Active Feedforward Wave Control of a Rectangular Panel Using a Wave filter Constructed with Smart Mode Sensors*

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Abstract
This paper is concerned with an active wave control method of a rectangular panel. It is the purpose of this paper to present a wave filtering method for the panel using smart modal sensors and its application to an adaptive feedforward control system. Firstly, a wave solution of a rectangular panel is derived to describe the wave dynamics of the structure. This is followed by the proposition of the design procedure of the wave filter using smart mode sensors. Then, from a viewpoint of numerical analyses, accuracy of the proposed method is verified using condition numbers of a filtering matrix. Furthermore, a multi-rate technique, which reduces the computational burden, is introduced for approximation of the sub-filters in the wave filter. Finally, experiment on an adaptive feedforward control system using the proposed method is carried out. It is found that the reflected wave absorbing control enables the inactivation of vibration modes.

Key words: Active Wave Control, Wave Filter, Rectangular Panel, Smart Mode Sensor, Multi-Rate Technique

1. Introduction

Many methods of active vibration control employing either feedforward or feedback have been proposed. The main ones are based on models of superimposed vibration modes(1)-(4). Research on active wave control based on wave analysis has recently been attracting considerable interest(5)-(17). Such methods utilize a mechanism in which vibration modes are excited and the waves that are constituent components of the vibration modes are removed to obtain unique control effects of an inactive vibration mode or the generation of a vibration-free state. The first proposed wave control method was termination impedance control proposed by Vaughan(5). The realization of a wave control system by placing an inertial damper at a cantilever tip was attempted by von Flotow et al(6). Tanaka and Kikushima proposed an active sink method in which the main point is to provide a finite beam that has the characteristics of a semi-infinite beam. The effectiveness of this method was verified both theoretically and experimentally(7). In addition, the authors extended active wave control theory to two-dimensional problems by proposing a feedforward wave control method for thin rectangular panels, clarifying that a control effect similar to that of a flexible beam can be obtained when the panel has a boundary condition that constrains its displacement(8).

Because of a general vulnerability to modeling errors, feedforward control systems are
often realized using an adaptive control method\(^{(18)}\), typically a filtered-x LMS algorithm. This method minimizes the error signals obtained in the field. Consequently, a means for obtaining information about wave propagation in structures (i.e., a wave filtering method) is required to construct an adaptive wave control system. Wave filtering methods that have been proposed can be classified into two general categories: those that use point sensors and those that employ smart sensors. Point sensor-based methods first started as a vibration intensity measurement method in a flexible beam\(^{(19)}\). Subsequently, the authors\(^{(9)-(11)}\) and Halkyard and Mace\(^{(12)}\) proposed a wave filtering method for a flexible beam and a thin rectangular panel and demonstrated its validity as a control sensor. On the other hand, the authors have found that the shaping function for a smart sensor used to measure wave amplitudes can be expressed by a complex exponential function, and proposed a method based on smart sensors that target a single frequency\(^{(14)-(15)}\). Also, as an evolved form, by introducing a phase shifter capable of accommodating a certain bandwidth, the authors have constructed a wave control system that is able to handle band-limited random noise, experimentally demonstrating its effectiveness\(^{(16)-(17)}\).

A main problem of the smart sensor-based methods\(^{(14)-(17)}\) is that a shaping function is dependent on frequency. Consequently, in these methods, a single frequency for which wave measurements can be made with 100\% theoretical accuracy is set in advance. However, as noted above, introducing a phase shifter that supports a certain frequency band means that wave measurements can be made with a certain degree of accuracy in that band only. Point sensor-based methods do not suffer from this problem. However, when the target structure is a rectangular panel\(^{(11)}\), many sensors are required to filter the group of vibration modes associated with the wave to be measured.

The aim of the present study is to develop a simple implementation of a smart wave filter. A simple wave measurement method is proposed by introducing multiple one-dimensional smart sensors and by applying appropriate signal processing to each sensor output. First, a wave solution is derived for a thin rectangular panel and then a filtering equation is presented that is based on one-dimensional smart sensors that extract only the target mode group. Next, the accuracy of the proposed method is evaluated based on the condition number of the matrix used in the filtering equation and the center of the sensor area is identified as the optimal measurement point. It is then revealed that a multi-rate signal processing technique\(^{(20)}\) can reduce the computational load when approximating the sub-filters that compose a wave filter. Finally, an adaptive feedforward wave control system using the proposed wave filter is constructed, demonstrating the validity of the control method for suppressing the vibration.

2. Wave filtering method using smart mode sensors

In this section, the design method for a wave filter that employs smart sensors is clarified. First, the solution for a thin rectangular panel is derived. The equation of motion for the corresponding structure is given by

\[
DV^2 w(x,y,t) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t),
\]

where \(D\) is the flexural rigidity, \(V^2\) is the Laplacian operator, \(\rho\) is the density, \(h\) is the thickness, and \(f\) is the external force. Here, when the external force is a harmonic input with an angular frequency \(\omega\), the variables of the panel displacement can be separated as \(w(x,y,t) = w(x,y)e^{j\omega t}\). Assuming that the eigenfunction component \(\varphi_{x,y,m}(y)\) in the \(y\) direction is periodic, \(w(x,y)\) can be expressed as follows, based on mode expansion theory:

\[
w(x,y) = \sum_{m,n=1}^{\infty} w_{x,m}(x)\varphi_{x,y,m}(y),
\]
where
\[
\frac{d^2 \phi_{y,n}(y)}{dy^2} = -\beta^2 \phi_{y,n}(y).
\] (3)

Here, \( \beta \) expresses the wave number in the \( y \) direction. From Eq. (2), the vibration modes of a rectangular panel are generally called \( (m, n) \) modes. Substituting Eqs. (2) and (3) into the homogeneous equation of Eq. (1) yields
\[
\sum_{m=1}^{\infty} \left[ \frac{d^4 w_{x,m}(x)}{dx^4} - 2\beta^2 \frac{d^2 w_{x,m}(x)}{dx^2} + \left( \beta^2 - k^2 \right) w_{x,m}(x) \right] = 0,
\] (4)

where
\[
k^4 = \rho h \omega^2 / D.
\] (5)

In Eq. (4), wave solution in the \( x \) direction can be expressed as a superposition of modes as
\[
w_x(x) = \sum_{m=1}^{\infty} w_{x,m}(x).\]

If this is the case, \( w_x(x) \) is derived as
\[
w_x(x) = c_{1,n} e^{a_n x} + c_{2,n} e^{b_n x} + c_{3,n} e^{-a_n x} + c_{4,n} e^{-b_n x},
\] (6)

where \( c_{1,n}, c_{2,n}, c_{3,n}, \) and \( c_{4,n} \) are coefficients determined by factors such as external forces, boundary conditions, and the exciting frequency. In addition, \( a_n \) and \( b_n \) are given by
\[
a_n = \sqrt{\beta^2 + k^2}, \quad b_n = \sqrt{\beta^2 - k^2}.
\] (7)

This equation reveals that \( b_n \) becomes imaginary above a certain frequency and consequently the second and fourth terms on the right-hand side of Eq. (6) become traveling wave components. Thus, unless the exciting frequency exceeds a certain value, a traveling wave will not propagate through the structure. This frequency is called the cut-on frequency and is defined by
\[
\omega_{e,n} = \sqrt{D / \rho h \beta^2}.
\] (8)

Substituting Eq. (2) into Eq. (6), the wave solution for a rectangular panel is obtained as
\[
w(x,y) = \sum_{n=1}^{\infty} \left( c_{1,n} e^{a_n x} + c_{2,n} e^{b_n x} + c_{3,n} e^{-a_n x} + c_{4,n} e^{-b_n x} \right) \phi_{y,n}(y).
\] (9)

The above equation indicates that the mode components in the \( x \) direction are bundled in the form of a traveling wave solution and that each wave term depends on the index \( n \) of the eigenfunction components in the \( y \) direction.

Next, as shown in Fig. 1, consider the case in which \( N \) polyvinylidene fluoride (PVDF) sensors\(^{(21)}\) that have been shaped by a shaping function \( F(y) \) are attached parallel to the \( y \) axis to an \( L_x \times L_y \) thin rectangular panel. In this case, the charge output of the \( i \)th PVDF sensor is expressed by
\[
q_i = \Gamma \int_{x_i}^{x_i + F(y)} \left( e_{31} \frac{\partial^2 w(x,y)}{\partial x^2} + e_{32} \frac{\partial^2 w(x,y)}{\partial y^2} \right) dx dy,
\] (10)

where \( \Gamma \) is the sensor constant, and \( e_{31} \) and \( e_{32} \) are the piezoelectric stress constants in the \( x \) and \( y \) directions, respectively (in this paper, these constants are fixed regardless of the sensor). Next, substituting Eq. (9) into Eq. (10) yields
\[
q_i = \Gamma \sum_{n=1}^{\infty} \int_{x_i}^{x_i + F(y)} \left( e_{a,n} c_{1,n} e^{a_n x} + e_{b,n} c_{2,n} e^{b_n x} + e_{c,n} c_{3,n} e^{-a_n x} + e_{d,n} c_{4,n} e^{-b_n x} \right) \phi_{y,n}(y) dx dy,
\] (11)

where
\[
e_{a,n} = e_{31} a_n^2 - e_{32} \beta_n^2, \quad e_{b,n} = e_{31} b_n^2 - e_{32} \beta_n^2.
\] (12)

Here, if \( F(y) << 1 / |b_n| \) \((1 / |b_n|)\) corresponds to the wavelength at frequencies above the
cut-on frequency), the finite-difference method can be used to approximate Eq. (11) as follows.

\[
q_l \approx \Gamma \sum_{n=1}^{\infty} \left( a_n e^{-x_n a_n} + b_n e^{-x_n b_n} + c_n e^{-x_n c_n} + d_n e^{-x_n d_n} \right) \int_0^{L_y} F(y) \phi_{y,n}(y) dy
\]

Next, consider the application of mode orthogonality to Eq. (13) to extract only specific \( y \) direction mode components. The orthonormality of the \( y \) direction mode components is expressed by

\[
\int_0^{L_y} \phi_{y,p}(y) \phi_{y,q}(y) dy = \delta_{pq},
\]

where \( \delta \) represents the Kronecker delta function. Using Eq. (14), a smart sensor can be designed for the \( p \)th mode. In other words, if the sensor shape function is \( d \phi_{y,p}(y) \), then Eq. (1) becomes:

\[
q_l = \gamma_{1,p} c_1, p e^{a_p y} + \gamma_{2,p} c_2, p e^{b_p y} + \gamma_{1,p} c_3, p e^{-a_p y} + \gamma_{2,p} c_4, p e^{-b_p y},
\]

where

\[
\gamma_{1,p} = \Gamma d \left( a_p^2 - b_p^2 \beta_p^2 \right),
\]

\[
\gamma_{2,p} = \Gamma d \left( a_p^2 - b_p^2 \beta_p^2 \right).
\]

Also, \( d \) represents the amplitude of the shape function \( F(y) \) and it has a sufficiently small value. Equation (15) reveals that by using a smart sensor, the \( y \) components of the displacement are cancelled so that only components relating to the traveling wave solution are extracted.

The method for filtering wave components that propagate in the \( x \) direction is described below. Generally, filtering of a wave propagating through a structure requires using multiple sensors and processing their outputs. Specifically, as shown in Eq. (15), the smart sensor output equation contains four wave terms and thus a minimum of four smart sensors are required for proper filtering. In this paper, the case is considered in which four smart sensors, having been shaped by the shaping function \( d \phi_{y,p}(y) \), are attached at regular intervals to a rectangular panel. If the sensor separation is \( 2L_s \) and the measurement point \( x_0 \) is at the midpoint of the sensor array, the charge output from each sensor is expressed by

\[
q_1 = \gamma_{1,1} c_1, 1 e^{a_p (x_0 - 3L_s)} + \gamma_{2,1} c_2, 1 e^{b_p (x_0 - 3L_s)} + \gamma_{1,1} c_3, 1 e^{-a_p (x_0 - 3L_s)} + \gamma_{2,1} c_4, 1 e^{-b_p (x_0 - 3L_s)}
\]

\[
q_2 = \gamma_{1,1} c_1, 1 e^{a_p (x_0 - L_s)} + \gamma_{2,1} c_2, 1 e^{b_p (x_0 - L_s)} + \gamma_{1,1} c_3, 1 e^{-a_p (x_0 - L_s)} + \gamma_{2,1} c_4, 1 e^{-b_p (x_0 - L_s)}
\]
\[ q_3 = \gamma_1 p c_1 p a_r (x_0 + L_s) + \gamma_2 p c_2 p b_r (x_0 + L_s) + \gamma_3 p c_3 p a_r (x_0 + L_s) + \gamma_4 p c_4 p b_r (x_0 + L_s), \]

\[ q_4 = \gamma_1 p c_1 p a_r (x_0 + 3L_s) + \gamma_2 p c_2 p b_r (x_0 + 3L_s) + \gamma_3 p c_3 p a_r (x_0 + 3L_s) + \gamma_4 p c_4 p b_r (x_0 + 3L_s). \]

The above set of equations can be written in matrix form as follows:

\[ \mathbf{q} = \mathbf{L}_p(\omega)\mathbf{w}_p(x_0), \]

where

\[ \mathbf{q} = (q_1 \ q_2 \ q_3 \ q_4)^T, \]

\[ \mathbf{L}_p(\omega) = \begin{bmatrix}
\gamma_1 p c_1 p e^{-3a_r L_s} & \gamma_2 p c_2 p e^{-3b_r L_s} & \gamma_1 p c_3 p e^{-3a_r L_s} & \gamma_2 p c_4 p e^{-3b_r L_s} \\
\gamma_1 p c_1 p e^{-a_r L_s} & \gamma_2 p c_2 p e^{-b_r L_s} & \gamma_1 p c_3 p e^{-a_r L_s} & \gamma_2 p c_4 p e^{-b_r L_s} \\
\gamma_1 p c_1 p e^{3a_r L_s} & \gamma_2 p c_2 p e^{3b_r L_s} & \gamma_1 p c_3 p e^{3a_r L_s} & \gamma_2 p c_4 p e^{3b_r L_s} \\
\gamma_1 p c_1 p e^{a_r L_s} & \gamma_2 p c_2 p e^{b_r L_s} & \gamma_1 p c_3 p e^{a_r L_s} & \gamma_2 p c_4 p e^{b_r L_s}
\end{bmatrix}, \]

\[ \mathbf{w}_p(x_0) = \begin{bmatrix}
c_1 p c_1 p e^{-a_r x_0} & c_2 p c_2 p e^{b_r x_0} & c_3 p c_3 p e^{-a_r x_0} & c_4 p c_4 p e^{-b_r x_0}
\end{bmatrix}^T. \]

If Eq. (24) is solved for the wave vector \( \mathbf{w}_p(x_0) \), the wave filtering equation can be derived as follows.

\[ \mathbf{w}_p(x_0) = \mathbf{L}_p^{-1}(\omega)\mathbf{q} = \mathbf{G}_p(\omega)\mathbf{q} \]

Accordingly, the wave component at \( x = x_0 \) can be filtered from the sensor output at \( x = x_1, x_2, x_3, \) and \( x_4 \). Figure 2 shows a schematic diagram of the wave filter for this case. As is clear from Eq. (26) and Fig. 2, the wave filter is configured from elements with frequency characteristics indicated by \( \mathbf{G}_p(\omega) \). Therefore, when actually constructing a control system, \( \mathbf{G}_p(\omega) \) must be realized in the target frequency area. This is achieved by using a digital filter; a detailed description of the method is given in Section 4.

In the above theoretical development, the wave measurement point was set at the center of the sensor region; however, the wave measurement point may be set at an arbitrary point in this method. In such a case, each sensor point is described in relation to the measurement point and its distance from that measurement point, and the theory can be redeveloped from Eq. (18). However, as shown in the next section, the selection of the wave measurement point greatly affects the filtering accuracy. To simplify the discussion below, the case when the wave measurement point is at the center of the sensor array is defined as wave filter type...
1 and the case when the wave measurement point is at point \( x = x_1 \) is defined as wave filter type 2.

### 3. Evaluation of filtering accuracy

In this section, the accuracy of the wave filtering proposed in the previous section is verified using numerical analyses. The specifications for the rectangular panel given in Table 1 are used. The simple support around the edge of the panel is the boundary condition and the traveling wave components belonging to \( n = 1 \) are the filtering (control) targets. In this case, the shaping function is defined as

\[
F(y) = d \sin \frac{\pi y}{L_y},
\]

where \( L_y \) represents the length of the rectangular panel in the \( y \) direction. A 40-\( \mu \)m-thick PVDF film with \( e_{31} = 7.5 \ \text{mC/m}^2 \) and \( e_{32} = 75 \ \text{mC/m}^2 \) is used for all the smart sensors in this study. In addition, the amplitude \( d \) of the shaping function is set to 0.01 m. The condition number of the \( L \) matrix is used as an index for evaluating the filtering accuracy. The condition number of the matrix provides the upper limit of the error generated when calculating the inverse of the matrix; it is used in this study as an index for the effect on filtering accuracy due to sensor calibration error, installation location error, observation noise, and similar effects\(^{(1)(13)}\).

Figure 3 shows the change in condition number with respect to frequency. Types 1 and 2 both have two peaks when the sensor separation is 0.08 m. The peak at 77.37 Hz matches

<table>
<thead>
<tr>
<th>Table 1 Specification of the rectangular panel</th>
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<tbody>
<tr>
<td>Length in the ( x ) direction, ( L_x )</td>
</tr>
<tr>
<td>0.33 m</td>
</tr>
<tr>
<td>Young's modulus</td>
</tr>
<tr>
<td>( 1.97 \times 10^{11} \ \text{N/m}^2 )</td>
</tr>
</tbody>
</table>

![Condition number of \( L \) versus sensor separation of the wave filter](image)

Fig. 3 Condition number of \( L \) versus sensor separation of the wave filter
the cut-on frequency for \( n = 1 \). In other words, as is clear from Eqs. (6) to (9), at the relevant frequency, \( c_{2,n} e^{ib_x x} \) and \( c_{4,n} e^{-ib_x x} \) are constant in the \( x \) direction; such filtering is physically impossible because no spatial difference is expressed between them. Accordingly, as is clear from Figs. 3(a) to (c), the peak frequency is independent of the sensor separation. Moreover, a peak appears at the half wavelength of the \( x \)-direction wave that depends on the \( n = 1 \) mode group. Thus, by reducing the sensor separation, the corresponding peak can be shifted to a higher frequency. Furthermore, the height of the asymptotic line of type 2 is about 10 to 100 times that of type 1. Accordingly, type 1 will give better performance. These results are discussed below.

First, to simplify the problem, the case is considered in which two smart sensors are attached in the far-field region where the effect of near field components can be neglected. In this case, wave filtering equation of the type 1 becomes

\[
\begin{align*}
\begin{pmatrix}
 c_{2,p} e^{b_p x_0} \\
 c_{4,p} e^{-b_p x_0}
\end{pmatrix} &= \frac{1}{\gamma_{2,p}} \begin{pmatrix}
 e^{-b_p L_x} - e^{b_p L_x} \\
 -e^{b_p L_x} + e^{-b_p L_x}
\end{pmatrix} \begin{pmatrix}
 q_1 \\
 q_2
\end{pmatrix}.
\end{align*}
\] (28)

When the exciting frequency exceeds the cut-on frequency, \( b_p \) is given by

\[ b_p = j k_p , \] (29)

where \( k_p \) represents the wave number in the \( x \) direction. Substituting Eq. (29) into Eq. (28) and introducing Euler’s formula, the wave component propagating in the \(+x\) direction, for example, is given by

\[
\begin{align*}
& c_{4,p} e^{-b_p x_0} = \frac{j}{2 \gamma_{2,p}} \sin 2k_p L_x \left( -e^{j k_p L_x} q_1 + e^{-j k_p L_x} q_2 \right).
\end{align*}
\] (30)

Furthermore, when \( L_x \) is sufficiently small that the relationship \( 2k_p L_x \ll 1 \) is satisfied, Eq. (30) becomes

\[
\begin{align*}
& c_{4,p} e^{-b_p x_0} = \frac{1}{2 \gamma_{2,p}} \left( q_1 + q_2 - \frac{1}{2 j k_p} q_2 - q_1 \right).
\end{align*}
\] (31)

The expression on the right-hand side of Eq. (31) enclosed in the parentheses reveals that the displacement component at the measurement point (the first term) and slope component in the \( x \) direction (the second term) are calculated based on the central difference. In other words, the wave components are calculated from the displacement components at the measurement point and its derivatives.

On the other hand, the wave filtering equation of the type 2 is given by

\[
\begin{align*}
\begin{pmatrix}
 c_{2,p} e^{b_p x_0} \\
 c_{4,p} e^{-b_p x_0}
\end{pmatrix} &= \frac{1}{\gamma_{2,p}} \begin{pmatrix}
 e^{-2b_p L_x} - 1 \\
 -e^{2b_p L_x} + 1
\end{pmatrix} \begin{pmatrix}
 q_1 \\
 q_2
\end{pmatrix}.
\end{align*}
\] (32)

Alternatively, by performing a similar process as for the type 1, the wave component propagating in the \(+x\) direction can be derived as

\[
\begin{align*}
& c_{4,p} e^{-b_p x_0} = \frac{1}{2 \gamma_{2,p}} \left( q_1 - \frac{1}{j k_p} \frac{q_2 - q_1}{2 L_x} \right).
\end{align*}
\] (33)

The above equation shows that in the type 2, information about the displacement component of the measurement point and its derivative are obtained using the one-sided difference. Accordingly, by using multiple sensors to measure the wave amplitude, if the sensor separation is sufficiently small, the state quantity estimated by the finite difference method can be considered to be equivalent to a coordinate transformation. However, it is difficult to determine whether the sensor separation in Figs. 3(a) and (b) is sufficiently small with respect to the wavelength (e.g., the wavelength at 300 Hz is 0.21 m). In such a case, the accuracy of the difference scheme becomes a dominant factor and type 1, which is based on the central difference, will be more accurate than type 2, which is based on one-sided
Next, consider the case in which the sensor area is relatively small. Figure 3(c) shows that when the sensor separation is 0.01 m, there is almost no difference between types 1 and 2. The values of both exceed 100 in the target frequency band and it is not meaningful to note the difference between them. This phenomenon is caused by the L matrix approaching a singular matrix as the sensor separation approaches zero. This result demonstrates that when using the finite difference method, there will be a high sensitivity to measurement errors if the sensor separation is too small.

Therefore, when designing a wave filter, the measurement position must be set in the center of the sensor area and the sensor separation must be determined based on the condition number of the L matrix.

4. Realization of a wave filter

This section describes the realization of a wave filter \( G_p(\omega) \) in Eq. (26) using digital filter technology. First, in this paper, the traveling wave component for \( n = 1 \) is chosen as the object to be filtered (\( p = 1 \)) and settings are implemented for a sensor separation of 0.04 m, a DSP sampling rate of 1500 Hz, and target frequency bandwidth of 500 Hz. In addition, a FIR filter whose stability does not need to be investigated is used in this study. For brevity, only the result for \( G_{1,41} \) (i.e., the element in row 4 and column 1 of \( G_1 \)) and the corresponding element are given in this paper.

Problems encountered in the realization of wave filters are that the constituent sub-filters (elements of \( G_p(\omega) \)) have non-causal properties and are not described by a rational function of the Laplace variable s and that the wave term properties change from a evanescent wave to a traveling wave near the cut-on frequency. Therefore, it is difficult to approximate the sub-filters with sufficient accuracy in a wide frequency band that contains the cut-on frequency. Figure 4 shows ideal but non-causal \( G_{1,41} \) frequency characteristics and the results of approximations using FIR filters with different orders. This figure shows that even when the order of the FIR filter is increased to 512, the results do not converge to the ideal characteristics and the accuracy of the approximation is not sufficient.

Therefore, the following three measures are implemented to overcome these problems:

I. Division of the frequency band

Consider the case in which the band below the cut-on frequency is defined as band 1, the band above is defined as band 2, and sub-filters are designed in each of these bands. In such a case, low-pass and high-pass filters are required to divide the frequency components of the sensor signal into two parts; Butterworth and elliptic filters are simultaneously used in this study. Butterworth filters have gradual cutoff characteristics, but their gain decreases with increasing distance from the cutoff frequency. As described later, this characteristic is necessary for the design of the sub-filter in band 2. Moreover, elliptic filters exhibit ripples

![Fig. 4 Comparison between the ideal \( G_{1,41} \) and approximated FIR filters](image-url)
over the entire band, but a low-order elliptic filter can have an extremely sharp cutoff. By using these filters, signal processing can be performed independently with the cut-on frequency as a boundary. The same filters must be used for all four sub-filters that construct the wave filter.

II. Estimation of velocity wave amplitude

As in Eq. (26), the proposed wave filter estimates the displacement wave amplitude based on strain information from the four smart sensors. However, Mace et al. have reported that if the velocity wave amplitude \( v_p(x_0) \) is estimated, the accuracy of the approximation will improve slightly. Therefore, in this paper, the velocity is estimated using

\[
v_p(x_0) = j \omega p(x_0) = j \omega G_p q_p = H_p q_p.
\]

III. Introduction of time delay

Non-causal frequency characteristics are generally difficult to approximate accurately in a certain bandwidth. However, the non-causality can be eliminated by introducing a certain time delay \( \tau \) into those characteristics. For example, when the approximation of \( H_{1,41} \) (i.e., the element in row 4 and column 1 of \( H_1 \)) is considered, \( H_{1,41} e^{j\omega \tau} \) will be approximated instead of \( H_{1,41} \). In this paper, \( \tau \) is set to the product of half the number of FIR filter taps and the sampling period, and the time delays in bands 1 and 2 are respectively defined as \( \tau_1 \) and \( \tau_2 \). In this case, real-time wave measurement is not achieved in the strict sense, but if the disturbance is a stationary signal such as white noise, it may be used for the error signal in an adaptive feed forward control system. However, as with the aforementioned low-pass and high-pass filters, the same time delay must be provided to all four sub-filters.

The order of the FIR filter selected for band 1 (band 2) is the smallest even number in the range for which the error is within ±1 dB (±0.3 dB) of the gain and ±10° (±5°) of the phase for the ideal characteristics. The orders of the Butterworth and elliptical filters used for the low-pass and high-pass filters are set within the range whereby the maximum gain in the stopband is at least 30 dB lower than the ideal gain characteristics in the other bands.

First, the ideal sub-filter in band 2 (77.37 to 750 Hz) is defined as \( H_{1,41,h} \). Figure 5 shows the frequency characteristics of \( H_{1,41,h} e^{j\omega \tau} \) and of an FIR filter (128th order) that has been approximated with a frequency weighting of 1 for frequencies between 77.37 and 740 Hz and of zero otherwise. Figure 5 shows that the approximation has good accuracy at the target frequencies, but it exhibits a very high gain in band 1 (0 to 77.37 Hz). As mentioned above, to eliminate this effect, a high-pass filter that combines a Butterworth filter and elliptic filter was used in this study. Figure 6 shows the frequency characteristics when this high-pass filter is applied. It shows that the gain decreases sharply immediately above the cut-on frequency; this represents the elliptic filter (23rd order) characteristics. Additionally, the gain falls off on the left-hand side as the frequency approaches 0 Hz; this
is a manifestation of the Butterworth filter (14th order) characteristics.

Next, the sub-filter in band 1 (0 to 77.83Hz) is realized. In this band, the ideal characteristics are defined as $H_{1,41,1}e^{j\omega\tau}$. Figure 7(a) shows the frequency characteristics of $H_{1,41,1}e^{j\omega\tau}$ and of an FIR filter (48th order) that has been approximated with the frequency weighting set to 1 in the frequency range 4 to 77.37 Hz and to 0 otherwise. As in the case when $H_{1,41,1}e^{j\omega\tau}$ was approximated, the gain is very high outside the target frequency band. To eliminate this effect, both a Butterworth filter and an elliptical filter were used, as for the case of $H_{1,41,0}e^{j\omega\tau}$. This also enhances the stopping performance of the low-pass filter; however, this paper proposes a method of downsampling\(^{(20)}\) to reduce the Nyquist frequency and to limit the maximum gain. This enables to reduce the order of the low-pass filter. Figure 7(b) shows the approximation results when the sampling rate is 166.7 Hz (1500/9 Hz). It shows that the maximum gain has been reduced to 15 dB. Thus, unlike in the case of $H_{1,41,0}e^{j\omega\tau}$, a Butterworth filter is not needed, and the desired low-pass filter can be constructed using a relatively low-order (14th order) elliptic filter. Additionally, if up-sampling is performed, a low-pass filter must also be used in a later stage and the same elliptic filter can also be used here. Figure 8 shows the frequency characteristics for this case. It shows that the band 1 gain characteristics are wavy; this is the ripple effect of the elliptic filter. However, because the purpose of the wave filter is to obtain information proportional to the amplitude of the wave to be controlled, this ripple is not problematic provided that the same elliptic filter is used in all sub-filters.

Figure 9 shows a block diagram of $H_{1,41}$ based on the above design method. In the figure, the number 9 represents the sampling rate conversion constant and the numbers enclosed in parentheses below each block represent the respective filter order. In contrast to the 37th order high-pass filter in band 2, the orders of filters in band 1 (with the exception of the sub-filter itself) are limited to the 28th order or less. Since these are IIR filters, the
calculations can be reduced by 18 iterations. Furthermore, because $H_{1,41,\omega}^\mathrm{l}\tau$ only has to be computed once for every nine samples, the introduction of a multi-rate signal processing method reduces the computational load.

5. Experiment

5.1. Control system configuration and experimental method

Figure 10 shows a photograph of a simply supported rectangular panel with smart sensors attached (with the same specifications as those listed in Table 1) and Fig. 11 shows a block diagram of the control system. In this experiment, the wave measurement point is located at $x = 0.18$ m. In addition, an actuator that includes an electromagnetic coil to generate shear force is used. The disturbance point is set to $(x_d, y_d) = (0.3$ m, $0.04$ m) and control points are set at $y$ coordinate locations of $y_1 = 2x/3$ and $y_2 = x/3$ (i.e., on the nodal line of the $(\ast, 3)$ mode group). The $x$ coordinate of the control points is $x_c = 0.03$ m. As mentioned above, since waves dependent on the $n = 1$ mode group are targeted in this study, two control actuators are driven in-phase. An adaptive feedforward control scheme\(^{(18)}\) based on a filtered-$x$ LMS algorithm is used as the control method. There are 256 taps in the controller and cancelation paths. In this experiment, the driving point compliance and the absolute displacement distribution of the panel are measured. For the former, the measurement is performed by applying a white noise disturbance, whereas in the latter, measurement is performed by applying a harmonic excitation disturbance at the resonance frequency with a laser Doppler vibrometer at the points of intersection of a mesh consisting of thirteen lines in the $y$ direction and seven lines in the $x$ direction (total of 91 points). The numerical values are then input to graph-generating software, which outputs the absolute displacement distribution.
5.2. Experimental results

Figure 12 shows the driving point compliance with and without control. For the case without control, seven vibration modes exist in the frequency band up to 500 Hz. However, the overall damping is large and almost no peak is observed for the (1, 2) mode. On the other hand, when reflective wave absorbing control is implemented, the resonant peak in the \( n = 1 \) mode group is suppressed. Especially in the vicinity of the (1, 1) and (3, 1) modal frequencies, the gain characteristics nearly converge as asymptotic lines (curves that form the base for the gain characteristics). This is a typical result of reflected wave absorbing control and indicates that the vibration mode has been inactivated.

Next, the characteristics of reflected wave absorbing control are clarified in terms of the displacement distribution. Figure 13 shows the absolute displacement distributions obtained with and without control. The exciting frequency in this case is 171 Hz, the so-called (2, 1) modal frequency. When no control is applied, a nodal line appears due to the formation of standing waves. In such a case, because the standing wave has a similar shape as the vibration mode, the vibration mode is excited and the maximum displacement is 176 µm.

On the other hand, if reflective wave absorbing control is implemented, the difference between the maximum and minimum amplitudes near the center of the panel is not as pronounced as for the case without control. This shows that the formation of a standing wave is inhibited due to the absorption of the reflected wave. The maximum amplitude is reduced to 55 µm, which is 31.3% of the uncontrolled amplitude.

For the experimental displacement distribution, it should be noted that a sufficient control can be obtained even though the wave is not fully absorbed (if the reflected wave is...
 completamente absorbed, there is almost no difference between the maximum and minimum amplitudes near the center of the panel). As the authors have previously reported, this is because the error signal is the wave amplitude. That is, when constructing a wave control system, the control effect will transition between traveling wave control and standing wave control. Thus, even if a filtering error occurs, the control performance will not be fatally deteriorated.

6. Conclusions

This paper proposed a wave filtering method for a rectangular panel using smart sensors and applies this method to an adaptive feedforward control system. The main results are summarized as follows.

1. Wave filter for a rectangular panel is based on modal filter in a certain direction and wave decomposition in the orthogonal direction. In this study, the former was realized using smart sensors and the latter was realized by signal processing.

2. The wave filtering accuracy was evaluated in terms of the condition number. The optimal wave measurement point was found to be at the center of the sensor area. This is because the wave filtering method is essentially the same as a coordinate transformation of the state variables that consist of displacement and its derivatives.

3. A method for realizing four sub-filters to decompose the four waves was realized. Because the filter characteristics are different before and after the cut-on frequency, an approximation method was proposed for each frequency band by using low-pass and high-pass filters. Moreover, down-sampling performed in the band below the cut-on frequency clearly reduced the amount of computation.

4. An adaptive feedforward wave control system was constructed by applying a wave filter using smart mode sensors to the error sensor in an adaptive control system based on a filtered-x LMS algorithm. The target mode group was inactivated by the proposed method. In addition, even if the wave is not fully absorbed, it was found that sufficient control could be obtained by disrupting the balance between traveling and reflected waves.

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References


