Sky-Hook with Adaptive Disturbance Cancellation Control for Flexible Structures with an Active Vibration Control Unit

Yohei HOSHINO**, Kyohei KATAYAMA**, Yukinori KOBAYASHI**, Takanori EMARU** and Yohsuke NAKANISHI***

** Faculty and Graduate School of Engineering, Hokkaido University, N13W8, Kita-ku, Sapporo, Hokkaido, 060–8628, Japan
E-mail: hoshinoy@eng.hokudai.ac.jp

*** Hokkaido Industrial Research Institute, Hokkaido Research Organization, N19W11, Kita-ku, Sapporo, Hokkaido, 060–0819, Japan

Abstract
This study presents an Active Wheel Damper (AWD) unit that can be easily mounted on flexible structures such as cantilever beams. The AWD uses a gyro sensor to directly measure the absolute angular velocity, which is the rotational velocity of the slope angle of a deflection curve caused by the flexible vibration of the structure, and can be used to realize a skyhook control by the direct feedback of the angular velocity. This study proposes a Sky-hook With Adaptive Disturbance Cancellation (SWADC) control—which is constructed as a combined skyhook control and an adaptive disturbance cancellation control system—to enhance the capability of the AWD unit to suppress vibrations. An adaptive algorithm, which estimates the frequency of vibration of a structure in real time, is derived on the basis of an adaptive notch filter algorithm. The estimated frequency is used to model a disturbance observer that estimates harmonic disturbances. This study demonstrates the vibration-suppression performance of the AWD unit by numerical simulations and experiments.

Key words : Skyhook Control, Active Mass Damper, Flywheel, Adaptive Disturbance Cancellation

1. Introduction

An Active Mass Damper (AMD) measures vibrations by sensors and realizes a feedback control to suppress vibrations in structures by using the reaction of the force that accelerates an inertial mass. An AMD with straight line motion needs a large inertial mass and a range of movement sufficiently long to generate a large control force to suppress vibrations in large, flexible structures. For suppressing vibrations in long, flexible structures such as cantilever beams, it is difficult to use an AMD with straight line motion, because the large inertial mass causes a corresponding large static deflection of the structures and it is difficult to ensure a long range of movement of the mass on the AMD.

A rotational vibration absorber which absorbs vibrations in the flexible beams has been proposed by some research papers(1) – (3). The rotational vibration absorber generates a bending moment by an appendage consisting of a rotational inertia and a rotational spring for passive damping, in addition, a pair of piezo actuators are used to realize an active damper. Still, such absorber needs a large inertia mass and have a limitation of output power of the piezo actuators for suppressing the vibrations in large flexible structures. An Active Wheel Damper (AWD) was proposed previously(4). The AWD uses a torque-unit(5), which is proposed as an actuator for driving robot manipulators consisting of rigid links, instead of the appendage mass and the piezo actuators. The AWD, as shown in Fig. 1, consists of a motor, flywheel, sensor, and microcomputer. And the AWD uses rotational motion to suppress vi-
brations in long, flexible cantilever beams. In this study, we applied the AWD to cantilever beams mounted on a vehicle, which simulates the booms used for spraying agrochemicals, and observed that the AWD effectively suppresses vibrations in the booms through numerical simulations and experiments. An AWD uses the reaction $M$ to the torque $-M$, which is generated by the motor, to accelerate a flywheel as an inertial mass, and provides damping to flexible structures. By utilizing the rotational motion of the flywheel, a large inertia (moment of inertia $J_w$) can be realized on the flywheel despite the light mass. Furthermore, the range of movement of the inertial mass is not an issue because the AWD uses the rotational motion of the flywheel. We can integrate the inertial mass (the flywheel), actuator, sensor, and controller into an AWD unit that can be easily mounted on flexible structures. Because of these advantages, an AWD can be used and mounted on long, flexible structures such as agricultural sprayer booms, as shown in Fig. 2. In addition, skyhook control can be easily realized using the proportional feedback of the absolute angular velocity obtained from the gyro sensor on the AWD unit. Some researches proposed the active control method to absorb vibrations in a flexible sprayer booms. These methods, however, require modification of the design of boom structure to realize the active damper, and have disadvantage of higher cost compared with the AWD.

This study assumes that the vibration of a cantilever beam is caused by the forced excitation due to disturbance and utilizes an Adaptive Notch Filter algorithm to estimate the principal frequency of the disturbance, which also realizes an Adaptive Disturbance Cancellation (ADC) control, a form of disturbance cancellation control, by updating the frequency of the harmonic disturbance model to that estimated by the adaptive notch filter. The ADC control is combined with skyhook control to improve the damping capability of the AWD unit in comparison with that obtained using only the skyhook control of the AWD unit. This also makes the AWD able to be simply mounted on flexible beam structures with different specifications such as the natural frequencies. We call this combined controller the Sky-hook With Adaptive Disturbance Cancellation (SWADC) control. The capability of the developed AWD unit with SWADC control is demonstrated by numerical simulations and experiments of vibration suppression by the AWD unit mounted on a cantilever beam.

2. Mathematical Model of a Cantilever Beam

The deflection of a beam at position $x$ is represented by $y(x, t)$, as shown in Figs. 3 and 4. When a distributed force $F(x, t)$ acts at a position $x$ on a cantilever beam with a density $\rho$, cross-sectional area $A$, Young’s modulus $E$, and moment of inertia of the area $I$, the equation of motion for a Bernoulli–Euler beam is expressed as

$$\rho A y''(x, t) + EI y''''(x, t) = F(x, t),$$

where operators $(\cdot)'$ and $(\cdot)''$ represent the partial derivatives of the variable $(\cdot)$ with respect to time $t$ and position $x$. By the superposition of vibration modes, the deflection of the beam
The equation of motion for the angle of deflection $\theta_i(t)$ of the beam due to the $\phi_i(x)$ of the cantilever beam yields an equation of motion for the $i$th modal coordinate, $\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{\int_0^L F(x,t) \phi_i(x) dx}{\rho A \int_0^L \phi_i^2(x) dx}$, because of the orthogonal relation of vibration modes, where $\omega_i$ is the $i$th circular natural frequency of the cantilever beam. The input moment $M$, which is generated by the motor as a reaction to the torque $-M$ acting on the flywheel, acts on the beam, as shown in Fig. 3. And $M$ can be expressed as

$$M = f \Delta x$$

by a couple of force $f$, as shown in Fig. 4, where $\Delta x$ represents the infinitesimal length of the arm of the couple of force $f$. When the AWD unit is mounted at position $x = l_u$ and the center of $\Delta x$ is at the position of the unit, the couple of force can be decomposed into two concentrated forces $f$ and $-f$ that act at $x = (l_u + \Delta x/2)$ and $x = (l_u - \Delta x/2)$, respectively. Using the Dirac delta function $\delta(x)$, the distributed force $F(x,t)$ in Eq. (1) working on the beam is expressed as

$$F(x,t) = f(t) \delta\left(x - (l_u + \frac{\Delta x}{2})\right) - f(t) \delta\left(x - (l_u - \frac{\Delta x}{2})\right).$$

Next, multiplying $\phi_i(x)$ on both sides of Eq. (5) and integrating over the length $L$ of the beam yields the following equation,

$$\int_0^L F(x) \phi_i(x) dx = f(t) \left[ \phi_i(l_u + \frac{\Delta x}{2}) - \phi_i(l_u - \frac{\Delta x}{2}) \right] = M(t) \frac{\phi_i(l_u + \frac{\Delta x}{2}) - \phi_i(l_u - \frac{\Delta x}{2})}{\Delta x}.$$

because of the properties of the Dirac delta function. Because $\Delta x$ is infinitesimal, applying the limit as $\Delta x \to 0$ to Eq. (6) gives

$$\int_0^L F(x) \phi_i(x) dx = M(t) \frac{d\phi_i}{dx} |_{x=l_u} = M(t) \phi'_i(l_u).$$

When $M(t)$ is generated by the input torque $\tau(t)$ of the motor, Eq. (3) can be expressed as

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = \tau(t) \phi'_i(l_u) \rho A \int_0^L \phi_i^2(x) dx.$$

By multiplying $\phi'_i(l_u)$ on both sides of Eq. (8), the equation of motion for the angle of deflection $\theta_i(t)$ of the beam due to the $i$th order modal coordinate becomes

$$J_i \ddot{\theta}_i(t) + K_i \dot{\theta}_i(t) = \tau(t), \quad J_i = \frac{\rho A \int_0^L \phi_i^2(x) dx}{\phi_i^2(l_u)}, \quad K_i = \omega_i^2 \frac{\rho A \int_0^L \phi_i^2(x) dx}{\phi_i^2(l_u)}.$$
The vibration of the beam can be measured in terms of the absolute angular velocity $\dot{\theta}_G$ by the gyro sensor mounted at position $l_a$. $\dot{\theta}_G$ is expressed as

$$\dot{\theta}_G = g'(l_a, t) = \sum_{j=1}^{\infty} \phi_j'(l_a) q_j(t)$$

by the superposition of vibration modes.

### 3. Controller Design

#### 3.1. Adaptive notch filter

When the second-order notch filter is expressed as the following Infinite Impulse Response (IIR) filter described in $z$-transform as

$$H(z) = \frac{(\rho_{NF}^2 + 1) + 2a\rho\z^{-1} + (\rho_{NF}^2 + 1)\z^{-2}}{2(1 + a\rho\z^{-1} + \rho_{NF}^2\z^{-2})},$$

the circular frequency of the notch $\omega$ is determined by the parameter $a$, and an adaptive notch filter can be developed\(^{(9)},(10)\). The adaptive notch filter updates the parameter $a$ in real time to match the circular frequency of the notch $\omega$ to the frequency of the unknown input signal with a narrow bandwidth. Here the bandwidth of the notch filter is determined by the parameter $\rho_{NF}$. Using $k$ as the time step and rewriting parameter $a$ as $a_k$, Eq. (11) becomes

$$\frac{N(z)}{D(z)} = \frac{(\rho_{NF}^2 + 1) + 2a_k\rho\z^{-1} + (\rho_{NF}^2 + 1)\z^{-2}}{2(1 + a_k\rho\z^{-1} + \rho_{NF}^2\z^{-2})}.$$  

Representing the input and output of the notch filter as $u(k)$ and $y(k)$, respectively, and defining the output of the transfer function $1/D(z)$ as $x(k)$, Eq. (12) can be rewritten as

$$y(k) = N(z)x(k), \quad x(k) = \frac{1}{D(z)}u(k).$$

By calculating $N(z)x(k)$, the output $y(k)$ can be expressed by a difference equation of $x(k)$ as

$$y(k) = (\rho_{NF}^2 + 1)x(k) + 2a_kx(k - 1) + (\rho_{NF}^2 + 1)x(k - 2).$$

The adaptive notch filter algorithm can be derived by the Recursive Least-Squares (RLS) method that updates the parameter $a_k$ recursively to minimize the following evaluation function

$$E(k) = \sum_{i=0}^{k} A^k y^2(i),$$

where $\lambda$ ($0 \leq \lambda \leq 1$) is a forgetting factor. By defining the variables $A(k)$ and $B(k)$ as

$$A(k) = x(k - 1), \quad B(k) = x(k) + x(k - 2),$$

respectively, the output of the filter, Eq. (14) can be rewritten as

$$y(k) = (\rho_{NF}^2 + 1)B(k) + 2a_kA(k).$$

By substituting Eq. (17) in Eq. (15), the following expression

$$E(k) = \sum_{i=0}^{k} A^k \left[ (\rho_{NF}^2 + 1)B^2(i) + 4a_k(\rho_{NF}^2 + 1)A(i)B(i) + 4a_k^2A^2(i) \right]$$

is obtained. The parameter $a_k$ that minimizes the evaluation function $E(k)$ satisfies

$$\frac{\partial E(k)}{\partial a_k} = \sum_{i=0}^{k} A^k \left[ 4(\rho_{NF}^2 + 1)A(i)B(i) + 8a_k A^2(i) \right] = 0.$$  

Therefore, the parameter $a_k$ of the adaptive notch filter is expressed as

$$a_k = \frac{(\rho_{NF}^2 + 1)}{2} \frac{\sum_{i=0}^{k} A^{k-i} A^2(i)}{\sum_{i=0}^{k} A^{k-i} A^2(i)}.$$
Now, defining variables $\Phi(k)$ and $\eta(k)$ as

$$\Phi(k) = A\Phi(k-1) + A^2(k),$$

$$\eta(k) = A\eta(k-1) + A(k)B(k),$$

respectively, the parameter $a_k$ of the adaptive notch filter in Eq. (20) can be rewritten as

$$a_k = -\left(\frac{\rho^2_{\text{NF}} + 1}{2}\right) \cdot \frac{\eta(k)}{\Phi(k)},$$

Using the relation between the circular frequency of the notch $\omega_k$ and the parameter $a_k$, the circular frequency of the vibration of the structure can be recursively estimated as

$$\omega_k = \frac{1}{T} \cos^{-1}\left(\frac{-a_k}{\rho^2_{\text{NF}} + 1}\right)$$

in real time, where $T$ is the sampling period of the measurements.

In the following section, the estimated circular frequency $\omega_k$ is used as the central frequency of a bandpass filter (BPF) and the natural frequency of the model of the harmonic disturbance observer, and a controller is developed that can adapt to the variable frequency of vibration in a cantilever beam.

### 3.2. Control model of a cantilever beam

Figure 5 shows the mechanical relation between the cantilever beam and AWD unit. The deflection angle of the beam is represented by $\theta$, which is the slope of the deflection curve of the beam at the position where the AWD unit is mounted. The vibrational motion of the deflection angle, $\theta = \gamma'(x,t)$, arises from the translational flexible vibration of the deflection of the beam $\gamma(x,t)$, as shown in Fig. 5, because of Eqs. (2), (9), and (10). The equation of motion for the flexible beam expressed in terms of the deflection angle of the beam $\theta$ due to a specific vibration mode is obtained by introducing a damping term into Eq. (9) as follows:

$$J\ddot{\theta} + C\dot{\theta} + K\theta = -w + d.$$  

Here $C$ represents the damping coefficient. The equation of motion for the rotation of the AWD unit excluding the flywheel and the equation of motion for the flywheel are, respectively, expressed as

$$J_u\ddot{\theta_u} = w + \tau,$$

$$J_w\dot{\theta_w} = -\tau.$$  

The moment of inertia of the AWD unit, excluding the flywheel, around the rotation axis of the motor—which is passing through the center of mass of the unit and is parallel to the $z$-axis in Figs. 3 and 5—is represented by $J_u$. The moment of inertia of the fly-wheel around the rotation axis of the motor is represented by $J_w$. The disturbance is represented by $d$; the disturbance acting on the AWD unit is represented by $w$, and the disturbance $w$ arising from the vibration in the beam is the contact force between the AWD unit and the beam. Because the rotational angle $\theta_u$ of the AWD unit arises from the deflection angle of the beam $\theta$ at the position where the AWD unit is mounted, the relation $\theta_u = \theta$ is satisfied, and the angle of

![Fig. 5 Dynamics model of AWD and cantilever beam](image-url)
rotation of the flywheel is represented by $\theta_w$. Assuming the model of Eqs. (25)–(27) as the model around a static equilibrium state, such as the static deflection of the cantilever beams caused by gravity, gravitational effects need not be considered in the model of Eqs. (25)–(27) for suppressing vibration. Adding both sides of Eqs. (25) and (26), the equation for the motion of a beam with the AWD unit is obtained as

$$(J + J_u)\ddot{\theta} + C\dot{\theta} + K\theta = \tau + d. \quad (28)$$

Feedback control can effectively suppress vibrations due to transient disturbances (unexpected shocks), and feedforward control can effectively cancel out vibration due to continuous disturbances. This study uses the combined control of both feedback and feedforward controls\(^{(11)}\)–\(^{(14)}\), and the control input is expressed as

$$\tau = \tau_{fb} + \tau_{ff}, \quad (29)$$

where $\tau_{fb}$ and $\tau_{ff}$ represent the feedback and the feedforward control inputs, respectively.

### 3.3. Feedback control (Skyhook control)

The controller in the study utilizes skyhook control\(^{(6)}\) as the feedback control $\tau_{fb}$. The skyhook control achieves an ideal damping performance and reduces the amplitude of the vibration across the entire frequency region. A skyhook control system can be easily realized by mounting an AWD unit on a beam because the absolute angular velocity—which is necessary to realize the skyhook controller—can be directly measured using the gyro sensor on the AWD unit. Incorporating the skyhook controller into the model expressed by Eqs. (25)–(27), the control input $\tau_{fb}$ is given by

$$\tau_{fb} = -K_{sk}\dot{\theta}. \quad (30)$$

In a continuous-time system, the skyhook control provides damping to the higher vibration modes to ensure the stability of the entire system, because the AWD unit can realize almost an ideal collocation between the gyro sensor and the actuator on the AWD unit.

### 3.4. Feedforward control (Disturbance cancellation control)

Here a state equation considering only the motion of the AWD unit is developed. The disturbance $w$ is assumed as an harmonic excitation force in this study, and the circular frequency of the harmonic excitation force $\omega_h$ is estimated using the adaptive notch filter expressed by Eq. (24). The model of the harmonic excitation force is expressed as

$$\ddot{w} + \omega_h^2 w = 0. \quad (31)$$

The state equation model of the excitation force is expressed as

$$\dot{w} = A_0 w, \quad (32)$$

$$A_0 = \begin{bmatrix} 0 & 1 \\ -\omega_h^2 & 0 \end{bmatrix}, \quad w = \begin{bmatrix} \dot{w} \\ w \end{bmatrix}. \quad (33)$$

Assuming the disturbance of the AWD unit by the harmonic excitation force, the state equation of the combined model of the disturbance $w$ and the motion of the AWD unit $\theta$ is expressed as

$$\dot{X} = AX + B\tau, \quad (34)$$

$$X = \begin{bmatrix} \dot{\theta} \\ \dot{w} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1/J_u & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1/J_u \\ 0 \end{bmatrix} \quad (35)$$

by Eqs. (26) and (32). The measurement of the disturbance $w$ is necessary for the feedforward controller to achieve disturbance cancellation in the model (34); however, $w$ cannot be measured directly. Here a disturbance observer that can estimate the disturbance $w$ is introduced.
Next, the symbol $\hat{\omega}$ is defined as the estimated variable of $\omega$. Assuming that the angular velocity $\hat{\theta}$ can be measured directly by the gyro sensor on the unit and providing feedback of the error $(\hat{\theta} - \theta)$, the state equation of the disturbance observer is expressed by Eq. (34) as

$$\dot{\hat{X}} = A\hat{X} + B\tau + L(\dot{\hat{\theta}} - \hat{\theta}), \quad (36)$$

$$\hat{X} = \begin{bmatrix} \dot{\hat{\theta}} \\ \dot{\hat{w}} \end{bmatrix}, \quad L = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^T, \quad (37)$$

where the feedback gain $L$ is a matrix of the design parameters $l_1$, $l_2$, and $l_3$. Substituting Eq. (34) from Eq. (36), the state equation of the error $\hat{e}$ is derived as

$$\dot{\hat{e}} = A'\hat{e} + L(\hat{\theta} - \hat{\theta}) = A'\hat{e}, \quad (38)$$

$$A' = \begin{bmatrix} 1 & 1/J_u & 0 \\ l_2 & 0 & 1 \\ l_3 & -\omega_k^2 & 0 \end{bmatrix}, \quad e = \begin{bmatrix} \dot{\hat{\theta}} - \hat{\theta} \\ \dot{\hat{w}} - w \\ \dot{\hat{\theta}} - \hat{\theta} \end{bmatrix}, \quad (39)$$

The characteristics of the state vector $e$ is determined by the poles of the matrix $A'$ that are represented by $\lambda_1$, $\lambda_2$, $\lambda_3$. Using the pole placement method to determine the design parameter $L$ so the state equation (38) has the desired stable poles, then the disturbance can be estimated by Eq. (36). This study defines the feedforward input $\tau_{ff}$ as

$$\tau_{ff} = -a\hat{\omega} \quad (0 \leq a \leq 1) \quad (40)$$

to cancel out the disturbance $w$, and $a$ is the gain of feedforward control.

Next, the reliability of the proposed method is evaluated. Applying a Laplace transformation to Eqs. (25) and (26) and removing $\Theta(s)$ from these equations, the transfer characteristics from the disturbance $d$ acting on the boom to the disturbance $w$, which is input to the AWD unit by the motion of the boom, are derived as

$$W(s) = \frac{J_u s^2}{(J + J_u) s^3 + C s + K} D(s) - \frac{J s^2 + C s + K}{(J + J_u) s^3 + C s + K} T(s). \quad (41)$$

Assuming that the estimated disturbance $\hat{\omega}$ for the disturbance $w$ in Eq. (26) can be obtained as $\hat{\omega} \rightarrow w$ by the disturbance observer, and with the control input of the unit as $\tau = -a\hat{\omega}$ necessary to cancel out the disturbance, the transfer function from the disturbance $d$ to the estimated disturbance $\hat{\omega}$ is obtained as

$$\hat{W}(s) = \frac{J_u s^2}{(1 - a)(J + J_u) s^3 + (1 - a)C s + (1 - a)K} D(s) \quad (42)$$

from Eq. (41). Similarly, giving the control input as $\tau = -a\hat{\omega}$, the transfer function between the disturbance $d$ and the deflection angle $\theta$ of the boom is obtained as

$$\Theta(s) = \frac{1}{(J + J_u) s^3 + C s + K + \left(\frac{a}{1 - a}\right)J_u s^3} D(s) \quad (43)$$

from Eq. (42) and the Laplace transformation of Eq. (28). From this, the gain of the transfer function in Eq. (43) from the disturbance $d$ to the deflection angle of the boom $\theta$ is shown to decrease when the gain $a$ is increased from 0, and the response from the disturbance $d$ to the vibration of the boom $\theta$ will be close to 0 when $a$ is close to 1. This result suggests that when the right side of Eq. (28) becomes 0, the estimated disturbance $\hat{\omega}$ will be close to the disturbance $d$.

In this study, the combined control in Eq. (29) for skyhook control Eq. (30) and an adaptive disturbance cancellation control Eq. (40) is termed as Sky-hook With Adaptive Disturbance Cancellation (SWADC) control.
4. Simulation Results

This section demonstrates the performances and stability of the proposed AWD unit and SWADC control by numerical simulations. Here the parameter specifications for the simulations of a brass cantilever beam and AWD unit are shown in Tables 1 and 2, respectively. These parameters are the specifications for the experimental setup of a cantilever beam and an AWD unit used in following section. Table 3 shows the parameters for the vibration model obtained with Eq. (9) by using the natural frequencies and vibration modes of the cantilever beam calculated by the Finite Element Method (FEM). In this table, the damping coefficient $C$ is given by assuming a cantilever beam with a small amount of damping.

The moment of inertia of the disturbance observer model $J_{d}$ in Eq. (36) is the only parameter for the control model of the SWADC controller. First, the effects of varying the parameter $J_{d}$ on the stability of the system and the vibration-suppression ability of the AWD unit are investigated. Here considering the moment of inertia with respect to the deflection angle of the cantilever beam $J$, the moment of inertia $J_{d}$ of the disturbance observer model is set to a larger value than the actual moment of inertia of the unit $J_{0} = 0.0087 \text{ kg} \cdot \text{m}^2$. The sampling period of the controller is set to 10 ms, and the initial beam deflection conditions are set to $\theta = 0.0$ and $\dot{\theta} = 0.0$.

The simulation results when using no control, skyhook control, SWDC (without the adaptive notch filter) are shown in Figs. 6–11. In these simulations, the moment of inertia of the control model (28) is set to $J = J_{0} = 0.222863 + 0.0087 = 0.23156 \text{ kg} \cdot \text{m}^2$, and the moment of inertia $J_{d}$ of the disturbance observer model (36) is set to the parameters shown in Table 4. The time historical responses of the angular velocities of the deflection angle of the beam $\dot{\theta}$, and the disturbance inputs given by the harmonic excitation forces $d$ are shown in (a) and (b) of Figs. 6–11, respectively. The estimation results by a disturbance observer are also shown in these figures to compare the exact values. The estimation results of the disturbances are shown on a scale indicated in each legend. The frequency of the harmonic excitation force as the disturbance input is set to 13.942 rad/s (2.218 Hz), which is similar to the natural frequency of the cantilever beam, and the amplitude is set to 0.25 N·m. Figures 6–8 show the results in which the gains of the skyhook control $K_{sy}$ and the disturbance cancellation control $\alpha$ are set to 1, and Figs. 9–11 show the results in which the gains $K_{sy}$ and $\alpha$ are set to 0.7. In these simulations, no control inputs are given during 0–10 s, feedback inputs with skyhook control are given during 10–20 s, and the Skyhook With Disturbance Cancellation control (SWDC, without the adaptive notch filter) is applied after 20 s.

First, we ascertain the capability of skyhook control to suppress vibrations. The results with skyhook control during 10–20 s in Figs. 6–11 show that the skyhook control reduces the

\begin{table}[h]
\centering
\begin{tabular}{l l}
<table>
<thead>
<tr>
<th>Parameters of brass beam</th>
<th>\hspace{2cm}</th>
<th>Parameters of the AWD unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1.78 kg</td>
<td>Moment of inertia $J_{0}$ 0.0087 kg-m^2</td>
</tr>
<tr>
<td>Length</td>
<td>0.78 m</td>
<td>Moment of inertia $J_{d}$ 0.0106 kg-m^2</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.0025 m</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>Width</td>
<td>0.085 m</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>Density</td>
<td>8949.2 kg/m^3</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>100.0 MPa</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>First natural frequency</td>
<td>2.2189 Hz</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>(Experimental)</td>
<td>2.05 Hz</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>Second natural frequency</td>
<td>13.905 Hz</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>J</td>
<td>0.222863 kg-m^2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.0006 N-m-s/\text{rad}</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>43.3187 N-m/\text{rad}</td>
<td></td>
</tr>
<tr>
<td>Natural frequency</td>
<td>2.2189 Hz</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>9.655 \times 10^{-5}</td>
<td>\hspace{2cm} \hspace{2cm}</td>
</tr>
</tbody>
</table>
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{l l l l}
| Poles      & $J_{0}$ & 0.0087 | 0.63 | 1.00 |
|------------|---------|--------|-----|------|
| $l_{1}$    & -1.664 | -9.525 | -9.525 |
| $l_{2}$    & -4.1679 \times 10^{1} & -2.2374 \times 14.335j & -2.2374 \times 9.7081j |
| $l_{3}$    & -1.1008 \times 10^{-1} & -1.0018 \times 10^{-1} & -1.0018 \times 10^{-1} |
| $l_{4}$    & -2.7262 \times 10^{-3} & -2.7262 \times 10^{-3} & -2.7262 \times 10^{-3} |
\end{tabular}
\end{table}

Frequency of the harmonic disturbance model is 2.2 Hz. Feedback gains for observer $l_{1}$, $l_{2}$, and $l_{3}$ are –10.
amplitude of vibration; however, the vibration is not completely eliminated.

Next, this study investigates the effect of the moment of inertia $J_u$ of the disturbance observer model (36) on the capability to suppress vibrations. Comparing the results of Figs. 6–11, we see that accurate estimations are obtained by the disturbance observer when the moment of inertia of the observer model $J_u$ is set to $J_u = 0.0087$ kg·m², which is the same as the actual value for the moment of inertia of the AWD unit (Figs. 6 and 9). Here, the ‘accurate estimations’ mean that the estimations agree with the actual disturbances with respect to the phases as shown in Figs. 6–11 and are sufficiently accurate to realize the disturbance cancellation as shown in Figs. 7–11. In this study, the disturbance observer (36) is constructed considering only the AWD unit excluding the cantilever beam because the AWD is assumed to be mounted on the different beams with unknown specifications ($J$, $C$ and $K$). In the simulations, the actual disturbance $d$ oscillates the beam, and the vibration of the beam generates the disturbance $w$ acting on the AWD. This means that the disturbance $d$ cannot be estimated directly by the observer, but $\hat{w}$ is obtained as an estimation of $w$ generated by the disturbance $d$ through the transfer characteristics of the beam. Therefore, the estimations $\hat{w}$ with different amplitudes from the actual disturbances $d$ are obtained depending on the transfer characteristics of the beam as shown in Figs. 6 (b)–11 (b). A closed-loop system becomes unstable when the disturbance cancellation control is applied when the gains of the skyhook control $K_{sky}$ and the disturbance cancellation control $\alpha$ are set to 1 (Fig. 6). As shown in Fig. 9, we can see that the closed-loop system becomes stable when the gains $K_{sky}$ and $\alpha$ are decreased.
Fig. 9 Simulation result ($J_\alpha = 0.0087$, Harmonic disturbance with 2.218 Hz, 0.25 N·m, and $K_{sky} = 0.7, \alpha = 0.7$)

Fig. 10 Simulation result ($J_\alpha = 0.63$, Harmonic disturbance with 2.218 Hz, 0.25 N·m, and $K_{sky} = 0.7, \alpha = 0.7$)

Fig. 11 Simulation result ($J_\alpha = 1.00$, Harmonic disturbance with 2.218 Hz, 0.25 N·m, and $K_{sky} = 0.7, \alpha = 0.7$)

to 0.7. When the moment of inertia of the disturbance observer model is set to $J_\alpha = 0.0087$ kg·m², from Table 4, we can see that the gain of the angular velocity $l_{d1}$ in the gain vector $L_d = [l_{d1} \ l_{d2} \ l_{d3}]^T$, which is determined from $L$ by transforming the disturbance observer model (36) to a discrete-time system, is larger than in other cases. Because the control input of skyhook control is obtained by multiplying the angular velocity $\dot{\theta}$ and feedback gain $K_{sky}$, the input $\tau$—consisting of the angular velocity $-K_{sky}\dot{\theta}$ and the feedback obtained by multiplying $\dot{\theta}$ and $l_{d1}$—is given to the disturbance observer (36), when the skyhook and the disturbance cancellation controls are applied simultaneously. As a result, the large feedback gain with respect to $\dot{\theta}$ on the disturbance observer can make the feedback system unstable. When the parameter is set to $J_\alpha = 0.63$ kg·m² (Figs. 7 and 10), we see that the vibration amplitudes are reduced by disturbance cancellation control after 20 s compared with the other cases, and superior damping is obtained. From Figs. 6–11, when setting $J_\alpha$ to a large value, we see that the amplitudes of the estimated disturbances become large, as shown in figures (b); as shown in figures (a), the responses with large amplitudes arise in the angular velocities of the deflection angle of the beam at 20 s when the disturbance cancellation control is activated. In this case, because the disturbance and the cancellation force do not balance initially when the disturbance cancellation is started, the amplitudes of the vibration become transiently large, and eventually the vibrations can be canceled out, and thus, they converge. When the parameter $J_\alpha$ is set to a larger value ($J_\alpha = 0.63, 1.0$) than $J_\alpha = 0.0087$ kg·m², we can set the gains of skyhook and disturbance cancellation controls to 1; thus, the closed-loop system is not only

---

The figure captions and the text describe the simulation results for different values of $J_\alpha$, showing the effect of disturbance cancellation control on the system's response to a harmonic disturbance. The figures illustrate the angular velocity and disturbance over time, highlighting the differences in response when $J_\alpha$ is set to 0.0087, 0.63, and 1.00 kg·m². The analysis emphasizes the impact of the disturbance observer model's moment of inertia on the system's stability and performance, particularly in the context of skyhook control and disturbance cancellation.
stable, but also the disturbance cancellation control indicates a higher capability to suppress vibrations.

This study also ascertains the stability of an AWD unit with SWDC control by using root locus analysis. The stability of the closed-loop system is investigated by varying the SWDC parameters \(K_{sk}, \alpha, \) and \(J_u\). The state equation of the closed-loop system is obtained as

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\dot{\theta}} \\
\dot{\omega} \\
\dot{\dot{\omega}}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -\frac{K}{J*} & -C + \frac{K_{sk} J}{J*} & 0 \\
0 & -l_1 + \frac{K_{sk} J}{J*} & l_3 & 0 \\
0 & -l_2 & l_3 & 0
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta} \\
\dot{\omega} \\
\dot{\dot{\omega}}
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
-\frac{\alpha}{J} \\
0
\end{bmatrix} \cdot d
\tag{44}
\]

from Eqs. (28), (29), (30), (36), and (40). The root locus is shown in Figs. 12 and 13 as the parameters \(K_{sk}\) and \(\alpha\) in Eq. (44) are varied. Figure 12 shows the results when setting the moment of inertia of the beam model \(J\) in Eq. (44) to the same value used in the time historical analysis \(J = 0.222863\), and Fig. 13 shows the results when \(J\) in Eq. (44) is set 10 times larger than the value used in the time historical analysis. The results in Figs. 12 (b), (c) and 13 (b), (c) show that the variations in the poles of the closed-loop system become small when the moment of inertia of the model for the disturbance observer \(J_u^*\) is set to a larger value than the actual value of the moment of inertia of the AWD unit. The results for when \(J_u^*\) is set to 0.0087, as shown in Figs. 12 (a) and 13 (a), show that a couple of poles are on the right plane when the gains of the controller \(K_{sk}\) and \(\alpha\) are set to 1 so that the closed-loop system is unstable. We can also see that all poles are on left plane, and the closed-loop system becomes stable by setting \(K_{sk}\) and \(\alpha\) to a value of less than 0.7 even if the parameter \(J_u^*\) is
set to 0.0087. These results show that the stability of the closed-loop system is ensured, and an AWD unit with the SWDC can suppress vibrations in beams that have a wide range of parameters $J$ when the moment of inertia of the disturbance observer $J_0$ is set to a larger value than the actual value, and the gains of the SWDC ($K_{xy}$ and $\alpha$) are set to a value less than 0.7.

5. Experimental Results (Free-Vibration Tests)

This study shows the performances of vibration suppression achieved by an AWD unit via free-vibration experiments (initial value response). The experimental setup is shown in Figs. 14 and 15. In the experiments, the unit suppresses the vibration of a 0.78-m-long brass cantilever beam, as shown in Fig. 16. A servomotor (SGMAV-04ADA61, YASKAWA Electric Corporation) is mounted on the AWD unit and is controlled by a motor driver (SGDV-2R8F01A, YASKAWA Electric Corporation). Since the goal of this study is to conduct experiments for suppressing vibration in booms\(^{(4),(15)-(17)}\), which are 4-m-long flexible cantilever beams mounted on the left and right sides of a vehicle simulating an agrochemical sprayer, as shown in Fig. 17, a 400 W servomotor is used. The output voltages from the gyro sensors and controller are measured using a midi LOGGER (GL500, GRAPHTEC Corporation). The values of the variables for the microcomputer program are obtained through serial communication between a Windows PC and a microcomputer (H8/3664, Renesas Electronics Corporation). The gyro sensor (ADXRS6141Analog Devices) on the AWD unit has 25 mV/(deg/s) of sensitivity. The accelerations of the cantilever beam are measured using an acceleration pickup (LS-10C, RION Co., Ltd.) and amplified by a sensor amplifier (LF-20, RION Co., Ltd.). These accelerations are not used for vibration control, but are measured to ascertain vibration-suppression performances.

This study compares initial value responses in which 10 cm of an initial displacement is applied at the cantilever beam tip. In the experiments, the circular frequency of the notch $\omega_n$ of the adaptive notch filter obtained through the serial communication from the microcomputer, high- and low-pass cutoff frequencies of the BPF ($f_H$ and $f_L$, respectively), amplified angular velocity (Fig. 15 (c)), control input (Fig. 15 (i)), and acceleration obtained by the ac-
Controllers with adaptive filters are realized on the microcomputer by C language and assembler language program. The high- and low-pass cutoff frequencies of the BPF for skyhook control are set to \( f_H = 0.3 \) Hz and \( f_L = 2.3 \) Hz, respectively. The adaptive disturbance cancellation controller estimates the principal frequency of disturbances in real time by an adaptive notch filter algorithm to update the circular frequency of the harmonic disturbance model of the disturbance observer \( \omega_k \) and the cutoff frequencies of the adaptive BPF \( (f_H, f_L) \). First, the adaptive notch filter estimates the frequency to obtain the natural frequency of the cantilever beam. The parameters for the adaptive notch filter are set to \( \rho_{NF} = 0.97 \) and \( \lambda = 0.97 \) based on the results of preliminary experiments. The cutoff frequencies of the adaptive BPF are updated in real time based on the frequency estimated by the adaptive notch filter. When the estimated frequency by the adaptive notch filter does not reach 1.2 Hz, these cutoff frequencies are set to \( f_H = 0.1 \) Hz and \( f_L = 2.2 \) Hz. In addition, an update is not performed when the amplitude of the vibration is smaller than the constant threshold, because the accuracy of the frequency estimation is deteriorated for a vibration with a small amplitude. Figure 18 shows the estimation results. The parameters for the BPF indicate \( f_H = 0.1 \) Hz and \( f_L = 2.2 \) Hz before 0.7 s when the estimation is started, and these parameters are updated to \( f_H = (f_k - 1) \) Hz and \( f_L = (f_k + 1) \) Hz, where \( f_k = \omega_k / (2\pi) \), while a frequency estimation is made. Although the estimated frequency indicates a value around 0 Hz while the output of the gyro sensor is small (after 7 s), the parameters for the BPF ensure the proper frequency band by stopping the update of cutoff frequencies \( (f_H, f_L) \). In addition, we can see that a cantilever beam with a mounted AWD unit has a first-order natural frequency of approximately 1.9 Hz. Compared with the first-order natural frequency of the beam before mounting the AWD unit obtained by FEM, which is 2.05 Hz, as shown in Table 1, an increase in the mass by mounting the AWD unit must decrease slightly the natural frequency of the cantilever beam. Based on this result, this study sets the central frequency of the BPF to 1.9 Hz. From the simulation results, the parameters for the controller are set to \( J_u = 0.63, l_1 = -10, l_2 = -10, l_3 = -10, K_{sky} = 1, \) and \( \alpha = 1 \). They resemble those used for the results shown in Fig. 7, which seems to indicate the best performance for vibration suppression in the simulations. Figure 19 shows the time historical responses of the acceleration measured by the accelerometer fixed at the beam tip, when applying no control, skyhook control, and SWADC control. The skyhook and the SWADC controls reduced the vibration in a shorter time than for the case without control, which indicates that these controllers have a superior capability to absorb vibration. However, no remarkable difference between skyhook and SWADC controls is observed from Fig. 19. Because the disturbance cancellation control is a controller that shows excellent performance for canceling harmonic disturbances, the SWADC control exhibits a performance similar to skyhook control for reducing the vibration of the initial value response. Note that through the experiments in which the flexible cantilever beams with AWD units are mounted on a vehicle pulled by a tractor, as shown in Fig. 17(15)–(17), we ascertained that the SWADC control has the most excellent capability to suppress the vibration caused by continuous disturbances. The time historical responses of control inputs with skyhook and SWADC controls are shown in...
Fig. 20. This figure shows the results during 0–4 s when control inputs are applied to compare the control inputs between skyhook and SWADC controls. Because these experiments indicate the responses against the initial beam displacement value, not a steady-state disturbance, the control input provided by disturbance cancellation control is superimposed on the control input by skyhook control; thus, the control input by SWADC control is larger than that by skyhook control after 1 s, as shown in Fig. 20.

6. Conclusions

This study developed an active mass damper unit for suppressing vibrations in structures such as flexible cantilever beams. The developed unit called an active wheel damper (AWD) uses the rotational motion of a flywheel and can be fixed simply onto flexible structures by mounting a gyro sensor and a servo motor on the unit. This study also developed a combined controller of skyhook control, which consists of the proportional feedback of the gyro sensor signal, and a disturbance cancellation control, which consists of the feedforward input of the estimated disturbances obtained by an adaptive observer for harmonic disturbances, to improve the capability to suppress vibrations. This study called the controller the Skyhook With Adaptive Disturbance Cancellation (SWADC) control. Using an adaptive notch filter to estimate the principal frequency of the vibration and realizing an adaptive disturbance observer by updating the natural frequency of the model for harmonic disturbance observer, an adaptive control system was constructed to cancel harmonic disturbances. By conducting numerical simulations of the vibration suppression of a cantilever beam with the developed AWD unit, this study evaluated the effects of the model parameters and the controller gain on the capability to suppress vibrations. Furthermore, this study confirmed the capability of the proposed AWD unit with SWADC by experiments. The SWADC control displayed an excellent capability to reduce vibration with suitable parameters estimated from the simulation results.

Acknowledgment

This research was supported by a Grant-in-Aid for Research on Priority Areas from Hokkaido prefecture, Japan.

References


