results

The testing of the designed control laws can be divided into two phases. The control performance is first analyzed with extensive simulations. Should the results prove satisfactory, the control law is then implemented in the test process. The results from the actual process can be used to validate the simulation results and the final performance analysis can be made. The model structure used in the simulations is illustrated in Fig. 7. In order to reproduce the real scenario, the process plant is simulated as a continuous-time model, while the controller is a discrete-time model with a 1kHz sampling rate. The analyses carried out for each of the control laws separately are given in the following subsections.
4.1. Linear control

The tests for the linear controller are carried out at a single operation point of the process with a static predefined rotation frequency. In order to analyze the control performance in a worst case scenario the rotation frequency is chosen as 39Hz, which is the same as the critical frequency of the process. Before any tests are carried out the theoretical closed loop sensitivity to disturbances is analyzed. This theoretical damping performance is given in Fig. 8.

![Figure 8 Theoretical damping ratio of the linear controller](image)

In the simulation scheme the process is subject to a 1000N sinusoidal disturbance. The process is first run without control for 10s after which the control is turned on, which clearly distinguishes the control performance. The simulation results are shown in Fig. 9.

![Figure 9 Simulation results of a process subject to a sinusoidal disturbance with control initially off and then turned on](image)

According to the simulations it is apparent that the control law is both stable and provides good damping with over 99,9% damping rate. As the controller performance has been validated in simulations it can be implemented on the actual rolling process. The control and process layout is show in Fig. 2. In the tests the process is subject to a sinusoidal disturbance at its critical frequency. The process is first run without control after which the control is turned on. The test results are given in Fig. 10. The control action has a clear impact on the amplitude of the vibrations. However some residual vibrations are still clearly visible. In order to analyze the actual control performance, the damping is evaluated in the frequency domain. The frequency spectra of the process output with and without control are given in Fig. 11.
According to the frequency spectra the controller provides over 97% damping at its design frequency. It is thus apparent that the linear control law yields very good performance in both simulation and actual implementation. The residual disturbances turn out to be some high frequency noise that cannot nor need to be controlled.

4.2. Nonlinear control

The tests for nonlinear controller are carried out with a process subject to a sinusoidal sweep disturbance. This corresponds to the varying rotation speed of the reel with constant width. The rotation frequency of the reel is swept over the whole feasible operation range of the process, namely from 5Hz to 50Hz. Before the tests are carried out the theoretical damping performance of the controller is analyzed. This performance is shown in Fig. 12.

The simulations are carried out in similar way as was for the linear controller. The real process is emulated by simulating the plant model in continuous time, while the controller is simulated in discrete time with 1kHz sampling rate. The disturbance force is generated as a sinusoidal sweep signal with 1000N amplitude. In addition to the force measurement, the disturbance frequency is fed into the controller. This corresponds to the measured rotation frequency of the reel. The simulation results are shown in Fig. 13.
The simulation results indicate that the nonlinear controller is effective over the whole operational frequency range with damping ratio of 99%. Finally the controller is implemented in the rolling process subject to a varying sinusoidal force disturbance. The test results are shown in Fig. 14.

The control action provides a significant decrease in vibrations with damping ratio of ~90% over the whole frequency range. The results are further validated in frequency domain by comparing the frequency spectra of the process with and without control. These
spectra are shown in Fig. 15.

According to the test results from both the time and frequency domain, the nonlinear controller yields very good performance in terms of vibration damping. The simulated results correspond well with the ones obtained from the practical tests. According to the theoretical damping ratio, the performance should be even better if the process is ran at some frequency for a longer period of time, instead of having a constantly changing frequency.

5. Conclusions

Active control of vibrations in an industrial rolling process is possible with a hydraulic actuator. The impact of the controller was clearly distinguishable in terms of the process output. The nonlinear extension of the linear controller performed well over the whole frequency band of operation. The restrictions of the linear controller do not apply for its the nonlinear counterpart. The nonlinear controller can be implemented in practice with ease as a traditional parameter varying discrete transfer-function form. Both of the controllers proved to be robust, highly effective and applicable for the given problem. Due to the generic formulation of the control scheme, the presented control design method is applicable for any vibration control problem that can be presented with the same model structure. Although the nonlinear controller performed well, some assumptions were made regarding the process variance in terms of the reel width. In practice it is possible and likely that the reel width cannot be assumed constant. This results in a process model with varying or unknown parameters. In order to control such system, the control design has to be altered to take these unknowns and nonlinearities into account. One possible choice for control of such systems could be based on the combined principles of the robust and optimal control, adjusted for the given problem. Currently there is ongoing research to take these additional requirements into account in control design. Regardless of the few limitations arising from the process properties, the nonlinear control algorithm presented in this paper is applicable for any process subject to a sinusoidal disturbance, when the process is linear over its range of operation.

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