LMI Approach to Robust Control of Rotary Cranes under Load Sway Frequency Variance*

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Abstract

Because horizontal motion of a rotary crane generates undesirable two-dimensional load sway, skillful operators are needed to control the crane's motion; for this purpose, various types of control schemes have been proposed. Because natural frequency of the rope-load oscillation system affects the stability and performance of the control system, the controller design should consider robustness with respect to rope length. If the control system considers the effect of rope length variance, the crane's motion can be controlled without a sensor system for measuring it. This paper presents a control method based on linear matrix inequality optimization for achieving robustness with respect to rope length variance. Numerical simulations and experimental results demonstrate the effectiveness of the proposed method.

Key words : Rotary Crane, Motion Control, Robust Control, State Feedback Control with Integrator, LMI

1. Introduction

Crane systems are widely used in industrial applications, such as construction sites, ports, railway yards, and other locations. A crane system should be controlled for rapid transport of load without excessive load sway at the final position. However, one-dimensional horizontal boom motion of a rotary crane typically generates undesirable two-dimensional load sway; therefore, crane operators must be highly skilled in order to manually stop the load sway immediately at the right position. Failures of crane control cause accidents, injure people, and damage surroundings.

To reduce the burden on human operators and increase their safety, various control schemes have been developed for crane control. These methods can be classified into two types: trajectory generation methods for the crane's motion that can suppress load sway without requiring load sway information (i.e., open-loop control)⁽¹⁾⁻⁽⁹⁾ and methods using load sway information measured by sensor systems in real time to suppress load sway (i.e., closed-loop control)⁽¹⁰⁾⁻⁽¹³⁾. The former methods are valid only for an ideal system without disturbances such as wind; therefore, they may not achieve good outdoor performance. In contrast, the latter methods may be used to construct a robust control system for cranes.

However, existing studies do not consider robustness with respect to variations in rope length. If the control system considers the effect of rope length variance, the crane's motion can be controlled without a sensor system for measuring it. This study aims to propose a method to achieve this.

Because the control system is expected to be robust with respect to varying parameters such as joint friction, we first consider a controller design on the basis of a disturbance observer to provide the linear dynamics of rotary crane motion. Next, we design a state feedback controller with an integrator to achieve robust control performance for a given range of variations in rope length. The proposed control system satisfies additional constraint conditions

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Fig. 1 Schematic model of rotary crane

Fig. 2 Control system with disturbance observer

on closed-loop pole placement. These conditions are expressed in terms of linear matrix inequalities (LMI)⁽¹⁴⁾, and controller gains are determined numerically via LMI optimization. Finally, simulations and experimental results demonstrate the effectiveness of the proposed method.

2. Rotary Crane Dynamics

In this section, we describe the dynamics of the rotary crane whose schematic model is shown in Fig. 1⁽¹³⁾. θ_1 and θ_2 are the load sway angles in the plane of vertical boom motion and the tangential direction of horizontal boom motion, respectively. θ_3 and θ_4 are the vertical and horizontal angles of the boom, respectively; l_r and l_b denote the lengths of the rope and boom, respectively; and (x, y, z) denotes the three-dimensional position of the load.

We assume that the rotary crane dynamics has the following characteristics:

- The load can be considered to be a point mass, and the torsion of the rope can be neglected.
- The angle and angular velocity (θ_{i+2} and $\dot{\theta}_{i+2}$, $i = 1, 2$, respectively) of vertical and horizontal boom motion and the load sway angles and their angular velocities (θ_i and $\dot{\theta}_i$, $i =$

1, 2, respectively) are measurable.

- The sway angle θ_i has a small magnitude, so that approximations $\sin \theta_i \simeq \theta_i$ and $\cos \theta_i \simeq 1$, ($i = 1, 2$) are reasonable.

The equations of motion for vertical and horizontal boom motion are

$$J_{i+2}\ddot{\theta}_{i+2} + d_{i+2} = K_{i+2}u_{i+2}, \quad (i = 1, 2) \quad (1)$$

where J_{i+2} , d_{i+2} , K_{i+2} , and u_{i+2} denote the inertia of vertical and horizontal boom motion, the nonlinear force term including disturbance, the plant gain, and the command voltage from the computer, respectively. Equation (1) assumes that inertia of vertical and horizontal boom motion J_{i+2} is constant, and the effect of time variance is included in disturbance d_{i+2} .

To compensate for the effect of disturbances, the following disturbance observer⁽¹³⁾ is applied to Eq. (1):

$$u_{i+2} = \frac{J_{i+2}}{K_{i+2}} \left\{ v_{i+2} + \frac{\omega_{i+2}}{s + \omega_{i+2}} \left(\frac{K_{i+2}}{J_{i+2}} u_{i+2} - s\dot{\theta}_{i+2} \right) \right\} \quad (2)$$

where s , v_{i+2} , and ω_{i+2} represent the differential operator, the new virtual control input calculated by the feedback controller, and the cut-off angular frequency of a low-pass filter (LPF), respectively. A block diagram of this control system is shown in Fig. 2. The dynamics of the crane system in the low-frequency range is

$$\ddot{\theta}_{i+2} \simeq v_{i+2} \quad (3)$$

On the other hand, the dynamics of the rope-load system can be derived as follows:

$$l_r(1 + \theta_1^2)\ddot{\theta}_1 + l_r\theta_1\ddot{\theta}_2 + l_b(\cos \theta_3 - \theta_1 \sin \theta_3)\ddot{\theta}_3 - l_r\ddot{\theta}_4 + l_r\theta_1(\dot{\theta}_1^2 + \dot{\theta}_2^2) - l_b(\sin \theta_3 + \theta_1 \cos \theta_3)\dot{\theta}_3^2 - (l_b \sin \theta_3 + l_r\theta_1)\dot{\theta}_4^2 - 2l_r\dot{\theta}_2\dot{\theta}_4 + g\theta_1 = 0 \quad (4)$$

$$l_r\theta_1\ddot{\theta}_2 + l_r(1 + \theta_2^2)\ddot{\theta}_2 - l_b\theta_2 \sin \theta_3\ddot{\theta}_3 + (l_b \sin \theta_3 + l_r\theta_1)\ddot{\theta}_4 + l_r\theta_2(\dot{\theta}_1^2 + \dot{\theta}_2^2) - l_b\theta_2 \cos \theta_3\dot{\theta}_3^2 - l_r\theta_2\dot{\theta}_4^2 + 2l_b \cos \theta_3\dot{\theta}_3\dot{\theta}_4 + 2l_r\dot{\theta}_1\dot{\theta}_4 + g\theta_2 = 0 \quad (5)$$

where g is the gravitational acceleration. The derivation of Eqs. (4) and (5) is described in Appendix A.

We assume the variables of the crane system and their time derivatives to be represented as follows:

$$\theta_n = \theta_{nf} + x_n, \dot{\theta}_n = \dot{\theta}_{nf} + \dot{x}_n, \ddot{\theta}_n = \ddot{\theta}_{nf} + \ddot{x}_n, \quad (n = 1, 2, 3, 4) \quad (6)$$

where θ_{nf} and $\dot{\theta}_{nf}$ are the desired state variables, $\ddot{\theta}_{nf}$ is the desired acceleration, and x_n , $\dot{x}_n$, and $\ddot{x}_n$ are deviations of signals from the desired ones.

We linearize the crane dynamics around the desired position of the boom by assuming that θ_i and $\dot{\theta}_i$ ($i = 1, 2$) are small, such that $\theta_i\dot{\theta}_p \simeq 0$, $\dot{\theta}_i\dot{\theta}_p \simeq 0$, $\dot{\theta}_i\dot{\theta}_{p+2} \simeq 0$, $\theta_i\dot{\theta}_{p+2}^2 \simeq 0$, and $\dot{\theta}_i\dot{\theta}_{p+2} \simeq 0$ ($p = 1, 2$) are satisfied. In addition, $\dot{\theta}_{p+2}$ is assumed to be small such that $\dot{\theta}_3\dot{\theta}_4 \simeq 0$ and $\dot{\theta}_{p+2}^2 \simeq 0$ are satisfied. These assumptions lead to the following linear state equation:

$$\dot{x}_i = A_i x_i + b_i v_{i+2}, \quad (i = 1, 2) \quad (7)$$

$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, b_i = \begin{bmatrix} 0 \\ 0 \\ -\gamma_i \\ 1 \end{bmatrix}, x_i = \begin{bmatrix} x_i \\ x_{i+2} \\ \dot{x}_i \\ \dot{x}_{i+2} \end{bmatrix}$$

where $\xi = g/l_r$, $\gamma_1 = l_b \cos \theta_3/l_r$, and $\gamma_2 = l_b \sin \theta_3/l_r$.

Hence, we can use v_1 to control θ_1 and θ_3 , and v_2 to control θ_2 and θ_4 . Because the structures of the two state equations are the same, we can apply the same controller.

Fig. 3 Stable region

3. Robust Controller Design

This section presents a controller for achieving robust performance with respect to rope length variance on the basis of an LMI approach.

3.1. LMI Representation of Stable Region

Dynamical system $\dot{x} = Ax$ is stable if all eigenvalues of matrix A lie in the left-half of a complex plane. However, considering robustness with respect to variations in matrix A and control performance, control design requires a suitable subregion of the left-half of the plane. To achieve this, Mahmoud and Pascal proposed a method for representing the stable complex region, indicated by the shaded region in Fig. 3⁽¹⁴⁾. The condition that all eigenvalues of A lie in the shaded region in Fig. 3 is represented as follows:

$$\begin{aligned} AX + XA^T + 2\alpha X &< 0 \\ AX + XA^T + 2\beta X &< 0 \\ \begin{bmatrix} \sin \theta ((AX) + (AX)^T) & \cos \theta ((AX) - (AX)^T) \\ \cos \theta ((AX)^T - (AX)) & \sin \theta ((AX) + (AX)^T) \end{bmatrix} &< 0 \\ X &> 0 \end{aligned} \quad (8)$$

where θ , α , and β are the positive scalars, and X is the symmetric matrix.

3.2. State Feedback Control with Integrator

We consider the following linear control system with an integrator for a rotary crane:

$$\begin{aligned} \dot{x}'_i &= A'_i x'_i + b'_i v_{i+2}, \quad (i = 1, 2) \\ A'_i &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad b'_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad x'_i = \begin{bmatrix} x_i \\ x_{i+2} \\ \dot{x}_i \\ \dot{x}_{i+2} \\ \int_0^t x_{i+2} dt \end{bmatrix} \\ v_{i+2} &= k_i^T x'_i, \quad k_i^T = \begin{bmatrix} k_{i0} & k_{i1} & k_{i2} & k_{i3} & K_{fi} \end{bmatrix} \end{aligned} \quad (9)$$

The closed-loop system can be represented as follows:

$$\dot{x}'_i = (A'_i + b'_i k_i^T) x'_i, \quad (i = 1, 2) \quad (10)$$

Hence, we determine the controller gains by a pole placement and optimal regulator method.

3.3. LMI Formulation of Pole Placement Problem

In this section, we formulate the pole placement problem on the basis of LMI. If all poles of the system in Eq. (10) are located in the region shown in Fig. 3, the system is stable. Thus, the pole placement problem can be represented as finding X and M_i such that the following equations are satisfied,

$$\begin{aligned} A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T + 2\alpha X &< 0 \\ A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T + 2\beta X &< 0 \\ \begin{bmatrix} \sin \theta (A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T) & \cos \theta (A'_i X - X A_i'^T + b'_i M_i - M_i^T b_i'^T) \\ \cos \theta (-A'_i X + X A_i'^T - b'_i M_i + M_i^T b_i'^T) & \sin \theta (A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T) \end{bmatrix} &< 0 \\ X &> 0 \end{aligned} \quad (11)$$

where $M_i = k_i X$.

3.4. LMI Formulation of Optimal Regulator Problem

We consider the following variable and cost function $J^{(14)}$:

$$\begin{aligned} J &= \frac{1}{2} \int_0^\infty z_i^T z_i dt, \quad (i = 1, 2) \\ z_i &= Q_i^{1/2} x_i + R_i^{1/2} v_{i+2} \end{aligned} \quad (12)$$

where $Q_i \geq 0$ and $R_i > 0$.

This problem can be transformed into the problem of finding X_2 , Y , and k_i such that $\text{Trace}(Y)$ is minimal in the following constraints:

$$\begin{aligned} \begin{bmatrix} (A'_i + b'_i k_i) X_2 + X_2 (A'_i + b'_i k_i)^T & 0 \\ 0 & -I \end{bmatrix} &< 0 \\ \begin{bmatrix} Y & (Q_i^{1/2} + R_i^{1/2} k_i) X_2 \\ X_2 (Q_i^{1/2} + R_i^{1/2} k_i)^T & X_2 \end{bmatrix} &> 0 \end{aligned} \quad (13)$$

Setting $M_{i2} = k_i X_2$ yields

$$\begin{aligned} \begin{bmatrix} (A'_i X_2 + b'_i M_{i2}) + (X_2 A_i'^T + M_{i2}^T b_i'^T) & 0 \\ 0 & -I \end{bmatrix} &< 0 \\ \begin{bmatrix} Y & (Q_i^{1/2} X_2 + R_i^{1/2} M_{i2}) \\ (Q_i^{1/2} X_2 + R_i^{1/2} M_{i2})^T & X_2 \end{bmatrix} &> 0 \end{aligned} \quad (14)$$

Therefore, the optimal regulator problem can be transformed into the new problem of finding X_2 , Y , and M_{i2} that satisfies the above LMI such that $\text{Trace}(Y)$ is minimal.

3.5. Controller Design

In this study, we combine the pole placement and optimal regulator problems expressed using LMI to design the controller as follows:

Setting $X = X_2$ and $M_i = M_{i2}$ ($i = 1, 2$) yield

$$\begin{aligned} A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T + 2\alpha X &< 0 \\ A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T + 2\beta X &< 0 \\ \begin{bmatrix} \sin \theta (A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T) & \cos \theta (A'_i X - X A_i'^T + b'_i M_i - M_i^T b_i'^T) \\ \cos \theta (-A'_i X + X A_i'^T - b'_i M_i + M_i^T b_i'^T) & \sin \theta (A'_i X + X A_i'^T + b'_i M_i + M_i^T b_i'^T) \end{bmatrix} &< 0 \\ X &> 0 \\ \begin{bmatrix} Y & (Q_i^{1/2} X + R_i^{1/2} M_i) \\ (Q_i^{1/2} X + R_i^{1/2} M_i)^T & X \end{bmatrix} &> 0 \end{aligned} \quad (15)$$

$$\begin{aligned} \begin{bmatrix} Y & (Q_i^{1/2} X + R_i^{1/2} M_i) \\ (Q_i^{1/2} X + R_i^{1/2} M_i)^T & X \end{bmatrix} &> 0 \end{aligned} \quad (16)$$

This problem can be numerically solved by using LMI optimization⁽¹⁵⁾.

3.6. Robust Control Performance

In this section, we demonstrate that if the system in Eq. (10) satisfies the pole placement condition mentioned in §3.3 at both ends of the rope length variation region, the system also satisfies the pole placement condition with any rope length within the variation region.

Assuming that the rope length variation region is $l_{r1} \leq l_r \leq l_{r2}$, we have

$$\frac{1}{l_r} = \varphi_1 \frac{1}{l_{r1}} + \varphi_2 \frac{1}{l_{r2}} \quad (17)$$

where $\varphi_1 + \varphi_2 = 1$ and $\varphi_1, \varphi_2 > 0$.

The system in Eq. (10) can be represented as follows:

$$\begin{aligned} \dot{x}'_i &= \left(\sum_{p=1}^2 \varphi_p A_{ip} + \sum_{p=1}^2 \varphi_p b_{ip} k_i^T \right) x'_i, \quad (i, p = 1, 2) \\ A_{i1} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\xi_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, A_{i2} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\xi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \\ b_{i1} &= \begin{bmatrix} 0 \\ 0 \\ -\gamma_{i1} \\ 1 \\ 0 \end{bmatrix}, b_{i2} = \begin{bmatrix} 0 \\ 0 \\ -\gamma_{i2} \\ 1 \\ 0 \end{bmatrix} \end{aligned} \quad (18)$$

where $\xi_1 = g/l_{r1}$, $\xi_2 = g/l_{r2}$, $\gamma_{i1} = l_b \cos \theta_{3f}/l_{r1}$, and $\gamma_{i2} = l_b \sin \theta_{3f}/l_{r2}$.

Therefore, the condition in Eq. (11) can be represented as follows:

$$\begin{aligned} A_{i1}X + XA_{i1}^T + b_{i1}M_i + M_i^T b_{i1}^T + 2\alpha X &< 0 \\ A_{i1}X + XA_{i1}^T + b_{i1}M_i + M_i^T b_{i1}^T + 2\beta X &< 0 \\ \begin{bmatrix} \sin \theta (A_{i1}X + XA_{i1}^T + b_{i1}M_i + M_i^T b_{i1}^T) & \cos \theta (A_{i1}X - XA_{i1}^T + b_{i1}M_i - M_i^T b_{i1}^T) \\ \cos \theta (-A_{i1}X + XA_{i1}^T - b_{i1}M_i + M_i^T b_{i1}^T) & \sin \theta (A_{i1}X + XA_{i1}^T + b_{i1}M_i + M_i^T b_{i1}^T) \end{bmatrix} &< 0 \\ X &> 0 \end{aligned} \quad (19)$$

$$\begin{aligned} A_{i2}X + XA_{i2}^T + b_{i2}M_i + M_i^T b_{i2}^T + 2\alpha X &< 0 \\ A_{i2}X + XA_{i2}^T + b_{i2}M_i + M_i^T b_{i2}^T + 2\beta X &< 0 \\ \begin{bmatrix} \sin \theta (A_{i2}X + XA_{i2}^T + b_{i2}M_i + M_i^T b_{i2}^T) & \cos \theta (A_{i2}X - XA_{i2}^T + b_{i2}M_i - M_i^T b_{i2}^T) \\ \cos \theta (-A_{i2}X + XA_{i2}^T - b_{i2}M_i + M_i^T b_{i2}^T) & \sin \theta (A_{i2}X + XA_{i2}^T + b_{i2}M_i + M_i^T b_{i2}^T) \end{bmatrix} &< 0 \\ X &> 0 \end{aligned} \quad (20)$$

Because the coefficient matrices A'_i and b'_i ($i=1,2$) of the control system in Eq. (10) are linear functions about the term $1/l_r$, the conditions in Eqs. (19) and (20) are also linear functions about the term $1/l_r$. Hence, if there exist $X > 0$ and $M_i > 0$ that satisfy the condition in Eq. (11) for $A'_i = A_{ip}$, $b'_i = b_{ip}$ ($i, p = 1, 2$), then X and M_i also satisfy the condition in Eq. (11) for $A'_i = \sum_{p=1}^2 \varphi_p A_{ip}$, $b'_i = \sum_{p=1}^2 \varphi_p b_{ip}$ ($\varphi_p > 0$, $\varphi_1 + \varphi_2 = 1$).

Therefore, robust control performance with respect to rope length variance can be achieved.

4. Results and Discussion

4.1. Experimental System

The experimental system is shown in Fig. 4. Same DC servo motors are used for both vertical and horizontal boom motion. Vertical and horizontal boom angle θ_{i+2} , ($i = 1, 2$) is measured by rotary encoders, having angular measurement resolution of 1.8×10^{-3} [deg].

Fig. 4 Experimental system

Fig. 5 Sensor system for measuring load sway

Load sway angles θ_1 and θ_2 are measured by potentiometers 1 and 2 in Fig. 5, respectively. Potentiometer 1 is fixed to the boom. Potentiometer 2 is fixed to part 1, which rotates around the rotational axis of potentiometer 1. Part 2 rotates around the rod and the rotational axis of potentiometer 2. Part 3 rotates around the rod, and part 4 rotates around part 3. Part 4 slides smoothly along the rope. The angular measurement resolution is 6.4×10^{-2} [deg]. θ_1 and θ_2 can be calculated from the output voltage values of potentiometers 1 and 2, respectively. The parameters of the rotary crane are shown in Table 1.

4.2. Simulation and Experimental Conditions

The following cycloid curve is employed to describe the desired angular trajectory of the boom:

$$r_q = (\theta_{qf} - \theta_{q0}) \left\{ \frac{t}{t_s} - \frac{1}{2\pi} \sin \left(2\pi \frac{t}{t_s} \right) \right\} + \theta_{q0}, \quad (q = 3, 4) \quad (21)$$

where θ_{qf} is the final angle, θ_{q0} is the initial angle, and t_s is the settling time. We set $\theta_{30} = 30$ [deg], $\theta_{3f} = 50$ [deg], $\theta_{40} = 0$ [deg], $\theta_{4f} = 30$ [deg], and $t_s = 3$ [s]. Also, r_q is set as $r_q = \theta_{qf}$ for $t \in (t_s, t_f]$ and $t_f = 10$ [s]. The cycloid curve provides zero acceleration at the initial and terminal points, and is widely used in industrial applications.

To determine the controller gains, we set $\theta = 78$ [deg], $\alpha = 1$, and $\beta = 30$ in Fig. 3 for the robust stable region; we also set $Q_i = \text{diag}\{1500, 315, 1500, 315, 50\}$ and $R_i = 1$ in Eq. (12), and considered rope length $l_r \in [l_{r1}, l_{r2}]$. The gains were assigned as follows:

$$\begin{aligned} k_1 &= \begin{bmatrix} -124 & 39.2 & -0.86 & 25.0 & 23.2 \end{bmatrix} \\ k_2 &= \begin{bmatrix} -103 & 38.4 & -0.92 & 24.4 & 22.8 \end{bmatrix} \end{aligned} \quad (22)$$

Table 1 Parameters of rotary crane

$J_3[\text{kg} \cdot \text{m}^2]$	1.02	$K_3[\text{N} \cdot \text{m/V}]$	2.06	$J_4[\text{kg} \cdot \text{m}^2]$	1.02
$K_4[\text{N} \cdot \text{m/V}]$	2.06	$L[\text{m}]$	0.65	$l_{r1}[\text{m}]$	0.25
$l_{r2}[\text{m}]$	0.75	$g[\text{m/s}^2]$	9.81	$m[\text{kg}]$	0.20

Fig. 6 Pole locations (Proposed controller)

In addition, we conducted a simulation to ensure that all poles of the closed-loop system in Eq. (18) lie in the proposed stable region as shown in Fig. 3. The initial poles of the vertical directional subsystem (i.e., under condition $l_r = 0.25[\text{m}]$) were set to $p_{11} = -10.8 + 7.74j[1/\text{s}]$, $p_{12} = -10.8 - 7.74j[1/\text{s}]$, $p_{13} = -1.57[1/\text{s}]$, $p_{14} = -1.53 + 0.97j[1/\text{s}]$, and $p_{15} = -1.53 - 0.97j[1/\text{s}]$; those of the horizontal directional subsystem were set to $p_{21} = -10.8 + 7.63j[1/\text{s}]$, $p_{22} = -10.8 - 7.63j[1/\text{s}]$, $p_{23} = -1.57[1/\text{s}]$, $p_{24} = -1.50 + 0.99j[1/\text{s}]$, and $p_{25} = -1.50 - 0.99j[1/\text{s}]$. The results shown in Fig. 6 confirm that the proposed controller achieves robustness with respect to rope length variance in the interval $[l_{r1}, l_{r2}]$.

4.3. Simulation and Experimental Results

To compensate for frictional effects, we conducted a simulation by using the following proportional-derivative (PD) controller with the disturbance observer shown in Fig. 2.

$$v_q = K_{qp}e_q + K_{qv}\dot{e}_q, \quad e_q = r_q - \theta_q, \quad \dot{e}_q = \dot{r}_q - \dot{\theta}_q, \quad (q = 3, 4) \quad (23)$$

where K_{qp} and K_{qv} are the controller gains; they were set to $K_{qp} = 50[1/\text{s}^2]$ and $K_{qv} = 10[1/\text{s}]$, respectively.

In this simulation, the disturbance d_{i+2} ($i = 1, 2$) in Eq. (1) includes not only the nonlinear force term but also the following friction term:

Fig. 7 Friction model

Fig. 8 Effectiveness of disturbance observer (DOB) (simulation)

Fig. 9 Proposed control results ($l_r=0.50[m]$)

Fig. 10 Proposed control results ($l_r=0.25[m]$)

Fig. 11 Proposed control results ($l_r=0.75[m]$)

Fig. 12 Pole locations (Conventional controller)

$$f_q = C_q \dot{\theta}_q + f_{qn}, \quad (q = 3, 4) \quad (24)$$

where C_q denotes the viscous friction coefficient and f_{qn} denotes the static friction or Coulomb friction term. The profile of f_{qn} is shown in Fig. 7, and can be represented as follows:

$$f_{qn} = \begin{cases} \operatorname{sgn}\{K_q u_q\} \min\{|K_q u_q|, f_{qs}\} & (\dot{\theta}_q = 0) \\ \operatorname{sgn}\{\dot{\theta}_q\} f_{qc} & (\dot{\theta}_q \neq 0) \end{cases} \quad (25)$$

where f_{qs} and f_{qc} are the magnitudes of static and Coulomb friction, respectively, and their numerical values are determined experimentally as $C_3 = 1.57[\text{N}\cdot\text{m}/(\text{rad/s})]$, $f_{3s} = 3.91[\text{N}\cdot\text{m}]$, and $f_{3c} = 3.91[\text{N}\cdot\text{m}]$; $C_4 = 1.39[\text{N}\cdot\text{m}/(\text{rad/s})]$, $f_{4s} = 3.71[\text{N}\cdot\text{m}]$, and $f_{4c} = 3.58[\text{N}\cdot\text{m}]$.

The simulation result is shown in Fig. 8. The cut-off angular frequency for the disturbance observer was set to $\omega = 40 [\text{rad/s}]$. All angular velocities were determined by calculating the backward difference between successive positional measurements. The steady-state tracking error shown in Fig. 8 was reduced by using the disturbance observer.

The robustness of the disturbance observer with respect to parameter variance, such as load mass and boom horizontal velocity, was demonstrated by numerical simulation, which is included in Appendix B.

To demonstrate the effectiveness of the proposed LMI-based design, we conducted simulations and experiments and set the rope length to $l_r = 0.50[\text{m}]$. The resulting vertical boom angle, horizontal boom angle, command voltages, and load sway angles are shown in Fig. 9. The controller gains obtained in Eq. (22) were used in these results. Although a slight overshoot appears in Figs. 9(a) and (b), load sway angles are suppressed in Figs. 9(c) and (d).

Fig. 13 Conventional control results ($l_r=0.50[m]$) (simulation)

Fig. 14 Conventional control results ($l_r=0.25[m]$) (simulation)

Fig. 15 Conventional control results ($l_r=0.75[m]$) (simulation)

To demonstrate the robust control performance of the proposed method, we conducted simulations and experiments in which we set the rope length to $l_r = 0.25[m]$ and $l_r = 0.75[m]$ with the same controller gains obtained in Eq. (22). The resulting vertical boom angle, horizontal boom angle, command voltages, and load sway angles are shown in Figs. 10 and 11. The results obtained in these setups were almost the same as those with condition $l_r = 0.50[m]$.

In addition, we conducted a comparative simulation to verify the effectiveness of the proposed method by conventional state feedback with an integrator controller. The controller gains were obtained by a conventional pole placement approach without considering robust control performance (i.e., under condition $l_r = 0.50[m]$)⁽¹⁶⁾.

In this simulation, the dominant poles were set to $-1.5 \pm 1.5j[1/s]$ on the basis of the poles obtained by LMI in § 4.2, and the non-dominant poles were set to $-10 \pm 1.5j[1/s]$, $-10[1/s]$ for both vertical and horizontal directional subsystem. The controller gains were designed as follows:

$$\begin{aligned} k_1 &= \begin{bmatrix} -181 & 226 & 72.1 & 93.3 & 235 \end{bmatrix} \\ k_2 &= \begin{bmatrix} -152 & 226 & 60.5 & 93.3 & 235 \end{bmatrix} \end{aligned} \quad (26)$$

Pole locations by using controller gains in Eq. (26) are shown in Fig. 12. When $l_r = 0.25[m]$, pole locations are $p'_{11} = p'_{21} = -2.98[1/s]$, $p'_{12} = p'_{22} = -1.39 + 1.49j[1/s]$, $p'_{13} = p'_{23} = -1.39 - 1.49j[1/s]$, $p'_{14} = p'_{24} = 16.5 + 21.8j[1/s]$, and $p'_{15} = p'_{25} = 16.5 - 21.8j[1/s]$, and include unstable ones. The closed system becomes unstable when $l_r = 0.34[m]$ as shown in Fig. 12.

The simulation results of vertical boom angle, horizontal boom angle, command voltage, and load sway angles are shown in Figs. 13-15. In Figs. 13 and 15, the results obtained were almost the same as those shown in Figs. 9 and 11, respectively. However, in Fig. 14, the rotary crane control system becomes unstable for the rope length of $l_r = 0.25[m]$ when the controller gains in Eq. (26) are used.

These results confirm the effectiveness of the proposed controller for achieving robustness with respect to rope length variance in rotary crane systems.

5. Conclusion

This paper proposed a controller for achieving robustness with respect to the rope length variance. First, we derived a linear model of a rotary crane by using a disturbance observer.

Fig. B1 Effectiveness of disturbance observer for load mass variance (simulation)

Next, we proposed a controller for achieving robust performance. The controller gains were obtained by solving a pole placement problem and an optimal regulator problem formulated through LMI optimization. The effectiveness of the proposed method was verified by both simulations and experimental results.

Appendix A Derivation of Rope-Load System Dynamics

This section briefly describes the derivation of the rope-load system dynamics in Eqs. (4) and (5). From Fig. 1, the position of the load is

$$\begin{cases} x = l_b \sin \theta_3 \cos \theta_4 + l_r \cos \theta \tan \theta_1 \cos \theta_4 - l_r \cos \theta \tan \theta_2 \sin \theta_4 \\ y = l_b \sin \theta_3 \sin \theta_4 + l_r \cos \theta \tan \theta_1 \sin \theta_4 + l_r \cos \theta \tan \theta_2 \cos \theta_4 \\ z = l_b \cos \theta_3 - l_r \cos \theta \end{cases} \quad (A1)$$

where θ is defined as shown in Fig. 1, and $\tan \theta = \sqrt{\tan^2 \theta_1 + \tan^2 \theta_2}$ is held. By assuming that θ_1 , θ_2 , and θ are small so that $\tan \theta_1 \approx \theta_1$, $\tan \theta_2 \approx \theta_2$, $\cos \theta \approx 1$ and $\theta = \sqrt{\theta_1^2 + \theta_2^2}$ are satisfied, the following equations are obtained:

$$\begin{cases} x = l_b \sin \theta_3 \cos \theta_4 + l_r \theta_1 \cos \theta_4 - l_r \theta_2 \sin \theta_4 \\ y = l_b \sin \theta_3 \sin \theta_4 + l_r \theta_1 \sin \theta_4 + l_r \theta_2 \cos \theta_4 \\ z = l_b \cos \theta_3 - l_r \cos \sqrt{\theta_1^2 + \theta_2^2} \end{cases} \quad (A2)$$

Noted that the height of the load as well as the potential energy is regarded to be unchanged if z is defined as $z = l_b \cos \theta_3 - l_r$ and θ_3 is fixed.

The kinetic energy T and potential energy U of the load are defined as follows:

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \quad (A3)$$

$$U = mgz \quad (A4)$$

The rope-load system dynamics in Eqs. (4) and (5) can be obtained by using the Lagrange's equations of motion from Eqs. (A2)-(A4).

Fig. B2 Effectiveness of disturbance observer for boom horizontal velocity variance ($\theta_{4f} = 15[\text{deg}]$) (simulation)

Fig. B3 Effectiveness of disturbance observer for boom horizontal velocity variance ($\theta_{4f} = 60[\text{deg}]$) (simulation)

Appendix B Simulation for Verifying Robustness of Disturbance Observer

This section presents simulation results to demonstrate the robustness of the disturbance observer shown in Fig. 2 with respect to parameter variance, such as load mass and boom motion velocity.

Simulation results in Figs. B1-B3 were obtained by the proposed controller without the friction term in Eq. (24). Tracking performance in transient period of θ_3 and θ_4 were unchanged even when load mass and boom horizontal velocity are largely changed. In addition, load sway frequency was unchanged because the same rope length is assumed. The proposed controller design is concluded to be applicable to practical systems with parameter variance, such as load mass and boom motion velocity.

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