Semi-Active Vibration Control of Smart Structures with Sliding Mode Control*

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Abstract
In this study, a semi-active vibration suppression system comprising piezoelectric elements is developed for flexible structures. The vibration suppression system comprises a cantilevered beam with bimorph piezoelectric ceramic tiles shunted by an RL electrical circuit with a switch. A general design method for vibration suppression of the beam is theoretically analyzed using mode analysis, wherein it is assumed that the piezoelectric elements are sufficiently thin and do not change the mode shape of the beam. With this assumption, the vibration suppression system for the beam is designed by tuning the optimal resistance and inductance parameters of the shunted RL network. In this paper, we propose a semi-active vibration control law to improve the damping effect while maintaining the stability of the passive control system. The proposed control law is similar to a sliding-mode control that accelerates the convergence of the system by using switching functions. As an example, numerical simulations have been performed for a cantilevered beam. This study shows that the resonant circuit functions as a type of a dynamic damper for mechanical systems and that sliding-mode control is very effective in damping the multi-mode responses. The results of the numerical simulations show that the semi-active vibration control system is practically more effective in damping vibrations than the passive control system.

Key words: Smart Structures, Piezoelectrics, Semi-Active Vibration Control, Sliding-Mode Control, Vibration Control

1. Introduction
In recent years, smart structures employing piezoelectric elements as embedded sensors and actuators and as elements of active structural vibration suppression systems have been studied. Two types of piezoelectric control systems—active and passive—are used for vibration damping. Active control systems generally produce a higher damping force than passive control systems; however, active control systems have problems with stability. In contrast, passive control systems are always stable. Therefore, semi-active vibration control systems, which produce a high damping force while being stable, are proposed as a method of damping vibrations in space applications.

In this paper, we propose a new vibration suppression system for smart structures using the sliding-mode control law. Results of numerical simulations show that this new semi-active vibration suppression system is more effective in damping vibrations than the passive vibration suppression system.
2. Characteristics of the piezoelectric element

As piezoelectric materials strain when an electric field is applied across them, they are suitable for use as actuators in control systems. Further, as they produce voltage when subjected to strain, they can be used for sensing strains. In general, piezoelectrics can efficiently transform mechanical energy into electrical energy and vice versa and, therefore, can be used as structural dampers [1].

Figure 1. Piezoelectric element with potential difference applied across top and bottom surfaces

Figure 1 shows a typical piezoelectric element. The fundamental constructive relations are the relations between the mechanical deflection and the electrical deflection and are written as

\[ q = e_{33}^T \cdot E_3 + d_{31} \cdot T_1 \]  \hspace{1cm} (1)

\[ S_1 = d_{31} \cdot E_3 + s_{11}^E \cdot T_1 \]  \hspace{1cm} (2)

where \( S_1 \) and \( T_1 \) denote the applied strain and stress in the \( x \)-direction, respectively; \( E_3 \) and \( q \), the applied electric field and charge density in the \( z \)-direction, respectively; and \( s_{11}^E \), \( d_{31} \), and \( e_{33}^T \), the elastic compliance in the \( x \)-direction, piezoelectric constant, and permittivity, respectively.

Figure 2. Resonant shunted piezoelectric with resistor and inductor in series

Consider a resonant circuit. Shunt the inherent capacitance of the piezoelectric element using a resistor and an inductor in series, thereby forming an LCR circuit. This circuit is
shown in Fig. 2. In this case, the electrical impedance $Z_{EL}$ of the entire circuit, including the piezoelectric element, is given by

$$Z_{EL}(s) = \frac{Ls + R}{LC_p s^2 + RC_p s + 1}$$

(3)

where $L$, $R$, and $C_p^T$ are the shunting inductance, shunting resistance, and inherent capacitance of the shunted piezoelectric, respectively. $s$ is the Laplace parameter. This circuit is resonant with some damping due to the resistance $R$, and it can be tuned in the vicinity of a mode of the underlying mechanical system, thereby increasing the modal damping ratio considerably. This effect is similar to that produced by the classic proof mass damper (PMD) or resonant vibration absorber. The elastic compliance $S_{SU}$ that relates the shunted piezoelectric to the underlying mechanical system can be expressed by using the non-dimensional electrical impedance $\bar{Z}_{EL}(s) = Z_{EL}(s)/Z_D(s) = Z_{EL} \cdot sC_p^T$ ($Z_D(s)$ is the electrical impedance of the piezoelectric) as

$$S_{SU}^{SU} = S_{11}^{E} \left(1 - k_{31}^2 \bar{Z}_{EL}(s)\right)$$

(4)

where $k_{31}$ denotes the electromechanical coupling constant of the piezoelectric material and is defined as follows:

$$k_{31}^2 = \frac{d_{31}^2}{S_{11}^E S_{33}^F}$$

(5)

$k_{31}$ represents the ratio between the electrical energy and the mechanical energy that can be stored in the piezoelectric elements. Equation (4) shows that the compliance of the shunted piezoelectric is equal to the short-circuit compliance of the piezoelectric modified by a term that depends on the electrical shunt circuit and the electromechanical coupling coefficient of the piezoelectric material.

3. Modeling of flexible structures with piezoelectric elements

Figure 3 shows a flexible beam model with a piezoelectric element. It is assumed that the mass and stiffness of the piezoelectric element are low and do not affect the mode shape of the beam. It is also assumed that a bending moment applied to the beam causes a constant variation in the deflection angle between the ends of the piezoelectric as follows:
The couple of the bending moments provided by the piezoelectric element can be given as

\[ M_p = -E_p(s)I_p \frac{d\theta}{dx} = -E_p(s)I_p \frac{\theta_j - \theta_i}{l_p} \tag{7} \]

where \( I_p \) is the moment of inertia of area of the piezoelectric, and \( E_p \) is the frequency-dependent Young’s modulus of the piezoelectric element that can be described as the inverse of the mechanical compliance, i.e., \( E_p(s) = 1/s_{11}(s) \).

The piezoelectric bending moment \( M_p \) is introduced into the equation of motion as the feedback force and is represented using a displacement vector. If we consider a single vibration mode, e.g., the first mode, then the equation of motion for the first mode including the feedback force of the piezoelectric element is written as

\[ \ddot{z}_1 + \omega_1^2 z_1 + E_p(s) \frac{I_p}{\mu_1 l_p} (\phi_{ij} - \phi_{ii}) \cdot z_1 = F_1 \tag{8} \]

where \( \omega_1 \), \( \mu_1 \), and \( \phi \) are the eigenfrequency, modal mass, and the element of the modal matrix, respectively, and \( F_1 \) is the modal force.

It has been shown in previous studies [2,3] that considering the similarities between a system containing resonant shunted piezoelectrics (RSPs) and a system containing a mechanical vibration absorber or a PMD can be instrumental in the optimal tuning and damping of the electric circuits of RSPs. The optimal tuning parameters of the resonant shunted piezoelectrics are provided in the reference section of this paper, and the optimal inductance and resistance of the resonant circuit are given as follows [2]:

\[ L_{opt} = \frac{1}{\left(\omega_1^2\right)^2 C_p s^2} \frac{1}{1 + K_{31}} \tag{9} \]
\[ R_{opt} = \frac{\sqrt{2K_{31}}}{\omega_1 C_p s} \frac{\sqrt{2K_{31}}}{1 + K_{31}} \tag{10} \]

4. Sliding-mode control for the semi-active vibration control system

Using equations (1) and (2) and approximations for the small elements, the feedback force can be expressed including voltage \( V_a \) generated by the strain of the piezoelectric element as

\[ M_p = -\frac{E_p(s)I_p}{l_p b_p |w_j - w_i|} x V_a \tag{11} \]
where $b_p$ is the output coefficient, and $w$ is the deflection. Using equations (7), (8), and (11), the equation of motion for the first mode is written as follows:

$$
\ddot{z}_i + \omega_i^2 z_i - \sum_{l=1}^{l=i} \phi_{ik} \frac{E_p(s)I_p}{\mu_l I_p b_p |w_j - w_l|} V_a = F_i
$$

(12)

If $V_a$ in equation (12) can be controlled, it will be possible to provide active control using the voltage as the control input.

In this paper, we propose a semi-active control law that is based on sliding-mode control. According to the sliding-mode control law [4], the switching function $\sigma_k$ is defined as

$$
\sigma_k \equiv S_k x_k = \left\{ \omega_k, \begin{bmatrix} z_k \\ \dot{z}_k \end{bmatrix} \right\}
$$

(13)

and the switching surfaces that satisfy $\sigma_k = 0 (k = 1, 2, \cdots, n)$ are obtained. If we can control $V_a$ so that it satisfies $\sigma_k \dot{\sigma}_k \leq 0$, it will be possible to control the system such that it converges to the origin on the phase surfaces. In order to achieve semi-active vibration control in this study, it is important to control the voltage indirectly rather than directly [5]. Using $\sigma_k$, a new function $L$ that evaluates the movement of the system is obtained as follows:

$$
L \equiv \sum_{k=1}^{n} \sigma_k^2
$$

(14)

The time derivative of $L$ is written as follows:

$$
\dot{L} = \sum_{k=1}^{n} \sum_{l=1}^{l=k} \phi_{lk} \frac{2E_p(s)I_p}{\omega_k \mu_k I_p b_p |w_j - w_l|} \sigma_k V_a + \text{(TermsNotIncluding}V_a) \tag{15}
$$

If we control the voltage and reduce the value of $\dot{L}$, i.e., reduce the value of the term including $V_a$, the system approaches the origin on the phase surfaces. Therefore, it is necessary to change certain system conditions; for example, $V_a$ can be made positive when $g \leq 0$, or it can be made negative when $g \geq 0$. The function $g$ is written as follows:

$$
g = \sum_{k=1}^{n} \sum_{l=1}^{l=k} \phi_{lk} \frac{2E_p(s)I_p}{\omega_k \mu_k I_p b_p |w_j - w_l|} \sigma_k
$$

(16)
By connecting a switch to the resonant circuit as shown in Fig. 4 and by switching properly, semi-active vibration control is achieved.

Now, we assume that the system satisfies $0 \leq g$ for the circuit shown in Fig. 4. When $V_a$ is positive, the switch should be turned on in order to generate the current and consume the vibration energy at the resistor. When $V_a$ is negative, the switch should be turned off. However, if the system satisfies $0 \geq g$, the switch should be turned on when $V_a$ is negative and off when $V_a$ is positive. Semi-active vibration control is achieved by using this switching mechanism.

5. Numerical Simulation

5.1. Simulation model

Numerical simulations were performed on the cantilevered beam with piezoceramics attached to its surface. The cantilevered beam with a resonant shunted piezoceramic element is shown in Fig. 5. The length, width, and thickness of the cantilevered beam were 45.00 cm, 3.00 cm, and 0.3 mm, respectively. A surface-mounted piezoceramic element was attached to one side of the beam and shunted by using an RL resonant circuit with a switch. The shunted piezoceramic element was a 0.2-mm-thick C-6 piezoceramic sheet manufactured by Fuji Ceramics Co. The specifications of the beam and the piezoceramic element are presented in Table 1.
Table 1. Specifications of beam and piezoceramic element

<table>
<thead>
<tr>
<th>Beam</th>
<th>Length</th>
<th>4.50 × 10⁻¹ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Width</td>
<td>3.00 × 10⁻² (m)</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>3.00 × 10⁻⁴ (m)</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>7.86 × 10³ (kg / m³)</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>206 × 10⁷ (N / m²)</td>
</tr>
<tr>
<td>Piezoceramic element</td>
<td>Length</td>
<td>3.00 × 10⁻² (m)</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>3.00 × 10⁻² (m)</td>
</tr>
<tr>
<td></td>
<td>Thickness</td>
<td>2.00 × 10⁻⁴ (m)</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>7.40 × 10⁻³ (kg / m³)</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>58.0 × 10⁹ (N / m²)</td>
</tr>
<tr>
<td></td>
<td>d₃₁</td>
<td>-1.95 × 10⁻¹⁰ (m / V)</td>
</tr>
<tr>
<td></td>
<td>k₃₃</td>
<td>39.0 × 10⁻²</td>
</tr>
<tr>
<td></td>
<td>ε₃₃</td>
<td>2130 × ε₀ (C · m / V)</td>
</tr>
<tr>
<td></td>
<td>bₚ</td>
<td>5.62 × 10⁷ (N / C)</td>
</tr>
</tbody>
</table>

Using equations (9) and (10), the optimal parameters for the first vibration mode were obtained as \( L_{opt} = 1.10 \times 10^5 (H) \) and \( R_{opt} = 6.92 \times 10^3 (\Omega) \). The efficiency of the multi-mode responses is considerably lower than that of the first mode response. Hence, in this study, we use the optimal parameters for the first vibration mode.

5.2. Results and discussion

![Bode Diagram](image)

Figure 6. Bode diagrams at optimal frequency tuning and various resistance values: Solid: optimal resistance tuning \( R = R_{opt} \); dotted: \( R = 1.2R_{opt} \); and dashed: \( R = 0.8R_{opt} \)
Figure 6 shows the Bode gain plots of the displacement of the tip of the cantilevered beam (the response to the power forcing at the root) for various values of resistances when the piezoceramic patches are placed at the root of the beam. These plots are very similar to the transfer functions for a 1-DOF system containing a PMD for various values of damping parameters. This similarity shows that the resistance of the RSPs functions as a damper for the PMD [6].

First, the simulations were carried out considering only the first mode. Figure 7 shows the impulse responses for the present semi-active system and the passive control system where the impulsive force is applied at the root [7]. The top is the time history from 0 s to 10 s, and the bottom is the enlarged time history from 5 s to 10 s. It is evident that the semi-active vibration control system is practically more effective in damping vibrations than the passive vibration control system. The attained first mode damping ratios are 0.134 (passive) and 0.174 (semi-active).

The responses of voltage $V_a$, current $I$, and function $g$ are shown in Fig. 8. It can be observed that the current was generated according to the control law obtained above.
Next, the simulations were carried out considering the first and second modes. Figure 9 shows the impulse responses for the semi-active and passive vibration control. The semi-active control with the single resonant circuit is effective in damping the both first and second modes, because the evaluation function $g$ takes both modes into account. Furthermore, Figure 10 shows the responses considering the first, second, and third modes. The semi-active vibration damping is also remarkable in this case; however, increasing the number of modes makes the evaluation function $g$ more complex and demands more intensive switching. Therefore, as shown later, it is effective to increase the number of resonant circuits and design the system parameters for each vibration mode.
Finally, we increased the resonant circuit for suppressing the second mode vibration. (See Fig. 11.) The parameters of each circuit are tuned optimally for damping each mode. For the semi-active control, the circuits are turned on and off separately according to the sign of the switching function for each mode. Figure 12 shows the impulse responses for the semi-active and passive vibration control. In this case, the passive control is effective to suppress both the first and second modes although the damping performance is lower than the semi-active control. The figure also compares the semi-active controls with (bold line) and without (broken line) the additional resonant circuit for the second mode. It is shown that the performances of these two semi-active controls are comparable to each other. This means that the semi-active control based on the multi-mode switching function is very efficient and effective without increasing hardware complexity.

![Resonant shunted piezoelectrics for the first and second modes](image1)

Figure 11. Resonant shunted piezoelectrics for the first and second modes

![Responses (with the first and second mode resonant circuits)](image2)

Figure 12. Responses (with the first and second mode resonant circuits)

6. Conclusion

The dynamics of a beam with resonant shunted piezoelectrics (RSPs) are analyzed assuming that the mass and stiffness of the piezoelectric elements are lower than those of the beam. The response transfer function of the beam with RSPs is similar to that of a system containing a mechanical vibration absorber or a proof mass damper (PMD), and the classic tuning theory can be used for optimal tuning of the circuit parameters of the RSPs.

In this paper, we proposed a semi-active control law based on sliding-mode control. The effectiveness of the semi-active damping system with RSPs and multi-mode switching function was shown through numerical simulations. The results of the numerical simulations show that the electrical system functions as a dynamic damper for the mechanical system, and the semi-active vibration control system is practically more effective in damping vibrations than the passive control system.
References


