Review of Research on the Cracked Rotor in Japan*

Tsuyoshi INOUE**
**Department of Mechanical Science and Engineering, School of Engineering, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8603, Japan
E-mail: inoue@nuem.nagoya-u.ac.jp

Abstract
The studies of the cracked rotor demonstrated by Japanese researchers started in early 1980s and many papers have been reported. The characteristics of these Japanese papers are: A group of papers investigated the vibration of the practical machinery and the estimation of the position and depth of the crack was tried in the quite early time, Another group of papers used a simple rotor model and investigated the vibration caused by the crack theoretically considering the nonlinearity involved in the crack’s characteristic. In the following, these studies of the cracked rotor are divided in the groups, the studies on the vibration analysis of the cracked rotor using simple rotor model, the studies on the vibration analysis of the rotor system with the breathing crack, the studies on the modeling and detection method of the crack and case study of the cracked rotor, and reviewed in detail.

Key words: Cracked Rotor, Modeling of the Crack, Breathing Crack, Detection

1. Vibration Analysis of the Cracked Rotor
Most of the studies about the cracked rotor consider the transverse crack which is caused by the fatigue and is perpendicular to the shaft’s center line. It is well known that the super-synchronous components, particularly 2X and 3X components, occur in the bending vibration of the cracked rotor.

Shiraki et al. (1) used the Jeffcott rotor model, and investigated the variation of the bending stiffness for the cases with the ideal crack and the open crack. The influence of the crack on the component of each order of the rotational speed, nX, was shown by expanding the variation of the bending stiffness in Fourier series. Then, the fundamental characteristics of the vibration due to the gravitational force, the vibration due to the unbalance, and the vibration in the case with both gravitational force and the unbalance were investigated. Also, the rotor system with a crack at an arbitrary position and arbitrary phase was considered for the quantitative analysis of the practical cracked rotor, and the numerical analysis using the transfer matrix was demonstrated. The anisotropic coupled characteristic of the journal bearing was considered for the modeling of the shaft's support, and 0X, 1X and 2X vibration components were analyzed under the action of both gravitational force and the unbalance. It was indicated that the rotor crack caused 1X and 2X vibration components under the action of gravitational force, the unbalance response was influenced by the phase of unbalance, and the unbalance affected on the open and closed behavior of the crack. The experimental system shown in Fig. 1 was used, and the numerical and experimental results were compared. Fig. 2 shows an example of those comparisons, and it was explained that the test rotor was the cracked rotor which had some amount of open and closed behavior.
from the observation of the change of the vibration characteristics for the phase of unbalance. It was also demonstrated that both 1X and 2X vibration components of the cracked rotor showed various kinds of characteristics as the results of mutual effects of the gravitational force (static load), magnitude and phase of the unbalance, the vibration mode of the rotor in the rotating condition and so on. As a result, the importance of the precise analytical method of the practical rotor system was emphasized.

Ichimonji et al. (2) noted that the torsional vibration easily occurs in DSS (Daily Start and Stop) operation or thyristor drive, and the fatigue crack may be caused by this torsional vibration. This crack caused by the torsional vibration appears showing 45 degree from the shaft center line, and behaves open-closed behavior relating to the torsional vibration as shown in Fig. 3. They derived the equation of motion of the cracked rotor caused by the torsional vibration as the parametric excitation system under the assumption that the shaft's bending stiffness varies in the frequency of the torsional vibration. The bending vibration of this rotor system was qualitatively investigated from the numerical calculation.

Iwata et al. (3)(4) developed the experimental setup (Fig. 4) of the cracked rotating shaft system in which the open-closed behavior of the crack was realized and its depth was controllable. The resonance curves were observed for the various depth of crack, and the super harmonic resonance of order 1/3 also occurs by the influence of the crack as well as
the super harmonic resonance of order 1/2 as shown in Fig. 5. They also reported the occurrence of the super sub harmonic resonance of order 3/2 when the crack is deep (Fig. 5). Furthermore, the open-closed behavior of the crack for the shaft rotation was modeled as
Fig. 8 Unstable ranges due to crack (4)

Fig. 9 Resonance curve of the cracked rotor (4)

Fig. 10 Theoretical model (5) Fig. 11 Natural frequency and unstable ranges (5)

shown in Fig. 6. The variation of shaft's bending stiffness as the function of the shaft rotation angle was theoretically represented and compared with the experiment (Fig. 7). Furthermore, the cracked rotor model with the derived bending stiffness model was used, and the unstable region due to crack (Fig. 8) and the resonance curve (Fig. 9) were analyzed, and the experimental results were explained.

Ikeda et al. investigated the vibration characteristics of the asymmetric shaft system. The asymmetric rotating shaft is not the cracked rotor, but it shows the same characteristics of the rotor system with an open crack. They also considered the asymmetric stiffness characteristic in the support system (Fig. 10). The unstable range around the major critical speed was clarified theoretically (Fig. 11), and confirmed experimentally. The influences of the asymmetry (corresponding to the depth of the crack), mass at the support base, and the stiffness of the base on the unstable range are clarified theoretically. The influences of the imbalance and the gravitational force on the resonance curve are also clarified theoretically, and compared with the experiment.
Ishida et al. utilized a simple rotor model considering the breathing crack, investigated a series of resonances caused or influenced by the crack using nonlinear theoretical analysis, and clarified their characteristics in detail (6)-(11). They firstly used the experimental setup as shown in Fig. 12(a), which is the vertical simple rotating shaft system in which an imitation breathing crack was realized (6). Fig. 12(b) shows the variation of the restoring force characteristic for the shaft deflection in the crack direction, and Fig. 12(c) shows the variation for the direction of the shaft deflection. The influence of the unbalance on the amplitude and the phase of the resonance curve at the major critical speed was investigated.
Ishida et al. also investigated the double frequency vibration (9). They used their own developed 2DOF inclination model shown in Fig. 15, and considered the characteristic of the breathing crack in the restoring force characteristic. The equation of motion of this model with the breathing crack was derived as shown in Eq.(1), and the underlined parts were the terms caused by the influence of the crack. They performed the nonlinear theoretical analysis of the double frequency vibration, and its theoretical result was confirmed by comparing with the numerical simulation as shown in Fig. 16. Then, they indicated that the unstable rotational speed range did not occur regardless of the phase between the unbalance and the crack direction, which was different from the case of the major critical speed (7).
Fig. 15  Theoretical model, coordinate systems, and the restoring force characteristic of the shaft with breathing crack \(^{(9)}\)

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\begin{align*}
\ddot{\theta}_y + i\omega\dot{\theta}_y + c\dot{\theta}_y + \Delta(\theta, C_y + \theta, S_x) + N_x &= M\cos(\omega t + \alpha) \\
\ddot{\theta}_y - i\omega\dot{\theta}_y + c\dot{\theta}_y + \Delta(\theta, S_x - \theta, C_y) + N_y &= M\sin(\omega t + \alpha) + M_0 \\
N_y &= 3\left(\pi_y^{(1)}C_y - \pi_x^{(1)}S_x + \pi_x^{(1)}C_x + \pi_y^{(1)}S_y\right)\theta_x^2 \\
&\quad + 2\left(\pi_y^{(1)}C_y + \pi_x^{(1)}S_x + 3\pi_y^{(1)}C_x + 3\pi_x^{(1)}S_y\right)\theta_y^2 \\
C_i &= \cos k\omega x, \\
S_i &= \sin k\omega x,
\end{align*}
\]

(1) \(^{(9)}\)

Fig. 16 Numerical and analytical results of the resonance curves and the variations of the center position (constant component in the vibration) of the double frequency vibration for the change of the rotational speed: (a)-(d) show the influence of the phase between the directions of crack and unbalance \(^{(9)}\)
Nonlinear resonances due to a breathing crack were also investigated theoretically and their characteristics were clarified by Ishida et al. (10). In the studies before this paper (10), the linearized models were used and only the unstable rotational speed range was investigated for such resonances (3). In this paper (10), the steady state oscillation of such resonances were analyzed theoretically by considering the nonlinearity due to the breathing crack, the characteristics of the resonance curves were explained, and confirmed by the experiment. As an example, Fig. 17 shows the resonance curve of the super sub harmonic resonance of order 3/2, and Fig. 18 shows the resonance curve of the sub harmonic resonance of order 1/2, which are both the resonances caused by the breathing crack. The qualitative correspondence of the numerical and theoretical results was indicated, and the influences of the magnitude of the unbalance and the phase between the unbalance and crack were explained.

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![Fig. 17](image1.png)
**Fig. 17** Theoretical result of the resonance curve of the super sub harmonic resonance of order 3/2 and the influence of the magnitude and the direction of the unbalance on it (10)

![Fig. 18](image2.png)
**Fig. 18** Theoretical result of the resonance curve of the sub harmonic resonance of order 1/2 and the influence of the magnitude and the direction of the unbalance on it (10)

![Fig. 19](image3.png)
**Fig. 19** Numerical simulation result of the rotor system with a breathing crack which shows the occurrence of various nonlinear resonances due to crack (11)
They also investigated numerically the various kinds of nonlinear resonances caused by the breathing crack and showed them in one figure as shown in Fig. 19 \(^{(11)}\). In this paper \(^{(11)}\), the influence of the internal resonance caused by the nonlinearity due to the breathing crack was also considered, these nonlinear resonances were analyzed theoretically, and the characteristics of them were clarified.

2. Modeling and Detection Method of the Crack

Shiraki et al. \(^{(1)}\) considered three crack models: the real crack model which represented smooth variation of the shaft stiffness for the shaft rotating angle, the ideal crack which represented the crack's open and closed behavior with the step function, and the open crack as shown in Fig. 20. The open and closed behavior and the variation of the bending stiffness for the shaft's rotational angle under the action of the gravitational force were shown. Two detection methods of crack size and axial position were explained; one was the method using natural frequency analysis and the excitation test, and the other was the method using the static deflection analysis and the statistic deflection test. The former is the modified method reported by Mayes and Davies \(^{(12)}\). Crack estimation of the test rotor were performed using these methods, and the estimation results of the size and position of the crack were explained.

Inagaki et al. \(^{(13)}\) diagnosed the crack in the practical machine of the wind-tunnel fan which showed the occurrence of large amplitude vibration. They compared the measured and calculated vibration responses of the rotor. Fig. 21 shows the figure of the wind-tunnel fan shaft and the modal response of 2X component, respectively. The maximum amplitude at the shaft center part reached to 4mm at 465rpm, and the dominant vibration component was 2X. Then, they measured the static deflection and natural frequency with rotating the shaft statically, and found that the shaft's bending stiffness decreased at a certain phase of the shaft's rotational angle, which indicated that the breathing crack occurred. After the detail inspection of the shaft, a deep transverse crack was detected with the angle and the depth shown in Fig. 22(a). They also estimated the position and the depth of the crack of this shaft by using the method by Shiraki et al. \(^{(1)}\), and the estimation result for the crack position shown in Fig. 22(b) corresponded to the measured actual position. The vibration response of the rotating shaft due to the gravitational force was theoretically obtained considering the breathing crack at the estimated size and position. The theoretical result of the 2X vibration component at around the critical speed of 2X was shown in Fig. 22(c) which corresponded to the experimental result shown in Fig. 21, and it clarified the validity of the theoretical analysis.
Fig. 21 Rotating shaft and modal response diagram of 2X vibration

Fig. 22 Detection of the crack (a) Detected crack’s depth and direction (b) Curve for detecting crack position (c) Modal response of 2X component at the resonance frequency of 2X
Toyota et al. \cite{14} proposed the detection method of the crack by investigating the measure of the complexity involved in the vibration of the cracked rotor. They utilized the method for chaotic dynamics in order to measure the complexity. The rotor model used in this study was the one used by Iwata \cite{3,4} and the simulated experimental data for various kinds of crack depth which also contained several levels of rate of noise were prepared. The detection capability of the crack by using Lissajous diagram, phase plane of vibration signal $(x - \dot{x})$, and the phase plane of vibration velocity $(\ddot{x} - \dot{y})$ were investigated as shown in Fig. 23. As a result, it is indicated that the detection capability of the crack using phase plane was high even if the data contains certain level of noise and it was useful for the detection of the crack in the early stage.

Iwatsubo et al. \cite{15} investigated the method of crack detection by adding the external excitation to the cracked rotor. They used the simple rotor model shown in Fig. 24 which was based on the Jeffcott rotor model and considered the crack term proposed by Mayes and Davis \cite{16}. That crack term involved the representation of the open and closed behavior of the crack. The vibration components due to crack, when the natural angular frequency was $\omega_0$, rotational angular speed of the shaft was $\dot{\omega}$, and the angular frequency of the external excitation was $\omega_1$, were investigated. As the result, it was clarified that the vibration components of $\omega_0 \pm \dot{\omega}$, $\omega_1 \pm 2\dot{\omega}$, $\omega_3 \pm 3\dot{\omega}$, etc occurred when the periodic external force was added, and the vibration components of $\dot{\omega}_0 \pm \omega$, $\dot{\omega}_3 \pm 2\omega$, $\dot{\omega}_5 \pm 3\omega$, etc occurred when the impact force was added. The numerical simulation using the FEM model shown in Fig. 24 and the experiment were performed to confirm these results. Fig. 25 and Fig. 26 show the numerical and experimental results, and both show the occurrence of the expected vibration components.

![Skewness characteristics for various cracked conditions (Noise rate = 5%)](image)

**Fig. 23** Skewness characteristics for various cracked conditions (Noise rate = 5\%) \cite{14}
Ishida and Inoue (17) investigated theoretically the resonance curve of the vibration component due to crack shown by Iwatsubo (15) under the action of the periodic external force. They used the simple 2DOF rotor system, the natural angular frequencies were $\omega_f > 0, \omega_h < 0$, the rotational speed was $\omega$, and the external force frequency was $\Omega$. They indicated that the nonlinear resonance due to crack occurred around at the excitation frequency $\Omega$ which satisfied the relationship $m\Omega + n\omega = p_f$ or $m\Omega + n\omega = p_h$ $(m,n = \pm 1, \pm 2, \ldots)$ as shown in Fig. 27. The vibration characteristic of each resonance was explained by the nonlinear theoretical analysis. It was clarified analytically that the resonance $\Omega + 2\omega = p_f, p_h$ occurred in the rotating shaft with the open crack (shown in Fig. 27 with the symbol of square), however, other resonances occurred only if there was the
breathing crack (shown in Fig. 27 with the symbol of open and solid circles).

Ishida and Lu \(^{(18)}\) investigated the detection method of the rotor crack by using the change of the characteristic due to crack in the nonstationary oscillation during the passage of the critical speed. They used 2DOF inclination model \(^{(9)}\) with acceleration terms and demonstrated numerical analysis. Fig. 28 shows the relationship between the maximum amplitude and the angular acceleration during the passage of the major critical speed and the critical speed of the subharmonic resonance of order 1/2. The former result explains that the maximum amplitude depends on the relative phase angle relationship between the unbalance and the crack. The later result explains that the maximum amplitude of the nonstationary oscillation during the passage of the critical speed of the subharmonic

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Fig. 27  Resonance curves at rotational speed \(\omega = 3.0\) and the frequencies variations of resonances \(^{(17)}\)

(a) major critical speed  (b) critical speed of the subharmonic resonance of order 1/2

Fig. 28  Variation of the maximum amplitude on the nonstationary oscillation during the passage through critical speed for the acceleration \(\lambda\) \(^{(18)}\)
oscillation or combination oscillation was affected by both the relative phase angle relationship between the unbalance and the crack and the initial angle of the unbalance in the acceleration. Therefore, it was indicated that several trials were necessary to detect the rotor crack if the vibration characteristics in these nonstationary oscillation during the passage of the critical speed of the subharmonic or combination oscillations were utilized.

Inoue et al. (19) applied the crack model which was reported by Christides and Barr (20) or Sinha et al. (21) in the rotor system, and developed the 1D-FEM rotor model with an open crack which was applicable for the quantitative analysis. Fig. 29(a) shows an example of the variation of the area moment of inertia around the crack position in such model, and its

![Fig. 29](image)

Fig. 29 Modeling of the Crack (a) Variation of the area moment of inertia around the crack position (b) representation of the parameter as the function of the depth of the crack (for crack direction) (19)

![Fig. 30](image)

Fig. 30 Comparison of the theoretical and experimental resonance curves of the double frequency vibration due to crack (22)
mathematical representation was considered. Here, the representation of the parameter, which is independent of the shaft diameter or depth of the crack, were investigated and derived by using the numerical data derived from the 3D-FEM software and confirmed by the experimental data (symbols) as shown in Fig. 29(b). As a result, the parameters of the crack model are represented in the function of the shaft diameter $d$ as:

$$l_{wey} = 2.10d, \ l_{wey} = 1.32d$$  \hspace{1cm} (2) \hspace{1cm} (19)$$

Furthermore, Inoue et al. (22)-(24) utilized the 1D-FEM crack model (19) and investigated the double frequency vibration due to crack quantitatively with considering the influence of the directional difference of the bearing support characteristic. They showed the analytical expression of the vibration, and validated it by comparing with the experimental data quantitatively as shown in Fig. 30. They also considered the resonance caused by the coexistence of the open crack and the external excitation force, which were reported qualitatively in their previous paper (17), and investigated its vibration characteristic quantitatively (25) by utilizing the 1D-FEM crack model (19). They consider the case with the directional difference in the bearing support characteristic and derived the analytical expression of the resonance due to crack, and validated it by comparing with the experimental data quantitatively (25).

3. Evaluation of the Fatigue Crack Propagation

Morita et al. (26) pointed out that the fatigue crack at the low pressure (LP) turbine’s rotor groove may has been caused by the synergy of both corrosion and the cycled tension-compression strain at the start and stop of machinery. They evaluated the corrosion fatigue crack propagation behavior at Christmas-tree-type LP turbine’s rotor groove shown in Fig. 31(a)(b) utilizing the FEM analysis and the theory of the fracture mechanics. They performed over 100 cases of FEM analysis for various kinds of numbers and length of crack, the gap size between the rotor groove and the inserted blade root, and the range of strain (in the case with no crack) and the stress intensity factor (in the case with crack) at the start and stop of machinery. They also developed the approximated expression of the strain range and the stress intensity factor for the values of the crack length and the gap size as parameters. Then, the initiation and the propagation behavior of the crack for various cases of gap sized were analyzed by using the developed expression. Furthermore, they investigated the influence of the gap size on the propagation behavior of the crack, and they developed an estimation method of the remaining life based on the propagation behavior of the crack. Fig. 31(c) shows the evaluated relationship between the crack length at the 3rd hook and the life consumption. In this figure, the life consumption is less than 0.45-0.5 if no crack is detected.
at the 3rd hook which means that the machinery has the remaining life longer than the consumed life time until then.

Matake et al. (27) focused the torsional load in the LP turbine rotor, and investigated the influence of the cyclic shear strain on the initiation and the propagation of the fatigue crack. They also investigated the shape of the fatigue crack caused by the torsional behavior. It is reported that the life time until the initiation of the fatigue crack due to the torsional motion is shorter than that due to the bending motion, but the life time until the fracture of the crack due to the torsional motion is longer than that due to the bending motion.

4. Case Studies and Other Topics

Arai (28) summarized and reported the examples of crack caused in the various practical machinery or mechanical elements. Fig. 32(a) shows the fatigue crack caused in the drive shaft of the large rotating machinery of bending fatigue test machine, and Fig. 32(b) shows the picture of the fatigue crack at the root of the assembled part of propeller which shows clear propagation of the crack. Fig. 33 shows the crack in the propeller shaft caused by the torsional vibration, and these cracks are 45 degree from the shaft center line due to torsional vibration as discussed in Fig. 3 (2).

Yoshida (29) researched the accidents and their countermeasures so far in the steam turbine rotor, and summarized them with classifying in the manufacturing time and operating time. The initiation of the crack caused by the low cycle heat fatigue and its prevention measure are also summarized (Fig. 34).

Fig. 32 Pictures of the drive shaft of the rotating machinery with cracked part, and Crack propagation from the assembled part of propeller (28)

Fig. 33 Crack of the propeller shaft caused by the torsional vibration (28)
Fig. 34 Crack occurred at the heat groove of the rotor and the hardness distribution around that (29)

References