Analysis and Experiment of Residual Load Sway Suppression in Rotary Crane Systems Using Simple Trajectory for Horizontal Boom Motion*

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Abstract

To suppress two-dimensional load sway caused by the horizontal boom motion of a rotary crane, both horizontal and vertical boom motions are generally used. However, it would be more energy efficient and safer if a control scheme is developed that only used horizontal boom motion, eliminating the need for any vertical boom motion. In addition, if we can suppress load sway without the need to measure it, reduction in the cost of sensors can be achieved. Furthermore, use of simple velocity trajectory patterns such as a trapezoidal velocity pattern and an S-curve acceleration/deceleration pattern, which are widely used in industrial automation systems, may provide cost-effective implementation of controllers. This study examines the analytical conditions for a simple S-curve trajectory of horizontal boom motion to suppress residual load sway without sensing it. Numerical simulation and experimental results demonstrate the effectiveness of the proposed conditions.

Key words : Rotary Crane, Motion Control, Residual Load Sway Suppression, Trajectory Generation, Tracking Control

1. Introduction

Rotary cranes are widely used to transport heavy loads and hazardous materials in various environments, such as shipyards, factories, nuclear installations, and construction sites. Recently, in Japan, there has been a decline in caregivers as a result of an aging society and declining birth rate. Hence, small crane systems are expected to be used to provide welfare and nursing services. However, the one-dimensional horizontal boom motion of a rotary crane typically generates undesirable two-dimensional load sway; therefore, skilled crane operators are required to manually control boom operation based on their experiences to arrest load sway immediately at the correct position.

To reduce the burden on human operators, automatic control systems have been widely investigated. Different types of control schemes have been developed for crane control, including optimal control⁽¹⁾⁻⁽³⁾, LMI based control⁽⁴⁾, gain scheduling control⁽⁵⁾⁻⁽⁷⁾, sliding mode control⁽⁸⁾⁻⁽¹⁰⁾, adaptive control⁽¹¹⁾, back-stepping control⁽¹²⁾, input-shaping control⁽¹³⁾, disturbance observer based control⁽¹⁴⁾, and nonlinear control based on the Lyapunov stability theorem⁽¹⁵⁾⁻⁽¹⁶⁾.

In most existing control schemes, both horizontal and vertical boom motions are used to suppress load sway. However, it would be less energy intensive and also safer if a control scheme were developed that used only horizontal boom motion, eliminating the need for vertical boom motion, which must overcome gravity.

Several studies present controller designs for load sway suppression that use only horizontal boom motion⁽¹⁷⁾⁻⁽²¹⁾. These methods can be classified into two types: trajectory gen-

Fig. 1 Schematic model of rotary crane

eration methods that can suppress load sway without requiring load sway information (i.e., open-loop control)^{(17),(18)} and methods that suppress load sway using load sway information measured by sensor systems in real time (i.e., closed-loop control)^{(19),(20)}. Because the latter methods require a sensor system, their hardware costs may be higher. In addition, because cranes are inherently stable system, feedback control seems undesirable, particularly when used in welfare and nursing environment. In contrast, although the controllers designed in Refs. (17) and (18) do not require load sway sensors, the trajectories generated for horizontal boom motion are complicated, and it is difficult to employ simple industrial controllers that accept only trapezoidal or S-curve acceleration/deceleration trajectories (hereafter S-curve trajectory). Use of a simple trapezoidal or S-curve trajectory would allow the use of cost-efficient industrial controllers.

In this study, we propose a control scheme that can suppress two-dimensional load sway using only horizontal boom motion. First, we consider a simple dynamical model of rotary crane motion that includes a centrifugal force term responsible for two-dimensional load sway. Next, a simple S-curve trajectory for horizontal boom motion that can suppress load sway is analytically generated. This trajectory can be applied to inexpensive industrial controllers. The simulation and experimental results demonstrate the effectiveness of the proposed method.

2. Rotary Crane Dynamics

In this section, we describe the dynamics of a rotary crane, a schematic model of which is shown in Fig. 1⁽²⁰⁾. Angles θ_1 and θ_2 denote the load sway angles in the plane of vertical boom motion and the tangential direction of horizontal boom motion, respectively. Angles θ_3 and θ_4 denote the vertical and horizontal angles of the boom, respectively. Parameters l and L denote the length of the rope and boom, respectively, and (x, y, z) denotes the three-dimensional position of the load.

We assume that rotary crane dynamics has the following characteristics:

- The load can be considered to be a point mass, and the torsion of the rope can be neglected.
- The sway angle θ_i has a small magnitude; hence the approximations $\sin \theta_i \approx \theta_i$ and $\cos \theta_i \approx 1$, ($i=1,2$) are reasonable, and $\dot{\theta}_i$ ($i=1,2$) are bounded.
- A controller that can achieve horizontal boom motion (i.e., θ_4 , $\dot{\theta}_4$ and $\ddot{\theta}_4$) has been implemented to track a given trajectory.

Fig. 2 S-curve trajectory for horizontal boom motion

The equations of motion for a rotary crane are

$$l(1+\theta_1^2)\ddot{\theta}_1 + l\theta_1\theta_2\ddot{\theta}_2 - l\theta_2\ddot{\theta}_4 + l\theta_1\dot{\theta}_1^2 + l\theta_1\dot{\theta}_2^2 - 2l\dot{\theta}_2\dot{\theta}_4 - (l\theta_1 + L\sin\theta_3)\dot{\theta}_4^2 + g\theta_1 = 0 \quad (1)$$

$$l\theta_1\theta_2\ddot{\theta}_1 + l(1+\theta_2^2)\ddot{\theta}_2 - l\theta_2\ddot{\theta}_4 + l\theta_2\dot{\theta}_1^2 + l\theta_2\dot{\theta}_2^2 + 2l\dot{\theta}_1\dot{\theta}_4 + (l\theta_1 + L\sin\theta_3)\dot{\theta}_4^2 + g\theta_2 = 0 \quad (2)$$

where g denotes the gravitational acceleration.

Because the centrifugal force caused by horizontal boom motion and gravitational force affect two-dimensional load sway significantly, to derive the trajectory by theoretical analysis easily, we linearize all the terms in Eqs. (1) and (2) except the centrifugal and gravitational force terms. For this purpose, we assume that θ_i and $\dot{\theta}_i$ ($i=1,2$) are small such that $\theta_i^2 \approx 0$, $\theta_i\theta_j \approx 0$, $\theta_i\dot{\theta}_j^2 \approx 0$ ($j=1,2$), $\dot{\theta}_i\dot{\theta}_4 \approx 0$, and $\theta_i\ddot{\theta}_4 \approx 0$ are satisfied.

$$\ddot{\theta}_1 = -\frac{g}{l}\theta_1 + \frac{L\sin\theta_3}{l}\dot{\theta}_4^2 \quad (3)$$

$$\ddot{\theta}_2 = -\frac{g}{l}\theta_2 - \frac{L\sin\theta_3}{l}\ddot{\theta}_4 \quad (4)$$

Considering Eqs. (3) and (4), we arrive at the following simple dynamical model of rotary crane motion:

$$\ddot{\theta}_1 = -\omega_n^2\theta_1 + b\dot{\theta}_4^2 \quad (5)$$

$$\ddot{\theta}_2 = -\omega_n^2\theta_2 - b\ddot{\theta}_4 \quad (6)$$

where $\omega_n = \sqrt{g/l}$ and $b = L\sin\theta_3/l$.

3. Trajectory Generation Analysis

In this section, we generate an S-curve trajectory for horizontal boom motion that can suppress load sway on the basis of the crane model in Eqs. (5) and (6). The S-curve trajectory shown in Fig. 2 is widely used in industrial applications. Parameters t_1 , t_2 , t_3 , and $\dot{\theta}_{4c}$ denote the acceleration period, constant-velocity period, deceleration period, and constant-velocity, respectively.

The S-curve velocity trajectory for horizontal boom motion can be represented as follows:

$$\dot{\theta}_4 = \begin{cases} A(1 - \cos\omega_1 t) & t \in [0, t_1) \\ 2A & t \in [t_1, t_1 + t_2) \\ A(1 + \cos\omega_2 t') & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \quad (7)$$

where $A = \dot{\theta}_{4c}/2$, $\omega_1 = \pi/t_1$, $\omega_2 = \pi/t_3$, and $t' = t - t_1 - t_2$.

We can obtain the acceleration trajectory $\ddot{\theta}_4$ as follows:

$$\ddot{\theta}_4 = \begin{cases} \omega_1 A \sin \omega_1 t & t \in [0, t_1) \\ 0 & t \in [t_1, t_1+t_2) \\ -\omega_2 A \sin \omega_2 t' & t \in [t_1+t_2, t_1+t_2+t_3] \end{cases} \quad (8)$$

By substituting Eq. (7) into Eq. (5), θ_1 and its time derivative for $0 \leq t < t_1$, θ_{11} and $\dot{\theta}_{11}$, are derived as follows:

$$\theta_{11} = C_{11} \cos \omega_n t + C_{12} \sin \omega_n t + B \left(3 + \frac{\omega_n^2 \cos 2\omega_1 t}{\omega_n^2 - 4\omega_1^2} - \frac{4\omega_n^2 \cos \omega_1 t}{\omega_n^2 - \omega_1^2} \right) \quad (9)$$

$$\dot{\theta}_{11} = -\omega_n C_{11} \sin \omega_n t + \omega_n C_{12} \cos \omega_n t + B \left(\frac{-2\omega_1 \omega_n^2 \sin 2\omega_1 t}{\omega_n^2 - 4\omega_1^2} + \frac{4\omega_1 \omega_n^2 \sin \omega_1 t}{\omega_n^2 - \omega_1^2} \right) \quad (10)$$

where $B = A^2 b / (2\omega_n^2)$. C_{11} and C_{12} can be determined from the boundary conditions $\theta_{11}(t) = 0$ and $\dot{\theta}_{11}(t) = 0$ at $t = 0$ as follows:

$$C_{11} = \frac{-12B\omega_1^4}{(\omega_n^2 - 4\omega_1^2)(\omega_n^2 - \omega_1^2)}, \quad C_{12} = 0 \quad (11)$$

Angle θ_1 and its time derivative for $t_1 \leq t < t_1 + t_2$, θ_{12} and $\dot{\theta}_{12}$, are given as follows:

$$\theta_{12} = C_{13} \cos \omega_n t + C_{14} \sin \omega_n t + 8B \quad (12)$$

$$\dot{\theta}_{12} = -\omega_n C_{13} \sin \omega_n t + \omega_n C_{14} \cos \omega_n t \quad (13)$$

where C_{13} and C_{14} can be determined from the boundary conditions $\theta_{12}(t) = \theta_{11}(t)$ and $\dot{\theta}_{12}(t) = \dot{\theta}_{11}(t)$ at $t = t_1$ as follows:

$$C_{13} = C_{11} + D \cos \omega_n t_1, \quad C_{14} = D \sin \omega_n t_1 \quad (14)$$

$$D = \frac{B(8\omega_n^2 \omega_1^2 - 20\omega_1^4)}{(\omega_n^2 - 4\omega_1^2)(\omega_n^2 - \omega_1^2)}$$

Angle θ_1 and its time derivative for $t_1 + t_2 \leq t \leq t_1 + t_2 + t_3$, θ_{13} and $\dot{\theta}_{13}$, are obtained as follows:

$$\theta_{13} = C_{15} \cos \omega_n t + C_{16} \sin \omega_n t + B \left(3 + \frac{\omega_n^2 \cos 2\omega_2 t'}{\omega_n^2 - 4\omega_2^2} + \frac{4\omega_n^2 \cos \omega_2 t'}{\omega_n^2 - \omega_2^2} \right) \quad (15)$$

$$\dot{\theta}_{13} = -\omega_n C_{15} \sin \omega_n t + \omega_n C_{16} \cos \omega_n t + B \left(\frac{-2\omega_2 \omega_n^2 \sin 2\omega_2 t'}{\omega_n^2 - 4\omega_2^2} - \frac{4\omega_2 \omega_n^2 \sin \omega_2 t'}{\omega_n^2 - \omega_2^2} \right) \quad (16)$$

where C_{15} and C_{16} can be determined from the boundary conditions $\theta_{13}(t) = \theta_{12}(t)$ and $\dot{\theta}_{13}(t) = \dot{\theta}_{12}(t)$ at $t = t_1 + t_2$ as follows:

$$C_{15} = C_{13} + E \cos \omega_n (t_1 + t_2) \quad (17)$$

$$C_{16} = C_{14} + E \sin \omega_n (t_1 + t_2) \quad (18)$$

$$E = \frac{B(20\omega_2^4 - 8\omega_n^2 \omega_2^2)}{(\omega_n^2 - 4\omega_2^2)(\omega_n^2 - \omega_2^2)}$$

By substituting Eq. (8) into Eq. (6), θ_2 and its time derivative for $0 \leq t < t_1$, θ_{21} and $\dot{\theta}_{21}$, are derived as follows:

$$\theta_{21} = C_{21} \cos \omega_n t + C_{22} \sin \omega_n t - \frac{Ab\omega_1 \sin \omega_1 t}{\omega_n^2 - \omega_1^2} \quad (19)$$

$$\dot{\theta}_{21} = -\omega_n C_{21} \sin \omega_n t + \omega_n C_{22} \cos \omega_n t - \frac{Ab\omega_1^2 \cos \omega_1 t}{\omega_n^2 - \omega_1^2} \quad (20)$$

where C_{21} and C_{22} can be determined from the boundary conditions $\theta_{21}(t) = 0$ and $\dot{\theta}_{21}(t) = 0$ at $t = 0$ as follows:

$$C_{21} = 0, \quad C_{22} = \frac{Ab\omega_1^2}{\omega_n(\omega_n^2 - \omega_1^2)} \quad (21)$$

Angle θ_2 and its time derivative for $t_1 \leq t < t_1 + t_2$, θ_{22} and $\dot{\theta}_{22}$, are obtained as follows:

$$\theta_{22} = C_{23} \cos \omega_n t + C_{24} \sin \omega_n t \quad (22)$$

$$\dot{\theta}_{22} = -\omega_n C_{23} \sin \omega_n t + \omega_n C_{24} \cos \omega_n t \quad (23)$$

where C_{23} and C_{24} can be determined from the boundary conditions $\theta_{22}(t) = \theta_{21}(t)$ and $\dot{\theta}_{22}(t) = \dot{\theta}_{21}(t)$ at $t = t_1$ as follows:

$$C_{23} = -C_{22} \sin \omega_n t_1, \quad C_{24} = C_{22}(1 + \cos \omega_n t_1) \quad (24)$$

Angle θ_2 and its time derivative for $t_1 + t_2 \leq t \leq t_1 + t_2 + t_3$, θ_{23} and $\dot{\theta}_{23}$, are given as follows:

$$\theta_{23} = C_{25} \cos \omega_n t + C_{26} \sin \omega_n t + \frac{Ab\omega_2 \sin \omega_2 t'}{\omega_n^2 - \omega_2^2} \quad (25)$$

$$\dot{\theta}_{23} = -\omega_n C_{25} \sin \omega_n t + \omega_n C_{26} \cos \omega_n t + \frac{Ab\omega_2^2 \cos \omega_2 t'}{\omega_n^2 - \omega_2^2} \quad (26)$$

where C_{25} and C_{26} can be determined from the boundary conditions $\theta_{23}(t) = \theta_{22}(t)$ and $\dot{\theta}_{23}(t) = \dot{\theta}_{22}(t)$ at $t = t_1 + t_2$ as follows:

$$C_{25} = C_{23} + F \sin \omega_n(t_1 + t_2) \quad (27)$$

$$C_{26} = C_{24} - F \cos \omega_n(t_1 + t_2) \quad (28)$$

$$F = \frac{Ab\omega_2^2}{\omega_n(\omega_n^2 - \omega_2^2)}$$

The following equations must be satisfied to suppress residual load sway:

$$\theta_{13}(T) = 0 \quad (29)$$

$$\dot{\theta}_{13}(T) = 0 \quad (30)$$

$$\theta_{23}(T) = 0 \quad (31)$$

$$\dot{\theta}_{23}(T) = 0 \quad (32)$$

where $T = t_1 + t_2 + t_3$. Considering Eqs. (5)-(8), (29) and (31), $\ddot{\theta}_{13}(T) = 0$ and $\ddot{\theta}_{23}(T) = 0$ are satisfied.

First, we consider conditions to suppress residual load sway θ_2 . Equations (31) and (32) can be written as follows from Eqs. (25) and (26):

$$R^{-1}(T) \begin{bmatrix} C_{25} \\ C_{26} \end{bmatrix} - \begin{bmatrix} 0 \\ F \end{bmatrix} = 0, \quad R(t) = \begin{bmatrix} \cos \omega_n t & -\sin \omega_n t \\ \sin \omega_n t & \cos \omega_n t \end{bmatrix} \quad (33)$$

We obtain the following equation from Eq. (33) by assuming $t_1 = t_3$ ($\omega_1 = \omega_2$):

$$M_1 M_2 \begin{bmatrix} 0 \\ -F \end{bmatrix} = 0 \quad (34)$$

$$M_1 = \begin{bmatrix} \cos \omega_n t_1 + 1 & -\sin \omega_n t_1 \\ \sin \omega_n t_1 & \cos \omega_n t_1 + 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} \cos \omega_n(t_1 + t_2) - 1 & -\sin \omega_n(t_1 + t_2) \\ \sin \omega_n(t_1 + t_2) & \cos \omega_n(t_1 + t_2) - 1 \end{bmatrix}$$

When the matrix M_1 in Eq. (34) is nonsingular, the necessary and sufficient condition to satisfy Eq. (34) for any F is:

$$\omega_n(t_1 + t_2) = 2k_1\pi, \quad k_1 = 1, 2, \dots \quad (35)$$

by which the matrix M_2 in Eq. (34) becomes a zero matrix. On the other hand, the matrix M_1 is singular if and only if

$$\omega_n t_1 = (2k_2 - 1)\pi, \quad k_2 = 2, 3, \dots \quad (36)$$

by which the matrix M_1 becomes a zero matrix, and Eq. (34) is satisfied. It is noted that F is not defined in the case $k_2 = 1$ (i.e., $\omega_n = \omega_1 = \omega_2$).

Next, we consider conditions to suppress residual load sway θ_1 . Equations (29) and (30) can be written as follows from Eqs. (15) and (16):

$$R^{-1}(T) \begin{bmatrix} C_{15} \\ C_{16} \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} = 0, \quad G = \frac{12B\omega_2^4}{(\omega_n^2 - 4\omega_2^2)(\omega_n^2 - \omega_2^2)} \quad (37)$$

Equations (11), (14), (17), (18) and (37) lead to

$$\begin{bmatrix} -G + D \cos \omega_n t_1 + E \cos \omega_n (t_1 + t_2) \\ D \sin \omega_n t_1 + E \sin \omega_n (t_1 + t_2) \end{bmatrix} + R(T) \begin{bmatrix} G \\ 0 \end{bmatrix} = 0 \quad (38)$$

Substituting the condition in Eq. (35) and the relation $T = 2t_1 + t_2$ into Eq. (38) lead to

$$\begin{bmatrix} -G + D \cos \omega_n t_1 + E \\ D \sin \omega_n t_1 \end{bmatrix} + R(t_1) \begin{bmatrix} G \\ 0 \end{bmatrix} = 0 \quad (39)$$

Equation (39) can be rewritten as

$$H(\cos \omega_n t_1 - 1) = 0, \quad H \sin \omega_n t_1 = 0 \quad (40)$$

$$H = \frac{8B\omega_1^2}{\omega_n^2 - 4\omega_1^2}$$

where the relation $H = D + G = G - E$ is used for derivation. Hence, the following condition is required to satisfy Eq. (40) for any H .

$$\omega_n t_1 = 2k_3\pi, \quad k_3 = 2, 3, \dots \quad (41)$$

Note that H in Eq. (40) is not defined for $k_3 = 1$ (i.e., $\omega_n = 2\omega_1 = 2\omega_2$).

Substituting the relation $T = 2t_1 + t_2$ and Eq. (36) into Eq. (38) lead to

$$\begin{bmatrix} -G - D - E \cos \omega_n t_2 \\ -E \sin \omega_n t_2 \end{bmatrix} + R(t_2) \begin{bmatrix} G \\ 0 \end{bmatrix} = 0 \quad (42)$$

Equation (42) can be rewritten as

$$H(\cos \omega_n t_2 - 1) = 0, \quad H \sin \omega_n t_2 = 0 \quad (43)$$

Hence, the following condition is required to satisfy Eq. (43) for any H .

$$\omega_n t_2 = 2k_4\pi, \quad k_4 = 0, 1, \dots \quad (44)$$

Considering Eqs. (35), (36), (41), (44) and the relation $t_1 = t_3$, we can finally obtain the following condition that enables the suppression of residual load sway:

$$t_1 = t_3 = k_5 \frac{\pi}{\omega_n}, \quad k_5 = 3, 4, \dots \quad (45)$$

$$t_2 = 2k_6 \frac{\pi}{\omega_n}, \quad k_6 = 0, 1, \dots$$

Because the above analysis does not include the case $k_5 = 1, 2$ (i.e., $\omega_n = \omega_1 = \omega_2$, $\omega_n = 2\omega_1 = 2\omega_2$, respectively) for which Eq. (9) cannot be defined, we consider this case in the analysis below.

When $k_5 = 1$, Eqs. (7) and (8), respectively, can be represented as follows:

$$\dot{\theta}_4 = \begin{cases} A(1 - \cos \omega_n t) & t \in [0, t_1) \\ 2A & t \in [t_1, t_1 + t_2) \\ A(1 + \cos \omega_n t') & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \quad (46)$$

$$\ddot{\theta}_4 = \begin{cases} \omega_n A \sin \omega_n t & t \in [0, t_1) \\ 0 & t \in [t_1, t_1 + t_2) \\ -\omega_n A \sin \omega_n t' & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \quad (47)$$

We have the following equations for load sway by applying Eqs. (46) and (47) to Eqs. (5) and (6), respectively.

$$\theta_{11} = C'_{11} \cos \omega_n t + C'_{12} \sin \omega_n t - \frac{B}{3} (-9 + \cos 2\omega_n t + 6 \cos \omega_n t + 6\omega_n t \sin \omega_n t) \quad (48)$$

$$\theta_{12} = C'_{13} \cos \omega_n t + C'_{14} \sin \omega_n t + 8B \quad (49)$$

$$\theta_{13} = C'_{15} \cos \omega_n t + C'_{16} \sin \omega_n t + \frac{B}{3} (9 - \cos 2\omega_n t' + 3 \cos \omega_n t' + 6\omega_n t \sin \omega_n t') \quad (50)$$

$$\theta_{21} = C'_{21} \cos \omega_n t + C'_{22} \sin \omega_n t + \frac{Ab}{2} t \cos \omega_n t \quad (51)$$

$$\theta_{22} = C'_{23} \cos \omega_n t + C'_{24} \sin \omega_n t \quad (52)$$

$$\theta_{23} = C'_{25} \cos \omega_n t + C'_{26} \sin \omega_n t + \frac{Ab}{4\omega_n} (\sin \omega_n t' - 2\omega_n t \cos \omega_n t') \quad (53)$$

Taking time derivative of Eqs. (48)-(53) and considering boundary conditions lead to the following coefficients:

$$C'_{11} = -\frac{2B}{3}, \quad C'_{12} = 0 \quad (54)$$

$$C'_{13} = \frac{8B}{3}, \quad C'_{14} = -2B\omega_n t_1 \quad (55)$$

$$\begin{bmatrix} C'_{15} \\ C'_{16} \end{bmatrix} = \begin{bmatrix} C'_{13} \\ C'_{14} \end{bmatrix} - R(t_2) \begin{bmatrix} \frac{13B}{3} \\ -2B\omega_n(t_1 + t_2) \end{bmatrix} \quad (56)$$

$$C'_{21} = 0, \quad C'_{22} = -\frac{Ab}{2\omega_n} \quad (57)$$

$$C'_{23} = \frac{Ab}{2} t_1, \quad C'_{24} = 0 \quad (58)$$

$$\begin{bmatrix} C'_{25} \\ C'_{26} \end{bmatrix} = \begin{bmatrix} C'_{23} \\ C'_{24} \end{bmatrix} - R(t_2) \begin{bmatrix} \frac{Ab}{2}(t_1 + t_2) \\ \frac{Ab}{4\omega_n} \end{bmatrix} \quad (59)$$

By considering Eq. (53) and its time derivative, and Eq. (59), Eqs. (31) and (32) for suppressing residual load sway θ_2 are written as follows:

$$R^{-1}(t_2) \begin{bmatrix} C'_{25} \\ C'_{26} \end{bmatrix} + \begin{bmatrix} \frac{Ab}{2} T \\ \frac{Ab}{4\omega_n} \end{bmatrix} = 0 \quad (60)$$

Hence, the necessary and sufficient condition to satisfy Eq. (60) is:

$$\omega_n t_2 = (2k_7 - 1)\pi, \quad k_7 = 1, 2, \dots \quad (61)$$

Substituting Eq. (56) into Eq. (50) and using the condition in Eq. (61), we obtain the following result:

$$\begin{bmatrix} \theta_{13}(T) \\ \frac{1}{\omega_n} \dot{\theta}_{13}(T) \end{bmatrix} = R^{-1}(t_2) \begin{bmatrix} C'_{15} \\ C'_{16} \end{bmatrix} + \begin{bmatrix} \frac{5B}{3} \\ -2B\omega_n T \end{bmatrix} = \begin{bmatrix} -\frac{16B}{3} \\ 0 \end{bmatrix} \quad (62)$$

Hence, residual load sway θ_1 cannot be suppressed in the case $k_5 = 1$.

When $k_5 = 2$, Eqs. (7) and (8), respectively, can be represented as follows:

$$\dot{\theta}_4 = \begin{cases} A(1 - \cos \frac{\omega_n}{2} t) & t \in [0, t_1) \\ 2A & t \in [t_1, t_1 + t_2) \\ A(1 + \cos \frac{\omega_n}{2} t') & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \quad (63)$$

$$\ddot{\theta}_4 = \begin{cases} \frac{\omega_n}{2} A \sin \frac{\omega_n}{2} t & t \in [0, t_1) \\ 0 & t \in [t_1, t_1 + t_2) \\ -\frac{\omega_n}{2} A \sin \frac{\omega_n}{2} t' & t \in [t_1 + t_2, t_1 + t_2 + t_3] \end{cases} \quad (64)$$

We have the following equations for load sway by applying Eqs. (63) and (64) to Eqs. (5) and (6), respectively.

$$\theta_{11} = C''_{11} \cos \omega_n t + C''_{12} \sin \omega_n t + \frac{B}{12} \times \left(36 - 64 \cos \frac{\omega_n}{2} t + 3 \cos \omega_n t + 6 \omega_n t \sin \omega_n t \right) \quad (65)$$

$$\theta_{12} = C''_{13} \cos \omega_n t + C''_{14} \sin \omega_n t + 8B \quad (66)$$

$$\theta_{13} = C''_{15} \cos \omega_n t + C''_{16} \sin \omega_n t + \frac{B}{12} \times \left(36 + 64 \cos \frac{\omega_n}{2} t' + 3 \cos \omega_n t' + 6 \omega_n t' \sin \omega_n t' \right) \quad (67)$$

$$\theta_{21} = C''_{21} \cos \omega_n t + C''_{22} \sin \omega_n t - \frac{2Ab}{3\omega_n} \sin \frac{\omega_n}{2} t \quad (68)$$

$$\theta_{22} = C''_{23} \cos \omega_n t + C''_{24} \sin \omega_n t \quad (69)$$

$$\theta_{23} = C''_{25} \cos \omega_n t + C''_{26} \sin \omega_n t + \frac{2Ab}{3\omega_n} \sin \frac{\omega_n}{2} t' \quad (70)$$

Taking time derivative of Eqs. (65)-(70) and considering boundary conditions lead to the following coefficients:

$$C''_{11} = \frac{25B}{12}, \quad C''_{12} = 0 \quad (71)$$

$$C''_{13} = \frac{32B}{12}, \quad C''_{14} = \frac{B}{2} \omega_n t_1 \quad (72)$$

$$\begin{bmatrix} C''_{15} \\ C''_{16} \end{bmatrix} = \begin{bmatrix} C''_{13} \\ C''_{14} \end{bmatrix} + R(t_2) \begin{bmatrix} -\frac{7B}{12} \\ -\frac{B}{2} \omega_n (t_1 + t_2) \end{bmatrix} \quad (73)$$

$$C''_{21} = 0, \quad C''_{22} = \frac{Ab}{3\omega_n} \quad (74)$$

$$C''_{23} = 0, \quad C''_{24} = \frac{2Ab}{3\omega_n} \quad (75)$$

$$\begin{bmatrix} C''_{25} \\ C''_{26} \end{bmatrix} = \begin{bmatrix} C''_{23} \\ C''_{24} \end{bmatrix} + R(t_2) \begin{bmatrix} 0 \\ -\frac{Ab}{3\omega_n} \end{bmatrix} \quad (76)$$

By considering Eq. (70) and its time derivative, and Eq. (76), Eqs. (31) and (32) for suppressing residual load sway θ_2 are written as follows:

$$R^{-1}(t_2) \begin{bmatrix} C''_{25} \\ C''_{26} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{Ab}{3\omega_n} \end{bmatrix} = 0 \quad (77)$$

Hence, the necessary and sufficient condition to satisfy Eq. (77) is:

$$\omega_n t_2 = 2k_8\pi, \quad k_8 = 0, 1, \dots \quad (78)$$

Substituting Eq. (73) into Eq. (67) and using the condition in Eq. (78), we obtain the following result:

$$\begin{bmatrix} \theta_{13}(T) \\ \frac{1}{\omega_n} \dot{\theta}_{13}(T) \end{bmatrix} = R^{-1}(t_2) \begin{bmatrix} C''_{15} \\ C''_{16} \end{bmatrix} + \begin{bmatrix} -\frac{25B}{12} \\ \frac{B}{2} \omega_n T \end{bmatrix} = \begin{bmatrix} 0 \\ B\omega_n t_1 \end{bmatrix} \quad (79)$$

Hence, residual load sway θ_1 cannot be suppressed in the case $k_5 = 2$.

Therefore, $k_5 = 3, 4, \dots$ is required in Eq. (45).

4. Results and Discussion

4.1. Experimental System

The experimental system is shown in Fig. 3. A DC servo motor drives the boom and base. The horizontal boom angle θ_4 is measured using a rotary encoder, whose angular measurement resolution is 1.8×10^{-3} [deg].

In Fig. 4, the load sway angles θ_1 and θ_2 are measured by potentiometers 1 and 2, respectively. Potentiometer 1 is fixed to the boom. Potentiometer 2 is fixed to part 1, which

Fig. 3 Experimental system

Fig. 4 Sensor system for measuring load sway

rotates around the rotational axis of potentiometer 1. Part 2 rotates around the rod and the rotational axis of potentiometer 2. Part 3 rotates around the rod, and part 4 rotates around part 3. Part 4 slides smoothly along the rope. The angular measurement resolution is $6.4 \times 10^{-2}[\text{deg}]$. Although angles θ_1 and θ_2 can be calculated from the output voltage values of potentiometers 1 and 2, the information of load sway angles θ_1 and θ_2 measured by the sensor system is not used for feedback control to achieve the load sway suppression. A similar structure has been presented in Ref. (22). The parameters of the rotary crane are shown in Table 1.

4.2. Tracking Controller Design

This section describes the tracking controller design needed for the boom to achieve the obtained S-curve trajectory described in § 3.

By integrating the velocity trajectory of Eq. (7), the angular trajectory θ_4 , which is employed for a desired angular reference r of the boom motion, can be obtained as follows:

$$r = \begin{cases} \frac{\dot{\theta}_{4c}t}{2} - \frac{\dot{\theta}_{4c}}{2\omega_1} \sin \omega_1 t & t \in [0, t_1) \\ \dot{\theta}_{4c}t - \frac{\dot{\theta}_{4c}t_1}{2} & t \in [t_1, t_1+t_2) \\ \frac{\dot{\theta}_{4c}t}{2} + \frac{\dot{\theta}_{4c}}{2\omega_2} \sin \omega_2(t-t_1-t_2) + \frac{\dot{\theta}_{4c}t_2}{2} & t \in [t_1+t_2, t_1+t_2+t_3) \\ \theta_f & t \in [t_1+t_2+t_3, t_f] \end{cases} \quad (80)$$

where θ_f is the final angle, t_f is the final time. In this study, we set $\theta_f = 45[\text{deg}]$, $t_f = 10[\text{s}]$ in Eq. (80), $k_5 = 3$ and $k_6 = 1$ in Eq. (45).

To achieve good tracking performance, we design a Proportional-Derivative (PD) controller equipped with a disturbance observer for horizontal boom motion⁽²¹⁾. This controller

Table 1 Parameters of rotary crane

$L[\text{m}]$	0.65	$l[\text{m}]$	0.40
$\theta_3[\text{deg}]$	45.0	$g[\text{m/s}^2]$	9.81

Fig. 5 Proposed S-curve trajectory result based on simple model ($l = 0.40[\text{m}]$)

assumes the first- and second-order derivatives $\dot{r}$ and $\ddot{r}$, respectively, of the reference signal r , are available to achieve good tracking performance. We have confirmed the effectiveness of this disturbance observer based controller under frictional disturbance⁽²¹⁾.

4.3. Simulation and Experimental Results

First, we apply the generated S-curve trajectory and the disturbance observer based controller to the simplified crane model in Eqs. (5) and (6), and the horizontal boom motion model with disturbance⁽²¹⁾. Simulation results are shown in Fig. 5. The residual load sway is suppressed completely as shown in Figs. 5(c) and (d), while the boom tracks the reference trajectory as shown in Fig. 5(a). These results confirm the effectiveness of the proposed S-curve trajectory generation for the assumed simplified model.

Next, the proposed trajectory and the boom motion controller are applied to the original model in Eqs. (1) and (2), and the results are shown in Fig. 6 accompanied with experimental results. In Fig. 6(c), the simulation result differs from that shown in Fig. 5(c) because of the effect of the ignored nonlinearity terms in the crane model in Eqs. (1) and (2). On the other hand, the experimental result in Fig. 6(c) also differs from the simulation result because of the effect of friction between the rope and the top of the boom, whereas good control performances are achieved in Figs. 6(a) and (d).

To demonstrate the effect of the friction between the rope and the top of the boom, we consider the following viscous friction term is in the left-hand side of Eq. (1):

Fig. 6 Proposed S-curve trajectory result ($l = 0.40[\text{m}]$)

Fig. 7 Proposed S-curve trajectory result with viscous friction ($l = 0.40[\text{m}]$)

Fig. 8 Step trajectory result ($l = 0.40[\text{m}]$)

Fig. 9 Cycloid curve trajectory result ($l = 0.40[\text{m}]$)

Fig. 10 Cycloid curve trajectory result ($l = 0.30\text{[m]}$)

Fig. 11 Proposed S-curve trajectory result ($l = 0.30\text{[m]}$)

Fig. 12 Cycloid curve trajectory result ($l = 0.50\text{[m]}$)

Fig. 13 Proposed S-curve trajectory result ($l = 0.50\text{[m]}$)

$$f_{v\theta_1} = C\dot{\theta}_1 \quad (81)$$

where C is the viscous friction coefficient, and is determined as $C = 0.04[\text{N}\cdot\text{m}/(\text{rad}/\text{s})]$ experimentally. The results are shown in Fig. 7. The experimental result in Fig. 7(c) almost coincides with the simulation one. Because the viscous friction changes the oscillation frequency little, the proposed method can be applied to real systems with friction.

To demonstrate the effectiveness of the proposed S-curve trajectory, we compared it experimentally with a conventional step trajectory and a cycloid trajectory for the desired angular reference of the horizontal boom motion.

The step trajectory is represented as follows:

$$r = \theta_f \quad t \in [0, t_f] \quad (82)$$

The cycloid trajectory is represented as follows:

$$r = \begin{cases} (\theta_f - \theta_0) \left\{ \frac{t}{t_s} - \frac{1}{2\pi} \sin\left(2\pi \frac{t}{t_s}\right) \right\} + \theta_0 & t \in [0, t_s] \\ \theta_f & t \in [t_s, t_f] \end{cases} \quad (83)$$

where θ_f is the final angle, θ_0 is the initial angle, t_s is the settling time, and t_f is the final time. We set $\theta_0 = 0[\text{deg}]$ and $t_s = 5.5[\text{s}]$.

The horizontal boom angle, command voltage, and load sway angles for the step trajectory are shown in Fig. 8. Significant residual load sway is confirmed.

The horizontal boom angle, command voltage, and load sway angles for the cycloid trajectory are shown in Fig. 9. The residual load sway remains in Fig. 9(d).

Load sway suppression in Figs. 9(c) and (d) is much better than that in Figs. 8(c) and (d), because the cycloid curve trajectory provides zero acceleration at the initial and terminal points. The proposed S-curve trajectory provides better performance. In particular, performance in Fig. 6(d) is better than that in Fig. 9(d), although the trajectory for θ_4 in Fig. 6(a) approaches θ_f faster than that in Fig. 9(a).

To demonstrate the effectiveness of the proposed method for different rope lengths (i.e., different natural angular frequencies of the rotary crane system), we conducted simulations and experiments by setting $l = 0.30[\text{m}]$ and $l = 0.50[\text{m}]$ and comparing the results with those of a cycloid curve trajectory. The results are shown in Figs. 10-13. The results in these setups were almost the same as those obtained with $l = 0.40[\text{m}]$.

5. Conclusion

Our objective was to suppress two-dimensional residual load sway without measuring it by using only horizontal boom motion. To achieve this, we first considered a simple model for rotary crane dynamics. Next, we derived the analytical conditions for an S-curve trajectory of horizontal boom motion to suppress residual load sway on the basis of the simple model. The proposed trajectory provides better control performance than a conventional step or cycloid trajectory. Because energy-efficient control is necessary owing to global environmental and energy resource issues, in future studies, we will verify the energy efficiency of the proposed method relative to that of the conventional method using vertical boom motion. In addition, in real crane operations, the rope length affects the stability and performance of the control system, we are extending the proposed method to the varying rope length case in future work.

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