Vibration Control using Harmonically-Varying Damping*

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Abstract

This study proposes a new vibration control method using harmonically-varying damping. Previous research has demonstrated that resonance could be induced by the combination of a sinusoidal base excitation (not at the natural frequency) with harmonically varying damping. The amplitude and phase of the resulting resonance can be controlled through appropriate choice of variable damping. This finding is used herein for response control. A single degree-of-freedom structure excited by an external sinusoidal input disturbance is considered. An ideal variable damper is employed as a semi-active control device, in conjunction with a secondary base displacement, resulting in cancellation of the input disturbance. In this paper, an expression for the ideal damping required, as well as the secondary base displacement, are developed; the control performance is then confirmed through numerical simulation.

Key words : Vibration Control, Semi-active Control, Variable Damping, Harmonically Varying Damping

1. Introduction

Recent research⁽¹⁾ has demonstrated a new method for the utilization of variable damping. The purpose of the method was not to decrease the vibration amplitude of the system, as many researchers have sought in the past⁽²⁾⁽³⁾, but rather to increase it. In those studies, the damping coefficient was harmonically varied. The force acting on the mass through the variable damper is equal to the product of the damping coefficient and the relative velocity between the base and the mass. As a result, the response contains a new component that has a modulated frequency different from the input frequency. Therefore, varying the damping coefficient at a suitable frequency can generate a new vibration that has an arbitrary frequency, amplitude, and phase. In other words, the variable damping device is not only an energy dissipation device, but also a frequency modulation device; this modulated input component can generate new resonances in the system. These results have been verified through simulation and experiment⁽⁴⁾. This phenomena was also investigated from a theoretical standpoint. The steady-state response was derived analytically and compared with the results obtained in numerical simulation to demonstrate the effectiveness of the proposed analysis method⁽⁵⁾.

This study proposes a new method for reducing the vibration response of base-excited structure using the resonance phenomenon created by harmonically varying the damping. Vibration mitigation is critical in a wide range of industries, including production machinery surrounded by multiple vibratory sources, rapidly moving vehicle on road or rail, civil infrastructure subjected to earthquakes, etc. Many ways are available to reduce vibration response; a suitable choice should be made among the diverse options. To date, a few approaches harnessing harmonically varying the damping have been reported. F. Dohnal has proposed a method for a stabilizing problem of a two-degree-of-freedom system with parametric excitation by

using the harmonically varying damping⁽⁶⁾. However, he considered a instability problem of the system with small disturbance and damping ratio studied by A. Tondl. The problem is not similar to an isolation structure whose damping ratio is relatively high and that has large input like earthquake.

This paper applies the concept of harmonically varying damping to achieve response control for a single-degree-of-freedom structure subjected to sinusoidal base excitation. An ideal variable damper is employed in conjunction with a secondary base excitation to achieve disturbance rejection. If a primary sinusoidal base excitation contains the natural frequency of the system, resonance is induced. However another resonance can also be generated by the modulated component to do with the variable damping device and the secondary base excitation at the same time. The additional resonance caused by the variable damping is adjusted to be out of phase with the response, resulting in effective control of the structural performance. Following derivation of an expression for the ideal variable damping, the control performance is confirmed by numerical simulation.

2. Harmonically Varying Damping

In this section, the effect of harmonically varying damping on structural response is elaborated, along with some key simulation results to illustrate the phenomenon.

2.1. Base-Excitated Model

In this study, a base-excited single-degree-of-freedom system is considered as shown in Fig.1. A mass m is supported by a spring k and a variable damper $c(t)$ that are connected in parallel. x and z are the absolute displacements of the mass and base, respectively. The

Fig. 1 SDOF base-excited vibration model.

dimensionless equation of motion is as follows.

$$\frac{d^2x(t)}{dt^2} + 2\zeta(t)\frac{dx(t)}{dt} + x(t) = 2\zeta(t)\frac{dz(t)}{dt} + z(t) \quad (1)$$

Here ζ is the damping ratio and is represented as $\zeta(t) = \frac{c(t)}{2\sqrt{mk}}$. The input-related terms are moved to the right side of the equation. The damping ratio $\zeta(t)$ can be controlled by the use of controllable devices such as MR or ER dampers⁽⁸⁾⁽⁹⁾.

Furthermore, the base displacement z would have two sinusoidal inputs: z_1 is the disturbance, and z_2 is the secondary base excitation. In this case, z is expressed as follow.

$$z(t) = z_1(t) + z_2(t) = Z_1 \sin(\omega_1 t + \gamma_1) + Z_2 \sin(\omega_2 t + \gamma_2), \quad (2)$$

where, Z_1, Z_2 are the amplitudes of the input, ω_1, ω_2 are the frequencies of the input, and γ_1, γ_2 are the phase of the input.

2.2. Variable Damping Force

The effect of harmonically varying damping on the response of the single-degree-of-freedom structure in equation (1) is explained in this subsection. The parameters of the base-excitation of equation (2) are considered as $Z_1 = 1, \omega_1 = \omega, \gamma_1 = \pi/2$ and $Z_2 = 0$, resulting in the following excitation.

$$z(t) = \cos \omega t \quad (3)$$

Then, the controllable damping ratio $\zeta(t)$ is considered to vary harmonically with a frequency ω_ζ , to yield:

$$\zeta(t) = \alpha \cos(\omega_\zeta t + \beta) + \zeta_{DC}, \quad (4)$$

where α is the amplitude; β the phase; and ζ_{DC} the non-zero center amplitude. The parameters are chosen to meet the constraint $\zeta(t) > 0$. The damping force on the right-hand side of equation (1) is given as the product of the damping ratio and the input velocity, with the associated damping force becoming:

$$\begin{aligned} f_{damp} &= 2\zeta(t) \frac{dz(t)}{dt} = -2\{\alpha \cos(\omega_\zeta t + \beta) + \zeta_{DC}\} \omega \sin \omega t \\ &= -2\omega \zeta_{DC} \sin \omega t - \alpha \omega \sin\{(\omega + \omega_\zeta)t + \beta\} - \alpha \omega \sin\{(\omega - \omega_\zeta)t - \beta\}, \end{aligned} \quad (5)$$

where ω_ζ is the frequency of the damping ratio $\zeta(t)$, and ω is the frequency of the velocity $dz(t)/dt$. From equation (5), the damping force f_{damp} is the addition of three sinusoidal waves with three different frequencies: the input frequency ω and two modulated frequencies $\omega_\zeta - \omega$, $\omega_\zeta + \omega$. By adjusting the frequency of the harmonically-varying damping, resonance can be induced, even if the frequency of the input z_1 is not at the natural frequency of the structure. In the next section, the resonance phenomenon is confirmed through numerical simulation.

2.3. Sinusoidal Simulation Results

The generation of resonance is confirmed by the numerical simulation. A Simulink model is developed for the equation of motion (1), and an explicit Runge-Kutta scheme was employed to integrate the differential equation. Simulation was performed until the response of the vibration system achieved the steady state response. In this simulation, the frequency of base excitation was defined as ω_1 ; the frequency of the oscillation of the variable damper as ω_ζ , and the absolute value of the difference between the two as $\lambda = |\omega_1 - \omega_\zeta|$.

2.3.1. Growth of Amplitude by Harmonically Varying Damping The input frequency is $\omega_1 = 6$, the frequency of the variable damping is $\omega_\zeta = 5$, and the amplitude, phase and DC offset of the damping ratio are $\alpha = 0.25$, $\beta = 0$, and $\zeta_{DC} = 0.25$, respectively. Figures 2 - 5 show the input primary base excitation, the response of the absolute displacement of the mass m , the damping ratio of the variable damper, and the actual damping force generated by the variable damper, respectively. Comparing Fig.2 with Fig.3 shows that the maximum amplitude of the response is larger than the amplitude of the input, because the natural frequency $\omega_n = 1$ of the vibration system is excited by the oscillating component $\lambda = |\omega_1 - \omega_\zeta| = 1$ of the newly generated wave. The phenomenon is similar to resonance; however, the main frequencies of the input and output are different. In addition, the output phase of the damping force at the natural frequency is controllable by changing the phase of the harmonically varying damping⁽⁷⁾.

2.3.2. Transmissibility of Proposed System The transmissibility from the base-input to the output for the structural model with the harmonically varying damping is illustrated in Fig. 6. The absolute maximum value of the steady-state displacement amplitude of the mass is presented versus frequency; this measure of the response is chosen because it includes several frequencies in the case of the proposed system, even though the input has just one frequency. With the exception of the frequency ω_1 for the sinusoidal base excitation, all other parameters are held constant at the values given in Section 2.3.1.

The Figure shows three dominant peaks. The first one is at around the frequency $\omega = 1$ that is the natural frequency of the system. However, two local maxima can also be seen at another frequencies $\omega = 4$ and $\omega = 6$, which are far away from the natural frequency but the differences between the input frequency and the damping frequency are the same as the natural frequency, i.e., $\lambda = |\omega - \omega_\zeta| = 1$. At the frequencies, the input whose frequencies are not the natural frequency of the system is modulated by the harmonically varying damping as shown in equation (5), and then the third term of the equation has the natural frequency

Fig. 2 Base-input excitation ($\omega_1 = 6$).

Fig. 3 Absolute displacement of the mass.

content. The modulation induces quasi-resonance, or two additional peaks. The most significant characteristic of the harmonically varying damping is that the resonance can be generated away from the natural frequency.

3. Semi-active Disturbance Cancellation Concept

This section proposes a new method for reducing vibration using the resonance phenomenon for harmonically varying damping that was described in the previous section. At first, the expression for the harmonically varying damping controller is developed for the ideal variable damping device; subsequently, an observer is described that is required for the control system.

3.1. Control Method of Harmonically Varying Damping

3.1.1. Disturbance from the Base From equation (2), the disturbance component on the RHS of equation (1) is given by

$$z_1(t) + 2\zeta_{DC}\dot{z}_1(t) = Z_1 \sqrt{1 + (2\zeta_{DC}\omega_1)^2} \cos(\omega_1 t + \gamma_1 - \phi), \quad (6)$$

where,

$$\phi = \tan^{-1} \frac{1}{2\zeta_{DC}\omega_1}. \quad (7)$$

3.1.2. Usable Damping Force by Damping Excitation and Base-input In this section, the usable damping force generated by the oscillating damping and the secondary base excitation z_2 is considered. The velocity corresponding to z_2 is

$$\frac{dz_2(t)}{dt} = Z_2\omega_2 \cos(\omega_2 t + \gamma_2). \quad (8)$$

Similar to equation (4), the harmonically varying damping force is obtained as

$$\begin{aligned} 2\zeta(t)\frac{dz_2(t)}{dt} &= 2\{\alpha \cos(\omega_\zeta t + \beta) + \zeta_{DC}\}Z_2\omega_2 \cos(\omega_2 t + \gamma_2) \\ &= 2Z_2\omega_2\zeta_{DC} \cos(\omega_2 t + \gamma_2) + Z_2\alpha\omega_2 \cos\{(\omega_2 - \omega_\zeta)t + \gamma_2 - \beta\} \\ &\quad + Z_2\alpha\omega_2 \cos\{(\omega_2 + \omega_\zeta)t + \gamma_2 + \beta\}. \end{aligned} \quad (9)$$

Fig. 4 Variable damping ratio ($\omega_\zeta = 5$).

Fig. 5 Variable damping force.

Fig. 6 Transmissibility of the system.

Where, the second term is extracted from the equation as follow.

$$u(t) = Z_2 \alpha \omega_2 \cos\{(\omega_2 - \omega_\zeta)t + \gamma_2 - \beta\} \quad (10)$$

In this study, the extracted damping force is considered applying to the control force. The third term of the equation (10) is also a candidate for the control force; however, both modulated damping forces cannot be specified simultaneously as the control force.

3.1.3. Condition for Cancellation of Input Effect To cancel the response due to the base-input component of the excitation given by (6) (i.e., $z_1(t)$) using the control force given in equation (10), the following two condition are required: (i) equalizing both amplitudes, and (ii) ensuring that the phases are 180° apart. The following equations are required for matching the amplitude and phase conditions, respectively.

$$Z_2 \alpha \omega_2 = Z_1 \sqrt{1 + (2\zeta_{DC}\omega_1)^2} \quad (11)$$

$$(\omega_2 - \omega_\zeta)t + \gamma_2 - \beta \pm \pi = \omega_1 t + \gamma_1 - \phi \quad (12)$$

From these condition to be satisfied, the amplitude and phase of the harmonically varying damping can be determined as follows

$$\alpha = \frac{Z_1 \sqrt{1 + (2\zeta_{DC}\omega_1)^2}}{Z_2 \omega_2}, \quad (13)$$

$$\omega_{\zeta}t + \beta = (\omega_2t + \gamma_2) - (\omega_1t + \gamma_1) + \phi \mp \pi. \quad (14)$$

In addition, because the damping ratio is positive, the following restriction must be satisfied

$$0 \leq \alpha \leq \zeta_{DC}, \quad (15)$$

where, $\zeta_{DC} = \frac{\zeta_{max}}{2}$, and ζ_{max} is the realizable maximum damping ratio of the system. This value of ζ_{DC} has been shown to provide the best performance for the system⁽⁵⁾.

3.2. Observer

As above, the control law for the variable damping ratio can be obtained in the same manner as was done for equations (13)-(14). However, because these equations contain information regarding z_1 , the primary base-input, this quantity needs to be estimated. In this subsection, an observer based on the assumption that the base-input is sinusoidal is constructed. The sinusoidal input from the base can be expressed as follow.

$$z_1(t) = Z_1 \sin(\omega t + \gamma_1) \quad (16)$$

The input model can be represented in the state-space form as

$$\begin{bmatrix} \dot{z}_1(t) \\ \ddot{z}_1(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & 0 \end{bmatrix} \begin{bmatrix} z_1(t) \\ \dot{z}_1(t) \end{bmatrix}. \quad (17)$$

Considering this relation, the equation of motion (1) can be expressed with the following augmented state-space equation

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{z}_1(t) \\ \ddot{z}_1(t) \\ \dot{z}_2(t) \\ \ddot{z}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -2\zeta(t) & 1 & 2\zeta(t) & 1 & 2\zeta(t) \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega_2^2 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ z_1(t) \\ \dot{z}_1(t) \\ z_2(t) \\ \dot{z}_2(t) \end{bmatrix}. \quad (18)$$

Moreover, if the acceleration of the mass should be measured, the output equation is obtained as

$$y(t) = \ddot{x}(t) = [-1 \quad -2\zeta(t) \quad 1 \quad 2\zeta(t) \quad 1 \quad 2\zeta(t)] \begin{bmatrix} x(t) \\ \dot{x}(t) \\ z_1(t) \\ \dot{z}_1(t) \\ z_2(t) \\ \dot{z}_2(t) \end{bmatrix}. \quad (19)$$

When an observer is constructed using these equation, the inputs z_1, z_2 can be estimated. Note that the damping ratio is fixed as $\zeta(t) = \zeta_{DC}$ for simplification and design the linear observer. Using the pole assignment method, the closed loop poles of the observer are located slightly to the left as compared to the original system.

Separating the time-invariant and the time-varying parts of the system matrix, equation (18) can be written as follow

$$\hat{\dot{x}}(t) = \bar{A}\hat{x}(t) + 2\zeta(t)A_{\zeta}\hat{x}(t) + L\{y(t) - \hat{y}(t)\} \quad (20)$$

where,

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega_2^2 & 0 \end{bmatrix}, \quad A_{\zeta} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (21)$$

The output equation is

$$\hat{y}(t) = \bar{C}\hat{x}(t) + 2\zeta(t)C_{\zeta}\hat{x}(t), \quad (22)$$

where,

$$\bar{C} = \begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \quad C_{\zeta} = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 1 \end{bmatrix}. \quad (23)$$

3.3. Parameter Extraction from the Estimated Value

If the state $\hat{z}_i$ is estimated, the amplitude, phase and frequency can be calculated as follows

$$\hat{Z}_i(t) = \left| \hat{z}_i(t) + j \frac{\dot{\hat{z}}_i(t)}{\omega_i} \right|, \quad (24)$$

$$\hat{\gamma}_i(t) = \angle \left(\hat{z}_i(t) + j \frac{\dot{\hat{z}}_i(t)}{\omega_i} \right), \quad (25)$$

$$\hat{\omega}_i(t) = \frac{d}{dt} \left\{ \angle \left(\hat{z}_i(t) + j \frac{\dot{\hat{z}}_i(t)}{\omega_i} \right) \right\}. \quad (26)$$

However, in this paper, the frequency ω_i is known; therefore, its value is not estimated for simplification. Using these estimated values, the desired damping ratio $\zeta(t) = \alpha \cos(\omega_{\zeta}t + \beta) + \zeta_{DC}$ is derived from equation (13) and equation (14).

4. Control Example

Finally, the performance of the proposed semi-active control system is confirmed by numerical simulation. Table 1 shows the parameters used in the simulation. The base-excitation z_1 includes the natural frequency of the system ω_n , therefore, resonance is induced. On the other hand, the frequency of the secondary base excitation z_2 is 6 which is a frequency that is attenuated in the vibration response, and therefore does not significantly impact system response.

Table 1 Specification of parameters.

Input amplitude	$Z_1 = 1, \quad Z_2 = 1$
Input frequency	$\omega_1 = 1, \quad \omega_2 = 6$
Natural frequency	$\omega_n = 1$
Damping ratio	$\zeta_{DC} = 0.25$

Additionally, the linear observer poles are located at $[-0.1 - 0.1 - 0.5 - 0.5 - 0.7 - 0.7]'$ left from the original system matrix of equation (18) which is replaced the damping ratio as $\zeta(t) = \zeta_{DC}$.

4.1. Time-domain Results

Figure 7 shows the time-history for the base excitation response that has two frequency. Figure 7 also compares the actual time history data of the input with the estimated data. The gray and black lines show the true value and the estimate, respectively. As is shown, the estimated value matches well with the true value for times greater than 5, demonstrating the good performance of the observer.

Next, comparisons between responses obtained from passive damping models and the semi-active damping model are presented. Figure 8 shows the absolute displacement response of the passive system corresponding to the damping ratio $\zeta_{DC} = 0.25$. In this figure, the amplitude of the response increases because of the natural frequency component of the input. Figure 9 shows the same response of the passive system corresponding to the damping ratio $\zeta_{DC} = 0.50$. In this case, the amplitude of the resonance decreases a little as the damping ratio increases. However, the amplitude of the high frequency response seems to increase, because the high damping ratio has a much greater impact on the isolation domain compared

Fig. 7 Base-input excitation.

Fig. 8 Passive output ($\alpha = 0, \zeta_{DC} = 0.25$).

Fig. 9 Passive output ($\alpha = 0, \zeta_{DC} = 0.5$).

Fig. 10 Semi-active controlled output.

Fig. 11 Harmonically varying damping ratio.

Fig. 12 Damping force.

to the low damping ratio. Figure 10 shows the response of the proposed semi-active control. As can be seen, the dynamics of the semi-actively controlled system have been improved significantly; The resulting response corresponding to the natural frequency of the system is clearly diminished compared to the responses of the passive damping systems. Note that the performance of the controller is not fully realized until the observer stabilizes. The semi-active control system at steady-state surpasses the performance of passive system (see Figs. 8-9).

While the specified damping ratio shown in Fig. 11 provides good performance, it does not entirely cancel the disturbance.

Figure 12 shows the damping force. As a result of the harmonically varying damping ratio, the damping force is superposition of several frequencies (e.g., at 1, 4, 6, 11); therefore, the time history is more complicated than other time histories.

Fig. 13 Comparison between passive and semi-active (Frequency 1).

Fig. 14 Comparison between passive and semi-active (Frequency 6).

4.2. Frequency-domain responses

Fourier transform of the time domain responses are computed, because the time history data does not provide accurate comparison result for analyzing response which includes multiple frequencies. Indeed, evaluation of the performance for the high frequency response is difficult to do from inspection of Fig.10. Figure 13 shows the frequency-domain results for the natural frequency component and Figure 14 shows the results for the frequency at 6. In these figures, the y-axis shows the amplitude of the wave, and the amplitude of the base-input for comparison. In Fig.13, the response amplitude corresponding to $\zeta = 0.25$ and $\zeta = 0.5$ are 2.24 and 1.41 times the input amplitude, respectively; meanwhile the response amplitude corresponding to the semi-active system is 0.21 times the input amplitude. Because the purpose of the proposed controller is reducing the resonance response, our goal is clear achieved. On the other hand, Figure 14 shows that the response with the low damping ratio $\zeta = 0.25$ has cut the input amplitude by 91%. While the performance of the high damping ratio $\zeta = 0.5$ at the frequency 6 is getting worse with only a 83% reduction, the semi-active controller is

maintaining the same performance at this frequency. Because this frequency is in the vibration isolation range, the amplitude increases as the damping ratio of the passive vibration system increases. However, the semi-active controller has not only significant performance at the natural frequency but also nearly identical performance as the case with the low damping ratio $\zeta = 0.25$ at the higher frequency.

5. Conclusion

This study proposed a new method for reducing vibration using harmonically varying damping. A single-degree-of-freedom structure subjected to sinusoidal base excitation was considered to illustrate the control theory. A variable damper was employed in conjunction with a secondary base excitation to achieve disturbance rejection. In the case where the primary base excitation includes the natural frequency of the system, resonance occurs. However, if the additional harmonic response caused by the variable damping and the secondary base excitation is at the same frequency as the resonance and with opposite phase, the resonance can be cancelled. An expression for the variable damping controller has been developed for an ideal variable damping device, and control performance has been confirmed by numerical simulation. The simulation has shown that the proposed control method is effective for reducing resonance response. The comparison between the passive system and the semi-active system validates the theoretical controller design. Therefore, the proposed control method provides another viable option among the diverse possibilities for vibration control.

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References

- (1) D. Iba, A. Masuda and A. Sone, *Generation of quasi-resonance by variable damping*, "Proceedings of SPIE, Active and Passive Smart Structures and Integrated Systems 2007", Vol.6525, No.6525-36, (2007), pp.1-9.
- (2) D. Karnopp, M. J. Crosby and R. A. Harwood, *Vibration control using semi-active force generator*, "Transactions of the ASME, Journal of Engineering for Industry", 96, (1974), pp.619-626.
- (3) H. P. Gavin, *Control of seismically excited vibration using electrorheological materials and Lyapunov methods*, "IEEE Transactions on Control Systems Technology", 9-1, (2001), pp.27-36.
- (4) D. Iba, A. Masuda and A. Sone, *Resonance by parametric excitation of variable damping*, "Journal of System Design and Dynamics", Vol.2, No.1, (2008), pp.331-341.
- (5) D. Iba, A. Masuda and A. Sone, *Frequency Response Analysis of Vibration System with Damping Parametric Excitation*, "Transactions of the Japan Society of Mechanical Engineers. C", 74-745, (2008), pp. 2145-2152, (in Japanese).
- (6) F. Dohnal, *Damping of Mechanical Vibrations by Parametric Excitation*, "Sudwest-deutscher Verlag Fur Hochschulschriften", (2009), pp.1-224.
- (7) D. Iba, A. Masuda and A. Sone, *Structural vibration control by damping parametric excitation*, "Proceedings of 2009 ASME Pressure Vessels and Piping Division Conference", PVP2009-78058, (2009), pp.1-9.
- (8) S. J. Dyke, B. F. Spencer, Jr., M. K. Sain and J. D. Carlson, *An Experimental Study of MR Dampers for Seismic Protection*, "Smart Materials and Structures: Special Issue on Large Civil Structures", 7, (1998), pp.693-703.
- (9) D. J. Peel, R. Stanway and W. A. Bullough, *Dynamic modelling of an ER vibration damper for vehicle suspension applications*, "Smart Materials and Structures", 5, (1996), pp.591-606.