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Abstract
In this paper, a new perturbation method is proposed, and is shown that the correction vector can be calculated shorter than ever. And, the computing efficiency of the response surface optimization method could improve greatly by applying perturbation method with complementary term to example analysis in the optimization of vibration characteristics by the response surface methodology and by finishing eigenvalue analysis which takes most computing time just once. Moreover, the validity and effectiveness is examined by examples by inducing approximately estimated formula of the vibration response value based on orthogonal polynomial. Lastly, it is shown that the computing time is shorten greatly compared with former method by applying this method to analysis of optimization problem of vibration characteristics.

Key words: Optimal Modal Analysis, Optimal Design, Perturbation Method, Vibration Control, Simulation, Complementary Vector

1. Introduction
These days, competition of product development get more and more intensified, and it has become more important to be applied into the development of product such as finite element method and optimal design method. In the step of optimal design of the structural vibration characteristics by using Response surface method, it is necessary to carry out vibration analysis at sampling points. These analysis time account mostly for computing time of optimizing proceeding, and so it has become striking topic how much shorten the analysis time of vibration in optimizing proceeding at sampling points.

There has been a method called perturbation(1,2) which predicts change of vibration characteristics in case of partial change by using the vibration analysis of the former structure. The accuracy of analysis can become worth in case of large structural change, because eigen mode vector before changing is used for expectation after changed.

To solving this problem, Yamazaki and Hagiwara proposed perturbation method with complementary term(3,4) by adding complementary vectors to modal vector used for the traditional perturbation method, which made it possible to estimate accurately vibration characteristic even in case of dramatics change of structure.
However, when changed part of structure is large, more complementary vectors are required, and as a result, the both times for calculating complementary vectors and response analysis increase dramatically and eigen mode and response analysis can get unstable extremely after structure changed.

Therefore, Terane and Hagiwara searched component modal vectors responding to changing parts of structure, and made the unit force vector in computing complementary vectors, and they proposed the method to search complementary vectors by the Ma-Hagiwara modal method.

As the result, they almost solved the problem that induce unstable property of results and reduce of computing time which are weak points for Yamazaki-Hagiwara method.

However, even in perturbation method with complementary term by Terane-Hagiwara, according to area change of modified structure, the number of complementary vectors increase and decrease, and the algorithm for calculating complementary vectors become complex.

In this paper, we propose a new method for calculating complementary vectors that is not depending on change of modified structure, and then try to derive approximated estimation formula of vibration response on the basis of orthogonal polynomial. Moreover, by using the property that high accuracy of vibration analysis of perturbation method with complementary term even if structure is changed drastically, the eigenvalue analysis for structure composed of average values of each design variable is carried out instead of eigenvalue analysis needed for each sample structure in the traditional response surface optimization method, and on the basis of the obtained mode vectors, calculation of complementary method and response analysis for each sample structure is respectively carried out. In this way, it is expected that the efficiency of response surface optimization method by carry out eigenvalue analysis only once which is carried out plural times in the conventional method.

2. Vibration analysis of sampling structure by perturbation method with Complementary Term

In optimization analysis of vibration characteristics by response surface method, it is necessary to carry out vibration analysis of plural sampling structures generated according to design variables and its change widths.

2.1 Calculating method of complementary vectors with former perturbation method with complementary term

Vibration equation is as follows

\[(\omega^2 M + K)X = F\]  \(\text{(1)}\)

Here, \(M\) is mass matrix, \(K\) is stiffness matrix, \(X\) is displacement vector, \(F\) is force vector, and \(\omega\) is circle frequency.

Eigen value analysis for Eq.(1) is carried out, and a small wide region modal vector \(\phi\) than interesting frequency region is preserved. And, by the following movement formula of following perturbation method with complementary term, the vibration response after modifying structure is calculated.

\[
\begin{bmatrix}
\phi_0^T \\
\phi_1^T \\
\end{bmatrix} \begin{bmatrix}
\omega^2(M + \delta M) + (K + \delta K) \\
\end{bmatrix} \begin{bmatrix}
\phi_0 \\
\phi_1 \\
\end{bmatrix} \begin{bmatrix}
X_0 \\
X_1 \\
\end{bmatrix} = \begin{bmatrix}
\phi_0^T \\
\phi_1^T \\
\end{bmatrix} F
\]  \(\text{(2)}\)

Here, \(\phi_0\) is complementary vector, \(x_0\) is displacement vector corresponding to complementary vector, \(\omega\) is given low-level or high-level frequency, the former one is used for complement in low-level side, and the latter is used for complement in high-level side. \(\delta M\) and \(\delta K\) are modified parts of mass matrix and stiffness matrix by the changed
structure.

As above, depending on calculated way of complementary matrix $\phi_a$, the efficiency and else are different largely.

In Ref.(3) unit input vector $F_i$ is defined for all degree of freedom of modified parts of structure, and in the following formula, existing excitation load is calculated by $F_h$

$$F_h = (I - M\phi\phi^T)F_i$$

and by using that the complementary vector $\phi_a$ is obtained as below.

$$\phi_a = (-\omega^2 M + K)^{-1}F_h$$

However, if many complementary vectors is used in this method, the computing time gets longer, If multiple complementary vectors which are similar in components are used, eigenvalue analysis and response calculation after modified structure gets much more unstable.

To solving this problem, in Ref. (6), the modified parts of structure is recognized as substructure, and the only parts responding to nodes included in substructure are extracted as $M_s$ and $K_s$ from matrix $M$ and $K$.

$$M_s = T^TMT \quad K_s = T^TKT$$

Here, $T$ is transforme matrix. Then eigen value analysis is carried out by matrix $M_s$ and $K_s$, the divided modal vector $\phi$ is calculated.

$$F_i = \{T\phi, (-\omega^2 \delta M + \delta K)T\phi_s\}$$

By the above formulas the input vector $F_i$ is calculated, and after being substituted into formula (3) and formula (4), the complementary vector $\phi_a$ is calculated.

By using above calculating method of complementary vector $\phi_a$, the problems about computing instability and speed decrease have been improved. but eigenvalue analysis problem of substructure mass matrix and stiffness matrix extracted from formula (5) according to change of modified structure region, the problem arise that the degree of freedom of extracted matrix $M_s$ and $K_s$, and the number of complementary vectors become different.

2.2 Proposed calculation method of perturbation method with complementary term

Using the property that parts other than one responding to structural change in $\delta M$ and $\delta K$ are all 0, vector $F_i$ is calculated required for computing complementary vector by the below formula.

$$F_i = (-\omega^2 \delta M + \delta K)\phi$$

In shown Fig.1, component of degree of freedom whose structure is unchanged of calculated vector $F_i$ are all 0.

Substitute $F_i$ to below formula the complementary vector $\phi_a$ is calculated.

$$\phi_a = \left[\omega^2(M + \delta M) + (K + \delta K)\right]^{-1}(I - M\phi\phi^T)F_i$$

Substitute the complementary vector $\phi_a$ to formula(2), the vibration response displacement after structural change is obtained.

In formula (7), the number of complementary vector $\phi_a$ always corresponds to eigenvalue vectors, so it is proved that is not related to change of degree of freedom. And, it is possible to analysis by comparatively few complementary vectors, so the calculation efficiency is improved.
2.3 Vibration analysis of sample structure by perturbation method with complementary term

Originally, though it is necessary to create movement formula for each sample structure and carry out vibration analysis, but in this paper, eigenvalue analysis for mean sample structure is carried out only once, and each sample structure is dealt as one after each structural change, and complementary vector $\phi_\phi$ is calculated by formula (7) and formula (8), next response analysis is carried out by formula (2).

Formula (7) is a simple matrix multiplication, and inverse matrix computation shown in formula (8) corresponds to solving once linear equation, so it is easily inferred that the calculation time is shorter than the one required for eigenvalue analysis of movement equation (1) in the real coordinate system.

3. Optimization of vibration property by proposed response surface method

In this paper, optimization design variables are united as designation of sample data 3 levels, and response surface is generated by orthogonal polynomial.

3.1 Approximate estimate formula of vibration response value

In considering approximation estimation formula related to design variable $a$, for creating that by using as few sample analysis results as possible, the vibration response value is given as the following orthogonal polynomial.

$$w = c_0 + c_1 A_1(a) + c_2 A_2(a)$$  \(9\)

Here, $w$ is estimated vibration response value, $c_0$, $c_1$, and $c_2$ are undetermined coefficients, $A_1(a)$, $A_2(a)$ are function terms of orthogonal polynomial related to design variable $a$. By using the following sample data at 3 level equal interval, the function terms of orthogonal polynomial are defined as formula (11).

$$a_1 = \bar{a} - h \quad a_2 = \bar{a} \quad a_3 = \bar{a} + h$$  \(10\)
\[ A_1(a) = \frac{a - \bar{a}}{h} \quad A_2(a) = \frac{(a - \bar{a})^2 - \frac{2}{3}}{h} \quad (11) \]

Here, \( \bar{a}, h \) are mean and interval value each of design variable \( a \). The formula (10) is assigned to the formula (11), and applying \( A_s(a) = 1 \), and as a result the orthogonal property of function term can be verified easily as below.

\[ \sum_{j=1}^{3} A_i(a_j)A_i(a_j) = 0 \quad i \neq k \quad i, k = 0, 1, 2 \quad (12) \]

Extracting the formula (12), it becomes the next formula.

\[ \sum_{j=1}^{3} A_i(a_j) = 0 \quad \sum_{j=1}^{3} A_i(a_j) = 0 \quad \sum_{j=1}^{3} A_i(a_j)A_i(a_j) = 0 \quad (13) \]

Here, by substituting sample data and analysis vibration response value into the formula (9), the formula below is obtained.

\[ c_0 + c_1A_1(a_1) + c_2A_2(a_1) = w_1 \]
\[ c_0 + c_1A_1(a_2) + c_2A_2(a_2) = w_2 \quad (14) \]
\[ c_0 + c_1A_1(a_3) + c_2A_2(a_3) = w_3 \]

Here, \( w_1, w_2, w_3 \) are vibration response mean values on each level. \( A_s(a_j) = 1, A_s(a_j), A_s(a_j) \) is each multiplied on both sides of those formula, and then summed up, and sample data of the formula (10) is assigned for that formula, and furthermore undetermined coefficient is calculated as below considering direct relationship of formula (13).

\[ c_0 = \frac{w_1 + w_2 + w_3}{3} \]
\[ c_1 = \frac{-w_1 + w_3}{2} \]
\[ c_2 = \frac{3w_1 + 9w_2 + 3w_3}{11} \quad (15) \]

When relative term in direct polynomial formula (11) and undetermined coefficient in formula (15) are assigned for formula (9), estimation formula is obtained regarding single variable \( a \).

Moreover, for improving approximate accuracy of estimation formula, the estimation formula considering the intersect terms of design variable \( a \) and \( b \) is defined as follows.

\[ w = c_{00} + c_{10}A_1(a) + c_{20}A_2(a) + c_{01}B_1(b) + c_{02}B_2(b) + c_{11}A_1(a)B_1(b) \quad (16) \]

Where, \( c_{00}, c_{10}, c_{20}, c_{01}, c_{02} \) are undetermined coefficients of constant term and single term, and \( B_1(b) \) and \( B_2(b) \) are function term of direct polynomial, and those are made up by using average value and interval value as formula (11). Regarding undetermined coefficient \( c_{11} \) in intersect term, it is possible to calculate in the next formula by using direct relationship of function term by the same procedure.

\[ c_{11} = \frac{w_{11} - w_{12} - w_{31} + w_{32}}{4} \quad (17) \]
Here, \( w_i \) is the vibration response value of \( a = a_i, \ b = b_j \). When function terms of each calculated undetermined coefficient and direct polynomial are assigned for formula (16), estimation formula is made up considering intersect term.

The approximate estimation formula regarding plural design variables is made up as the next procedure.

1. Calculate all average value of vibration response values, and define it as constant term \( c_0 \), and add it to estimation formula.
2. By using (15) of formula calculate undetermined coefficient \( c_1, c_2 \) of single term and add \( c_1 A_i (a) + c_2 A(a) \) to estimation formula.
3. By using (17) of formula calculate undetermined coefficient \( c_{11} \) of intersect term and add \( c_{11} A_i (a) B_j (b) \) to estimation formula.

The above calculation procedure is applied for all design variables, the approximate estimation formula is created regarding vibration response value.

3.2 Proceeding of optimize calculation of vibration property by proposed response surface method

The flow chart of vibration analysis using perturbation method with complementary term and optimize calculation using response surface method is shown in figure 2.

Firstly, plural sample data is made up by orthogonal intersect table. Next, eigenvalue analysis of the structure responding to average value of all sample data is carried out. The perturbation method with complementary term which is on the basis of average structured mode vector is carried out, and vibration response value is extracted from analysis result.

Furthermore, making approximate estimation formula by the relationship with vibration response value extracted from sample data, and then optimized calculation minimizing vibration response value is determine by using sample data and analysis results. However, approximate estimation formula is used for optimized calculation, so it is necessary to verify the obtained optimal structure.

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**Fig.2 The flowchart of optimal design with RSM**
4. Example of numeral calculation

Fig.3 shows the double plate model with two identical forms whose edges are connected each other by springs. Here, the spring which transformed only at axis direction is called scalar spring.

Setting the length of plate as 1.0m, the width as 0.75m, the thickness of upper plate as 1.2mm, and regarding material property of plate young elasticity coefficient is 210GPa, Poisson ratio is 0.29, mass density is 7820kg/m³, and spring axis direction elastic coefficient is 1750N/m.

Boundary condition is set as completely free, and a unit excitation load is given at normal direction of plate at A point as Fig.3, and then the frequency response function of normal direction of plate at excitation point is selected as evaluation for vibration property.

The calculation method of perturbation vector was compared to the traditional calculation method, however it using 83 eigenmodes from 0 to 75 Hz as same as before, that is calculating area of frequency value 60Hz multiplied by 1.25.

![Fig.3 Dynamic analysis modal of double plate](image)

The elastic coefficient at axis direction of 4 scalar springs and the part volume of the upper plate are designated as design variables. To examining the effect by the size difference of changed area of plate, the examination is conducted after dividing that into the case of larger change area and the case of smaller change area.

Applying quadrilateral shell element to create analytical mesh and based on the length of plate and direction of width, two kinds of analytical model is created: 10×8 equally divided coarse mesh with 198 nodes and minute 60×48 equally divided fined mesh with 5978 nodes as Fig.4, and then vibration analysis is to be carried out.

![Fig.4 Large and small change zoon on upper plate](image)
4.1 Accuracy verification of proposed perturbation method with complementary term

Here, eigenvalue analysis is carried out for initial structure shown in Fig.3, and complementary vector is calculated for each case as below by using formula (7) and (8), and eigenvalue analysis after structural change and frequency response analysis are carried out by equation of motion (2).

- case1: coarse mesh and large changed area
- case2: coarse mesh and large changed area
- case3: fine mesh and small changed area
- case4: fine mesh and small changed area

The maximum plate thickness after change is 0.16 mm that is 0.2 times as initial structure. The results of eigenvalue analysis and frequency response analysis are shown in Table.1-4 and Fig.5-8 respectively.

**Table1 Eigen frequency analysis result of Case1**

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM(Hz)</th>
<th>CPM(Hz)</th>
<th>Diff.(%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.152</td>
<td>21.208</td>
<td>0.265</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>21.658</td>
<td>21.667</td>
<td>0.038</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>22.593</td>
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<td>1.00</td>
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<td>4</td>
<td>25.216</td>
<td>25.220</td>
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<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>26.670</td>
<td>26.677</td>
<td>0.027</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>27.543</td>
<td>27.543</td>
<td>0.002</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>28.334</td>
<td>28.336</td>
<td>0.006</td>
<td>1.00</td>
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<td>8</td>
<td>29.538</td>
<td>29.537</td>
<td>0.003</td>
<td>1.00</td>
</tr>
<tr>
<td>9</td>
<td>31.283</td>
<td>31.283</td>
<td>0.001</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>32.643</td>
<td>32.641</td>
<td>0.007</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Table2 Eigen frequency analysis result of Case2**

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM(Hz)</th>
<th>CPM(Hz)</th>
<th>Diff.(%)</th>
<th>MAC</th>
</tr>
</thead>
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<td>1</td>
<td>21.060</td>
<td>21.060</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>23.011</td>
<td>23.011</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>23.174</td>
<td>23.174</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>24.163</td>
<td>24.164</td>
<td>0.004</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>25.218</td>
<td>25.218</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>25.790</td>
<td>25.790</td>
<td>0.000</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>26.503</td>
<td>26.503</td>
<td>0.000</td>
<td>1.00</td>
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<tr>
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<tr>
<td>9</td>
<td>27.425</td>
<td>27.425</td>
<td>0.001</td>
<td>1.00</td>
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<tr>
<td>10</td>
<td>28.894</td>
<td>28.894</td>
<td>0.000</td>
<td>1.00</td>
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</tbody>
</table>
Table 3 Eigen frequency analysis result of Case 3

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM (Hz)</th>
<th>CPM (Hz)</th>
<th>Diff. (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
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<td>20.78</td>
<td>20.77</td>
<td>0.05</td>
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</tr>
<tr>
<td>2</td>
<td>22.45</td>
<td>22.44</td>
<td>0.05</td>
<td>1.00</td>
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<tr>
<td>3</td>
<td>23.17</td>
<td>23.16</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>24.26</td>
<td>24.25</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>27.75</td>
<td>27.74</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>28.09</td>
<td>28.08</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>28.83</td>
<td>28.75</td>
<td>0.28</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>29.84</td>
<td>29.82</td>
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</tr>
<tr>
<td>9</td>
<td>31.81</td>
<td>31.79</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>32.49</td>
<td>32.47</td>
<td>0.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4 Eigen frequency analysis result of Case 4

<table>
<thead>
<tr>
<th>No.</th>
<th>FEM (Hz)</th>
<th>CPM (Hz)</th>
<th>Diff. (%)</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.05</td>
<td>20.05</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>20.63</td>
<td>20.62</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>22.51</td>
<td>22.50</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>23.75</td>
<td>23.74</td>
<td>0.05</td>
<td>1.00</td>
</tr>
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<td>5</td>
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<td>24.39</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>25.33</td>
<td>25.32</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>25.71</td>
<td>25.70</td>
<td>0.04</td>
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</tr>
<tr>
<td>8</td>
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<td>26.07</td>
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<tr>
<td>9</td>
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<td>0.05</td>
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</tr>
<tr>
<td>10</td>
<td>28.22</td>
<td>28.20</td>
<td>0.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In each table, the first row is the 10 eigen frequency numbers counted from 20Hz, the second row is the analytical results by using finite element method, the third row is the analytical results by perturbation method with complementary term, the fourth row is error, and the fifth row is MAC value denoting the identity of eigen modes.

From each table, it is proved that the eigen frequencies after structural change corresponds well to the analytical results by finite element method. And, the result of MAC value comparing both mode vectors are 1 which shows that eigen mode are also corresponding.

However, the eigen frequency results which has the same accuracy of analysis until 60Hz is obtained as shown in Fig.5-8, 10 eigenvalue analyzed results from 20Hz related directly to optimization objective function are displayed in Table 1-4 on the condition of the paper space.
Fig. 5 The results of frequency response of Case 1

Fig. 6 The results of frequency response of Case 2

Fig. 7 The results of frequency response of Case 3
Fig. 8  The results of frequency response of Case 4

On the frequency response shown in Fig.5-8, dotted line shows the result of recalculation by finite element method and solid line shows the result of perturbation method. It is verified by those figures that both of the results are corresponding well over the range of frequency 0-60Hz.

It is possible that eigen mode doesn’t appear in frequency response because of mode form and the observed point position. For example, 10 eigen vibration frequencies from 20Hz shown in Table.4, frequency 20.05Hz and 22.51Hz are not measured as max value in Fig.8, however the maximum point of the remaining 8 frequency response agreed with the eigen vibration frequencies in Table.4.

4.2 Optimization of vibration characteristics by response surface method

The optimize problem of vibration characteristics is defined as below.

\[
\text{Find } x = \{T, K_a, K_b, K_c, K_d\}^T \\
\text{Min. } W = f(x) \\
\text{S.T. } 0.4 \text{mm} \leq T \leq 1.2 \text{mm} \\
175 \frac{N}{m} \leq K_a, K_b, K_c, K_d \leq 8750 \frac{N}{m}
\]

Here, \( x = \{T, K_a, K_b, K_c, K_d\}^T \) is the design variable vectors, \( T \) is the plate thickness of structure of change area, and \( K_a, K_b, K_c, K_d \) are elastic coefficient of scalar springs, \( W = f(x) \) is the objective function, and response integral value of the designated frequency range is minimized where the frequency range is set as 20-23Hz as shown Fig.9.

For examination, Optimized calculation is carried out by dividing into 2 cases as coarse mesh and fine mesh.

The change of design variable in the case of coarse mesh is shown in Table5. As shown in table5, the thickness of changed part of optimum structured upper plate is 35.2% thinner, and elastic coefficient of scalar spring b, c, d are reduced until lower limit, on the other hand elastic coefficient of scalar spring a is increased more than twice.

Comparing to response value in the frequency range corresponding to objective function surrounded by the bold line of Fig.9, it is proved that optimum structure shown by solid line is obviously lower than the response value of initial structure shown by dotted line. As the result of conducting the confirmation analysis by using design variable in Figure.5, the
response integral value of optimum structure is 404.5, while response integral value of optimum structure is 71.6 which is improved around 82.3%. On the other hand, In the case of optimization analysis by fine mesh, the same result is obtained, and the response value of optimum structure is 126.6 while response integral value of initial structure is 399.5 which is improved around 68.3%.

From the above optimization result, it become revealed that perturbation method with complementary term which is applied for vibration characteristics optimization by response surface method has enough analysis accuracy and so very effective.

5. Discussion about calculation efficiency of proposed optimization method

The time of vibration analysis on the basis of mode method is divided roughly into 2 steps: creation of equation of motion and eigenvalue analysis time and response analysis time. For examination, here response analysis time in mode coordinate system is ignored, and the creation of equation of motion and the time of eigenvalue analysis is defined as $t_e$.

In the vibration analysis by proposed perturbation method with complementary term, the response analysis time in mode coordinate system is ignored, and the calculation time of complementary vector is defined as $t_c$, and the creation time of expanded equation of motion (2) is defined as $t_x$.

After defining the number of sample structure is $S$, in case that intersect table is applied for 3 level design variables, there are many cases, for example $S = 27$ and $S = 81$.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Original structure</th>
<th>Optimal structure</th>
<th>Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (mm)</td>
<td>0.8</td>
<td>0.5177</td>
<td>-35.2</td>
</tr>
<tr>
<td>Ka (N/m)</td>
<td>1750</td>
<td>6000</td>
<td>242.9</td>
</tr>
<tr>
<td>Kb (N/m)</td>
<td>1750</td>
<td>175</td>
<td>-90.0</td>
</tr>
<tr>
<td>Kc (N/m)</td>
<td>1750</td>
<td>175</td>
<td>-90.0</td>
</tr>
<tr>
<td>Kd (N/m)</td>
<td>1750</td>
<td>175</td>
<td>-90.0</td>
</tr>
</tbody>
</table>

Fig.9 Optimal and original of analysis result with coarse mesh
The calculation time $t_{FEM}$ in usual finite element method for 1 step optimization calculation using $S$ samples is estimated as below.

$$t_{FEM} \approx St_e$$ (19)

and the calculation time $t_{CPM}$ in proposed perturbation method with complementary term is estimated roughly as follows.

$$t_{CPM} \approx t_e + (S-1)(t_e + t_s)$$ (20)

In this paper, formula (7) and formula (8) are used for calculation of complementary vector. Formula (7) is simple matrix multiplication, and formula (8) corresponds almost to the time of one static analysis. The coefficient matrix of formula is expanded as below.

$$\begin{bmatrix}
\phi'
\end{bmatrix} \begin{bmatrix}
S + dS
\end{bmatrix} \begin{bmatrix}
\phi
\end{bmatrix} = \begin{bmatrix}
\phi' S \phi & \phi' S \phi_d
\end{bmatrix} + \begin{bmatrix}
\phi' \phi_d S \phi & \phi' \phi_d S \phi_d
\end{bmatrix}$$ (21)

The matrix $S$ and $\partial S$ in the above formula can be written as follows.

$$S = -\omega^2 M + K \quad \partial S = -\omega^2 \partial M + \partial K$$ (22)

The first term on the right side in formula (21) is multiplied only once by average structure $S$ and $\phi$ beforehand, and of second term on the right hand in formula (21) the term $\partial S$ corresponding to unchanged structure part is all 0, and so only the part of structural change should be calculated. By these devices the calculation time of coefficient matrix in formula (2) will be shorten.

After comparing the term on the right term in formula (19) and formula (20), the calculation time of complementary vector and the creation time of expanded motion of equation $t_e + t_s$ is much less than the time of motion of equation creation in real coordinate system and the time of eigenvalue analysis, and $t_{CPM} < t_{FEM}$ can be judged certainly.

For example, the numerical calculation example of the sample structure in the former program, the sample structure created by L27 intersect table for 5 design variables is 27. In formula (19) and formula (20), the calculation time of optimization calculation for one step is compared by finite element method and perturbation method with complementary term. The result is as follows.

In coarse mesh model, the structural degree of freedom $n$ is 1188, and the number of complementary vector $m$ is 83, while 27 times multiplied by 4.5 seconds equals to 121.5 seconds in case of finite element method and 4.5 seconds plus 26 times(1.1 seconds plus0.1 seconds) equals to 35.7 seconds in case of perturbation method with complementary term. While in fine mesh model, the structural degree of freedom $n$ is 35868 and complementary vector $m$ is 83, 27 times multiplied by 41.0 seconds equals to 1107.0 seconds in case of finite element method and 41.0 seconds plus 25 times(5.8sedonds plus 4.3 seconds) equals to 303.6 seconds.

The calculation efficiency is respectively improved as 121.5 seconds divided by 35.7 seconds equals to 3.40 times in coarse mesh model and 1107.0 seconds divided by 303.6 seconds equals to 3.65 times respectively by using perturbation method with complementary term, and that is not so related to the fineness of analytical mesh by which 3.5 times reduction of calculation time is obtained.

6. Conclusion

The examination of applying perturbation method with complementary term to sample vibration analysis in vibration characteristics optimization by response surface method was
carried out.

(1) In examining the calculation methods of complementary term, and the new calculation method of complementary vectors for comparable little time and regardless of change of changed area of structure was proposed, and the efficiency was improved.

(2) For sample vibration analysis in response surface method, the eigenvalue analysis taking most time is carried out for sample average structure only once, then method which applies perturbation method with complementary term for vibration analysis of each sample is proposed, and it is shown that calculation efficiency is improved largely as around 3.5 times with having analysis accuracy kept well.

(3) On the assumption that design variable is united as 3 levels, the approximate estimation formula on the basis of intersect polynomial is induced, and the efficiency is verified by numerical calculation example.

In the future, this perturbation method with complementary term is applied for optimization method other than response surface method, and the efficiency is to be confirmed.

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References

(2) A.Nagamatsu, Modal Analysis, pp.189-250, Baifusha (1985).