LQR Control for a Self-Balancing Unicycle Robot on Inclined Plane*

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Abstract
A self-balancing unicycle robot, which has a wheel for balancing and moving in the longitudinal plane (pitch angle) as well as a vertical flywheel for balancing in the lateral plane (roll angle), is studied in this paper. The non-linear dynamic equations of the unicycle robot on slope are analyzed using the Lagrangian dynamic formulation, then a linear model of the robot is derived at the equilibrium point, and linear quadratic regulators (LQR) are designed to control the robot on slopes with the angle of inclination varying from -11° to 11°. Simulation and physical experiment results validated that the unicycle robot can achieve good performance both on level plane and on slope. As far as we know, this is the first report of an autonomous unicycle robot moving and balancing on slope.

Key words: Self-Balancing Robot, Unicycle Robot, LQR, Modeling, Slope Climbing

1. Introduction
Human beings or other intelligent creatures can’t ride a unicycle if they are not trained, because a lot of skills are needed to ride a unicycle. The self-balancing unicycle robot is a kind of robot that simulates the behavior of a person riding a unicycle. Unicycle robot is a non-holonomic, non-linear, static unbalance system that has the minimal number of point contact to the ground, therefore it is a perfect platform for researchers to study motion and balance control.

Various unicycle robots have been developed in a number of studies, and several control systems for these robots have been proposed. All of the unicycle robots achieve both the movement and the pitch balance by driving the single wheel, but their mechanisms of lateral balance different from each other. In 1980, Ozaka et al invented a single wheel robot which has a long arm that stretches out right and left, by moving a mass along the arm it maintains the roll balance. This robot may be the first self-balancing unicycle robot though no approving experiment results achieved [1]. Schoonwinkel designed a self-balancing unicycle robot according to the principles of people riding a unicycle, which achieves motion and balance control of the robot by a horizontal flywheel simulating the movement of the rider twisting his torso and arms [2]. In 1990, Vos et al proposed a LQG controller to achieve the stability control of the robot of Schoonwinkel’s model [3]. In 2005, Dao et al developed a self-balancing unicycle robot, which achieves lateral stabilization by two gyroscopes acting as actuators [4]. In 2007, UniBot was developed in the University of California, San Diego, which combines both the ideas of "wheeled inverted pendulum" and "inertia wheel pendulum" and achieves lateral balance by controlling the vertical flywheel [5]. In 2009, Guo et al proposed a nonlinear dynamic model for a unicycle robot with a vertical flywheel and validated the model by simulations [6]. Ruan et al also proposed a dynamic model as well as a sliding mode control of a unicycle robot with a vertical
flywheel and validated them by simulations [7].

The self-balancing unicycle robot should be able to move not only on level plane but also on slanted ground, obviously the latter is much more complicated and difficult to realize. Seo et al studied the motion of a Wheeled Inverted Pendulum with considering tilted road as one of various road conditions, and validated their LQR control strategy by simulations [8]. Guo et al also proposed a gain-scheduling optimal fuzzy logic controller to control the same model of Seo’s and validated it by simulations [9]. In 2008, "Murata Girl" was introduced in the Japan Robot Exhibition (CEATECJAPAN2008) [10] and caught attention of the world, but now there is no report on her ability of slope climbing though her cousin “Murata Boy” can ride a bicycle on a slope.

In this paper, the motion and balancing control of a unicycle robot on an inclined plane is studied. Firstly, the unicycle robot is presented and its non-linear dynamic model is analyzed. Then a linear model of the robot is derived at the equilibrium point, and linear quadratic regulators (LQR) are designed to control the robot on slope with a large range of inclination. Finally, simulation and physical experiment results as well as conclusions will be provided.

2. The Unicycle Robot and its Dynamic Model

A. The Self-balancing Unicycle Robot

The self-balancing unicycle robot we studied is shown in Fig.1 [11]. The robot is controlled by the DSP2812. It has a Tin Woodman like appearance and consists of a wheel at its lower part and a vertical inertia flywheel at its upper part. The wheel is driven by a Maxon RE 35 DC servo motor plus a Planetary Gearhead with reduction 32:1, which can apply a nominal torque of 2.98N·m, to make the robot move and balance in the longitudinal plane, and the inertia flywheel is driven by another DC servo motor to achieve the lateral
balance of the robot, which imitates the action of a person swinging both arms from overhead down to the side he is falling to restore lateral balance when he is out of balance on a balance beam.

Fig. 2 shows a model of the unicycle robot on a slope. Coordinates are assigned to analyze its dynamics, in which, \((X,Y,Z)\) are global coordinates, and \((W1,W2,W3),(F1,F2,F3)\) and \((D1,D2,D3)\) are coordinates of the wheel, the frame and the flywheel respectively. \(\Phi\) is the point at which the wheel contacts the floor, and \(\alpha\) is the incline angle of the slope, and \(\beta\) and \(\phi\) is the roll angle and the pitch angle of the robot respectively. Other variables and parameters adopted in this paper are defined as follows: \(\omega\) and \(\eta\) are the rotating angle of the wheel and the flywheel respectively, and \(\Gamma_w\) and \(\Gamma_d\) are the torque put on the wheel and the flywheel respectively, and \(rm\) \(wm\) and \(l_d\) denote the mass of the wheel, the frame and the flywheel respectively, and \(r_r\) is the radius of the wheel, \(d_r\) is the distance between the center of gravity of the frame and the axis of the wheel, \(f_w\) is the Coulomb's friction between the rolling wheel and the concrete floor, and the friction factor is 0.022. The viscous friction in the joint of the unicycle robot in modeling is neglected because the wheel and the flywheel are well supported on the frame with ball bearings. The moment-of-inertia of the wheel about the axes \((W1,W2,W3)\), of the frame about the axes \((F1,F2,F3)\) and of the flywheel about the axes \((D1,D2,D3)\) are as follows:

\[
I_w = 0.0049kgm^2, \quad I_{w2} = 0.0098kgm^2, \quad I_{w3} = 0.0049kgm^2, \quad I_{f1} = 0.07kgm^2,
\]
\[
I_{f2} = 0.0072kgm^2, \quad I_{f3} = 0.0068kgm^2, \quad I_{d1} = 0.078kgm^2, \quad I_{d2} = 0.041kgm^2, \quad I_{d3} = 0.041kgm^2.
\]

B. Modeling of the Self-balancing Unicycle Robot

It is assumed that there is pure rolling and non-slipping between the wheel of the unicycle robot and the ground, and that the robot just moves forward and backward in line, i.e., the robot does not change its direction.

Taken \([\beta \phi \omega \eta x_p z_p]^T\) as the generalized coordinates, equations representing the motion of the unicycle robot on a slope with inclination \(\alpha\) can be derived using the Lagrange equations:

\[
\left[m_{all}r_u^2gc\sin\beta + m_{fi}\phi\sin\beta + \phi\sin\beta(2[m_{fi} + m_{fr})(c\phi\beta + I_{d1}\phi)]
\right]
\]
\[
= \left[I_{w1} + m_{fr}c^2\alpha + 2m_{fr}c\phi + (I_{f1} + I_{d1} + m_{fr})(c\phi\beta)^2
\right]
\]
\[
+ (I_{f2} + I_{d2} + m_{fr})(c\phi\beta)^2 + I_{d3}\phi\beta
\]
\[
\Gamma_w + \{m_{fr}c\alpha + (I_{d1} + m_{fr})(c\phi\beta)^2 + I_{d1}\phi\beta \phi + m_{fr}c\alpha \phi \phi
\}
\]
\[
+(I_{f2} + I_{fr} + m_{fr})(c\phi\beta)^2 + I_{d3}\phi\beta \]
\[
= \Gamma_w + f_wr_w
\]
\[
[I_{d4}\phi + I_{d3}\phi\beta - I_{d3}\phi\beta\phi = \Gamma_d
\]

Where,

\[
m_{all} = m_w + m_f + m_d, \quad m_d = m_f l_1 d_1, \quad m_f = m_f l_1^2 + m_f l_2^2, \quad m_c = (m_w + m_f + m_d)r_w^2,
\]
\[
I_A = I_{f1} - I_{f3} - I_{d1} - I_{d3}, \quad x_p - r_w\omega c\alpha = 0, \quad z_p - r_w\omega_s \alpha = 0, \quad s\theta \text{ and } c\theta \text{ denote } \sin\theta \text{ and } \cos\theta \text{ respectively.}
Define the state vector $X = \begin{bmatrix} \beta & \phi & \omega & \eta & \dot{\beta} & \dot{\phi} & \dot{\omega} & \dot{\eta} \end{bmatrix}^T$, and input vector $U = \begin{bmatrix} \Gamma_w & \Gamma_d \end{bmatrix}^T$, then the nonlinear equations of the unicycle robot can be rewritten as:

$$M(X)\dot{X} = F(X) + GU$$  \hspace{1cm} (1)

where,

$$M(X) = \begin{bmatrix} I_{4\times4} & \theta_{4\times4} \\ \theta_{4\times4} & M_{22} \end{bmatrix}, M_{22} = \begin{bmatrix} m_{11} & 0 & 0 & m_{14} \\ 0 & m_{22} & m_{23} & 0 \\ 0 & m_{32} & m_{33} & 0 \\ m_{41} & 0 & 0 & m_{44} \end{bmatrix}$$

$I_{4\times4}$ is a $4 \times 4$ unit matrix, and $\theta_{4\times4}$ is a $4 \times 4$ zero matrix.

$$m_{11} = I_{w1} + m_c c^2 \alpha + 2m_d r^c \phi c \alpha + (I_{f1} + I_{d1} + m_B) (c \beta)^2 + (I_{f3} + I_{d3}) (s \phi)^2$$

$$m_{14} = I_{d1} c \phi, m_{22} = I_{f2} + I_{d2} + m_B, m_{23} = m_d r^c c \phi c \alpha, m_{32} = r_m m_c c \phi c \alpha, m_{33} = I_{w2} + m_d l^c_r^2,$$

$$m_{41} = I_{d1} c \phi, m_{44} = I_{d1}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F(X) = \begin{bmatrix} m_d r^c g c \alpha s \beta + m_d g c \phi s \beta + \phi s \phi [2m_l r^c + (I_d + m_B) c \phi] \dot{\beta} + I_{d1} \phi \\ gm_c \beta s \phi - [m_l r^c c \alpha + (I_d + m_B) \phi] \dot{\beta} s \phi - I_{d1} \beta s \phi \\ m_d r^c c \phi c \alpha - m_d r^c g c \alpha + f \phi s \phi \\ I_{d1} \phi s \phi \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ \omega \\ \eta \end{bmatrix}$$

By linearizing the model at the equilibrium point and substituting the parameters with their value, we can get the linear model of the system in the standard form:

$$\dot{X} = AX + BU$$

$$Y = CX$$  \hspace{1cm} (2)

Though the matrices $A, B, C$ may vary a lot when different value of $\alpha$ is adopted, the controllability of the system can be validated.

3. LQR control and simulations

In the self-balancing unicycle platform, information of state vector $X = \begin{bmatrix} \beta & \phi & \omega & \eta & \dot{\beta} & \dot{\phi} & \dot{\omega} & \dot{\eta} \end{bmatrix}^T$ can be easily obtained by the attitude and heading reference system (AHRS) and the encoders attached on the DC servo motors, therefore, we can design a Linear Quadratic Regulator (LQR) to control the robot. By chosen weighting matrices $Q$ and $R$, we can design a full-state feedback control law $u(t) = -Kx(t) = -R^{-1}B^T P x(t)$ to minimize the objective function

$$J = \int_0^\infty (x^T Q x + u^T R u) dt$$

In the following simulations, $R = \text{diag}(1,1)$ is a constant matrix and $Q = \text{diag}(Q_{11}, Q_{22}, Q_{33}, Q_{44}, Q_{55}, Q_{66}, Q_{77}, Q_{88})$ will be specified according to the different range of the obliquity of the slope, i.e. $\alpha$. The robot using flywheel to control its balance in roll direction, its ability of restore balance in roll direction is limited, and the roll angular velocity $\beta$ is critical for the balance control of the robot, thus $Q_{55}$ should be much larger than the other elements in $Q$ and it will be adjusted to a correct value by simulations. The proposed control method is summarized in the schematic diagram shown in Fig.3. Finally, the full-state feedback control law designed according to the linear model will be
applied to the nonlinear model of the self-balancing unicycle robot in simulations and to the physical robot platform respectively.

A. Simulation of balancing and moving on level plane

Suppose that the unicycle robot is rest on a level plane (\(\alpha = 0^\circ\)) with the initial state \(X_0 = [0.2, 0.2, 0, 0, 0, 0, 0, 0]^T\), which means that the robot is statically placed on the ground and leaned forward and rightward 0.2rad respectively, and these two parameters (0.2rad) of lean are decided by the kickstand that is used to keep the unicycle robot stand when not operating. We can get the optimal state feedback matrix

\[
K = \begin{bmatrix}
53.6 & 1 & 0 & 0 & 10.1 & 1.5 & 0 \\
-812.2 & 0 & 0 & -213.8 & 0 & 0 & -2.8
\end{bmatrix}
\]

by specifying \(Q = diag(50, 50, 1, 1, 1, 1150, 5, 1, 1)\). The simulation result is shown in Fig.4.

From Fig.4 we can learn that the robot can achieve good balance and motion control performance on level plane. Subfigure (a) and (b) validate that the robot can achieve balance from the initial state to the upright position in 3.5 seconds and remain stable with the steady state error less than 2% by moving the wheel forward and back about 1 rad. Subfigure (c) and (d) validate that the robot can steadily move at the desired speed of 1rad/s (\(v=0.1m/s\)) while maintain longitudinal and lateral balance well.

B. Simulations of ascending and descending a slope with zero initial velocity

In this paper, we mainly study the motion and balance control of the unicycle robot on a slope. Simulations reveal that the robot can achieve good performance with the angle of the slope vary from \(-11^\circ\) (the robot descends a slope) to \(11^\circ\) (the robot ascends a slope) supposing that the robot is rest on the slope at the beginning of motion. The control performance will distinctly deteriorate when \(|\alpha| > 11^\circ\). This is caused by the large linearization error between the linear model and the non-linear model of the unicycle robot when \(|\alpha|\) is large, since we are designing the LQR controller according to the linear model and applying this controller to control the non-linear model.

A simulation result of the unicycle robot ascending a slope (\(\alpha = 5^\circ\)) is shown in Fig. 5. In this simulation, the initial state of the robot is supposed as \(X_0 = [0.2, 0.2, 0, 0, 0, 0, 0, 0]^T\), which means that the robot is statically placed on the slope and leaned forward and rightward 0.2rad respectively; and the matrix \(Q\) is specified as \(Q = diag(50, 50, 1, 1, 10000, 2, 1, 1)\), and the optimal state feedback matrix \(K\) is designed as

\[
K = \begin{bmatrix}
0 & -53.6 & -1 & 0 & 0 & -10.2 & -1.5 & 0 \\
-812.25 & 0 & 0 & -1.06 & -213.8 & 0 & 0 & -2.85
\end{bmatrix}
\]
Fig. 5 shows the process of the robot moving and balancing on a slope. In Subfigure (a) and (b), the robot achieves balance in the roll plane easily, and it achieves balance in the pitch plane from initial state by driving the wheel backward a special distance. Noting that the attitude of the robot balanced on slope is not in vertical, it leans about 0.24 rad forward instead. Subfigure (c) and (d) show the behavior of the robot moving up the slope at a given speed. It firstly keep balance in the roll plane, at the same time it drives the wheel backward a special distance to adjust its attitude. When its body is leaning forward about 0.34 rad, the robot will keep this attitude and move up the slop at the given speed ($\dot{\omega}=1$ rad/s, i.e. $v=0.1m/s$ in this example).
Fig. 5 Simulation of motion and balance when ascending a 5° slope

Fig. 6 is the simulation result of the unicycle robot descending a slope ($\alpha = -5^\circ$), in which, $X_0 = [0.2, 0.2, 0, 0, 0, 0, 0, 0]^T$, $Q = \text{diag}(50, 50, 1, 1, 10000, 2, 1, 1)$, and

$$K = \begin{bmatrix} 0 & -53.61 & -1 & 0 & 0 & -10.17 & -15 & 0 \\ -812.6 & 0 & 0 & -1 & -213.8 & 0 & 0 & -2.8 \end{bmatrix}$$

In Subfigure (a) and (b), the robot achieves balance in the roll plane easily, and it achieves balance in the pitch plane from initial state by driving the wheel forwards a special distance. Noting that the attitude of the robot balanced on slope is also not in vertical, it leans about 0.24 rad backward instead. Subfigure (c) and (d) show the behavior of the robot moving down the slope at a given speed from the initial state. It firstly keep balance in the roll plane, at the same time it drives the wheel forward a special distance to adjust its attitude. When its body is leaning backward about 0.34 rad, the robot will keep this attitude and move down the slope at the given speed ($\dot{\omega} = 0.1 \text{ rad } / \text{s}$, i.e. $v = 0.1 \text{ m } / \text{s}$).

Note that the attitude control of the unicycle robot ascending and descending a slope is similar to the attitude control of a person negotiating a slope, i.e. a person should also lean himself forward when he is walking up a slope and backward when he is walking down a slope. This result may validate our control strategy in a certain aspect.
C. Simulations of switching from slope to level plane and reverse

The unicycle robot should often ascend and descend slopes to fulfill a task, thus the performance of the robot ascending and descending a slope with changing gradient should be studied.

The simulation results shown in Fig.7 and Fig.8 are that the unicycle robot performs a 4-steps motion task, namely

1. The robot moves from level ground to a slope and ascends it;
2. The robot climbs up the slope and comes to a stage;
3. The robot leaves the stage and descends a slope;
4. The robot leaves the slope and downs to the ground.
In Fig.7 and Fig.8, \( t=0 \) is the time when the robot changes from one condition of the road to another. The desired velocity of the robot is \( v = 0.2 \text{ m/s} \) on both the ground and the stage, while the desired velocity of descending and ascending slope is \( v_s = 0.15 \text{ m/s} \). The desired velocity on a slope is specified slower than that on a level plane because the robot is more difficult to control on slope than on level plane. From the simulation results we can see that the robot can adjust its attitude according to the variation of the road conditions, and the varying curves of both the attitude and the velocity is smooth, and the varying range of both the attitude and the velocity is also acceptable.

Note that the attitude control of the unicycle robot is also similar to those of a person when the road condition is changed from level plane to a slope or vice versa. The robot can accordingly adjust its attitude when the road conditions are changed, for example, it will decelerate when it leaves the level plane to ascend a slope (Fig.7b), and accelerate when it leaves the slope and up to a stage (Fig.7d) as well as when it leaves a stage to descend a slope (Fig.8b), and brake and even go astern a little when it descends the slope to a level plane (Fig.8d). This result may validate our control strategy in a certain aspect.
4. Experiment on the physical robot platform

The aforementioned LQR control strategies are applied to the physical robot system (Fig.9), with the control parameters undergoes some fine-tuning in the experiment because of the modeling errors. The physical experiment results reveal that the robot can achieve good performance on condition that the angle of the slope vary from $-8.5^\circ$ (the robot descends a slope) to $8.5^\circ$ (the robot ascends a slope) on smooth surface of board, and the robot can ascend a concrete lane with the maximum inclination of $9^\circ$ gracefully.
A. Experiment of balancing and moving on level plane

The unicycle robot is rest on a level plane with the initial state leaned 0.2 rad backward and rightward respectively, and these two parameters of lean are decided by the kickstand used to keep the unicycle robot stand when not operating. We fine-tune the state feedback matrix as:

$$K = \begin{bmatrix} 0 & -53.5 & -1 & 0 & 0 & -10.1 & -1.6 & 0 \\ -806.2 & 0 & 0 & -1 & 212.8 & 0 & 0 & -2.6 \end{bmatrix}.$$  

In fact, the state feedback matrixes adopted in the following physical experiments are all the same, because we find that this set of parameters can achieve good performance both on level plane and on slope, though we supposed that different control parameters are needed for different inclination and had studied 3 controllers in simulation.

From Fig.10 we can learn that the robot can achieve good balance and motion control performance on level plane. Subfigure (a) and (b) validate that the robot can achieve balance from the initial state to the upright position in 2 seconds and remain stable with the steady state error less than 20% by moving the wheel forward and back about 0.05m. In fact, the unicycle robot can even keep balancing on the edge of an angle iron (Fig.1a). Subfigure (c) and (d) validate that the robot can steadily move on the level plane and maintain longitudinal and lateral balance well given a desired speed of 0.7 rad/s ($\nu = 0.07 \text{ m/s}$) at 4.5s. Different from the simulations, the robot can’t keep the pitch anger steadily and smoothly, it oscillates in the pitch direction instead, in the fact, the robot also oscillates in the roll direction but with smaller amplitude, because the robot keep balance dynamically with a small overshoot about 0.5°. From subfigure(c) we can see that the robot will lean forward about 1.5° when it moves forward, and this attitude will of course favor its moving.

B. Experiment of climbing slope to a stage and reverse

An experiment result of the robot climbing a slope of $\alpha = 3.5^\circ$ up to a stage and reverse is shown in Fig.11. Subfigure (a) and (b) validate that the robot can safely go to a stage from level plane by climbing a slope, while subfigure (c) and (d) validate that the robot can go to the level plane from the stage by descending a slope. The desired velocity of the robot is $\nu = 0.085 \text{ m/s}$ no matter on the ground, the stage, and slope. In subfigure (a) and subfigure (b), the robot was initially balancing on the ground, and it was given a desired velocity $\nu = 0.085 \text{ m/s}$ at 4.5s and began to move on the level plane until it
encountered the slope at 12s. Because the slope is made of a board, the thickness of the board also acts as an obstacle with height of 1.8cm at the edge of the board for the robot. As a result, the robot smoothly adjusts its attitude to lean forward about 9° and then began to ascend the slope at 18s. When the robot passed the slope and went to the stage, it stopped and kept balance on the stage. In subfigure (c) and (d) the robot was initially balancing on the ground, and it was given a desired velocity $v = 0.085 \, \text{m/s}$ at 6.5s and began to move on the stage until it encountered the slope $\alpha = -3.5^\circ$ at 17s, then the robot slightly slowed its speed and smoothly adjusted its attitude to $2.7^\circ$ backward to descend the slope. When the robot went down to the floor, it stopped according to the desired speed $v = 0\, \text{m/s}$ and kept balance on the floor. However the robot did not remain in an upright position, it leaned backward about $2.7^\circ$ and rightward about $2^\circ$ instead, though, the motion and balance performance of the robot still looks well.
In this experiment, the capability of the unicycle robot continuously ascending and descending slopes is studied, and the physical experiment results are presented in Fig.12, in which, the unicycle robot performed a 3-steps motion task, namely

1) The robot moved from level ground to a slope ( \( \alpha = 3.5^\circ \) ) and ascended it;
2) The robot climbed to the top of the slope and at once descended a slope with the same inclination;
3) The robot descended and left the slope and went down to the ground.

In Fig.12, the unicycle robot was initially balancing on the ground, and it was given a desired velocity \( v = 0.085m/s \) at 7s and began to move on the level plane until it encountered the slope, then, similar to the experiment results of (Fig.11a), the robot stopped and adjusted its attitude smoothly and began to climb the slope at the time of 17s. When the robot climbed up to the top of the slope it encountered another section of downhill slope, so it slowed down and adjusted its attitude backward smoothly and began to descend the slope. When the robot went down to the floor, it stopped according to the desired speed \( v = 0m/s \) and kept balance on the floor.
5. Conclusions

In this paper, a self-balancing unicycle robot, which has a wheel for balancing and moving in the longitudinal plane (pitch angle) as well as a vertical flywheel for balancing in the lateral plane (roll angle), is studied. The non-linear dynamic model of the unicycle robot on slope is analyzed using the Lagrangian dynamic formulation, and LQR optimal controllers are designed, and results of both simulations and physical experiments validated that the robot can achieve good performance of motion and balancing on level plane as well as slope with a large variation of inclination. As far as we know, this is the first report of an autonomous unicycle robot moving and balancing on slope.

It is noticeable that the robot can adjust its attitude according to the variation of the road conditions, and the varying curves of both the attitude and the velocity are smooth. Furthermore, the attitude control of the unicycle robot ascending and descending a slope is also similar to those of a person negotiating a slope, which will certainly favor its moving. In fact, the robot can not only negotiate a pathway with wavy surface, but also balance well on the edge of an angle iron. The robot shown its ability on the 6th China Beijing International Cultural & Creative Industry Expo and attracted attention of visitors and the news media, and China Daily and CCTV (China Central Television) news reported our unicycle robot on November 12, 2011.

There are still two drawbacks in the unicycle robot with vertical flywheel. Firstly, there is no direct force for changing the yaw direction of the robot and therefore it is difficult to make the robot to track a curve or to turn around; Secondly, the robot achieves lateral balance by the angular acceleration of the vertical flywheel, and its lateral balance ability is limited because the flywheel is easy to become saturated, as a result it can no longer provide lateral balance torque once the flywheel has reach its maximal angular velocity. These two are our primary research issues in future.
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