【Review Paper】

Review of Research on Nonlinear Rotordynamics in Japan*

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Abstract

This article introduces research on nonlinear rotordynamics in Japan. In most cases, the nonlinearity treated there was weak and it was approximated by a power series of low order. Firstly, the research on various nonlinear resonances due to the weak nonlinearity is introduced. Next, those on more complicated nonlinear phenomena, such as entrainment, internal resonances, chaotic vibrations, nonstationary vibrations, and more complicated systems, such as cracked rotors and continuous rotors, are explained. Then, research on phenomena due to strong nonlinearity is introduced. Various vibration suppression methods where the nonlinearity plays an important role are also explained. Finally, studies on the models representing helicopters and wind turbines are explained.

Key words: Rotordynamics, Nonlinear Resonance, Self-Excited Oscillation

1. Introduction

As rotors became lighter and their operational speeds became higher, the occurrence of nonlinear resonances became a serious problem. In the history of rotordynamics starting with Rankine's paper⁽¹⁾ in 1869, research on nonlinear rotordynamics is comparatively new. It started after World War II. There are three pioneers in this field. In 1955, Yamamoto reported on a subharmonic resonance due to ball bearings^{(2),(3)} and on resonance curves of a rotor with a radial clearance^{(4),(5)}. In the 1960's, Tondl⁽⁶⁾ studied nonlinear resonances due to oil film in journal bearings. In 1966, Ehrich⁽⁷⁾ reported on a subharmonic resonance observed in an aircraft gas turbine with squeeze-film damper bearings.

In this article, research on nonlinear rotordynamics in Japan is introduced. Concerning the works by foreign researchers, only the historically important pioneering works are introduced to show the start of the research in each field. Since many of the papers introduced in this review were written in Japanese, it is imagined that these papers were not widely read outside Japan. The author hopes that this article helps to introduce the development of nonlinear rotordynamics in Japan.

2. Resonances in Symmetrical Rotor Systems with Weak Nonlinearity

2.1 Steady-State Resonances

When an elastic rotor is supported by ball bearings, the restoring force has nonlinear spring characteristics due to the clearance in bearings. Figure 1 shows a measurement of a clearance in bearing⁽⁵⁾. In bearing engineering, the terminologies "a radial clearance" and "a thrust clearance" are generally used. However, Yamamoto used the terminology "an angular clearance" based on such measurements. When an elastic shaft is supported by ball

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bearings, the system has weak nonlinearity and the spring characteristics can be approximated by power series of low order. Figure 2 shows resonance curves obtained in such a system⁽⁸⁾. In this setup, an elastic shaft with a disk is supported by a double-row self-aligning ball bearing and a single-row deep groove ball bearing at both ends. The notations p_1, p_2, p_3, p_4 represent the natural angular frequencies and ω represents the rotational speed. The notations $\omega = p_2$, $\omega = 2p_2$ and $\omega = mp_i - np_j$ represent the relationship when harmonic, subharmonic and combination resonances occur, respectively. In this setup, nonlinear resonances caused by the second and third powers were observed. Yamamoto studied a subharmonic resonance⁽²⁾ and combination resonances⁽⁹⁾⁽¹⁰⁾. He named the latter a summed-and-differential harmonic resonance. He discussed these resonances analytically using normal coordinates⁽¹¹⁾⁻⁽¹³⁾, and investigated experimentally in detail⁽¹⁴⁾.

Thereafter, Yamamoto and his colleagues investigated subharmonic resonances and combination resonances⁽⁸⁾⁽¹⁵⁾⁽¹⁶⁾ and super-combination resonances⁽¹⁷⁾. The intensity of nonlinearity depends on the kind of bearings. They observed various nonlinear resonances due to the quadratic nonlinearity when they used a double-row deep-groove ball bearing⁽¹⁸⁾.

There are two notable results in these papers. One is the theoretical analysis using normal coordinates.⁽¹¹⁾⁻⁽¹³⁾ The transformation formula from physical coordinates to normal coordinates in rectilinear system cannot be applied to rotor systems with gyroscopic moments. Yamamoto derived the formula from a physical insight based on modal shapes. Let (x, y) and (θ_x, θ_y) be deflections and inclinations of a disk and (X_1, X_2, X_3, X_4) be normal coordinates. Then, the translation formulas are given by

$$\begin{aligned} x &= X_1 + X_2 + X_3 + X_4, & y &= -\frac{\dot{X}_1}{p_1} - \frac{\dot{X}_2}{p_2} - \frac{\dot{X}_3}{p_3} - \frac{\dot{X}_4}{p_4}, \\ \theta_x &= -\frac{1-p_1^2}{\gamma} X_1 - \frac{1-p_2^2}{\gamma} X_2 - \frac{1-p_3^2}{\gamma} X_3 - \frac{1-p_4^2}{\gamma} X_4 \\ \theta_y &= -\frac{1-p_1^2}{\mathcal{P}_1} \dot{X}_1 - \frac{1-p_2^2}{\mathcal{P}_2} \dot{X}_2 - \frac{1-p_3^2}{\mathcal{P}_3} \dot{X}_3 - \frac{1-p_4^2}{\mathcal{P}_4} \dot{X}_4 \end{aligned} \quad (1)$$

where γ is the spring constant of the coupling terms.

Fig.1 Measurement of clearance in a bearing⁽³⁾

Fig.2 Resonance curves obtained in an experiment⁽⁸⁾.

The other is the prediction of the occurrence of nonlinear resonances⁽¹⁵⁾⁽¹⁶⁾. Through many experiments, Yamamoto and his colleagues experienced that some kinds of nonlinear resonances appear often but other are not. They⁽¹⁵⁾⁽¹⁶⁾ explained this fact as follows. Let the deflection of the Jeffcott rotor be (x, y) . If the restoring forces are approximated by power series up to the third power, the corresponding potential energy is given by

$$V = \frac{1}{2}(x^2 + y^2) + \varepsilon_{30}x^3 + \varepsilon_{21}x^2y + \varepsilon_{12}xy^2 + \varepsilon_{03}y^3 + \beta_{40}x^4 + \beta_{31}x^3y + \beta_{22}x^2y^2 + \beta_{13}xy^3 + \beta_{04}y^4 \quad (2)$$

The distribution of this potential energy has the shape shown in Fig.3 which deviates slightly from the rotating paraboloid $V_0 = (x^2 + y^2)/2$ corresponding to a linear system. By $x = r \cos \varphi$, $y = r \sin \varphi$, Eq.(2) is transformed to a polar coordinate expression.

$$V = \frac{1}{2}r^2 + (\varepsilon_c^{(1)} \cos \varphi + \varepsilon_s^{(1)} \sin \varphi + \varepsilon_c^{(3)} \cos 3\varphi + \varepsilon_s^{(3)} \sin 3\varphi)r^3 + (\beta^{(0)} + \beta_c^{(2)} \cos 2\varphi + \beta_s^{(2)} \sin 2\varphi + \beta_c^{(4)} \cos 4\varphi + \beta_s^{(4)} \sin 4\varphi)r^4 \quad (3)$$

This shows that the potential energy consists of the components whose magnitudes change 1, 2, 3, 4 times, respectively, and the constant component. When only one of these components exists, the equipotential line (that is, the horizontal cross section) of V has the shapes shown in Fig.4. Yamamoto and Ishida⁽¹⁵⁾⁽¹⁶⁾ succeeded to predict the possibility of the occurrence nonlinear resonances using these components. For example, the resonance curve of a forward subharmonic resonance of order 1/2 is given by⁽¹⁶⁾

$$\left\{ 1 + i_p \omega \left(\frac{\omega}{2} \right) - \left(\frac{\omega}{2} \right)^2 + 4\beta^{(0)}(R_0^2 + 2P^2) \right\}^2 + \left(\frac{c\omega}{2} \right)^2 = 4\varepsilon^{(1)2}P^2 \quad (4)$$

where i_p is the ratio of the polar moment of inertia to the diametral moment of inertia of the rotor, R_0 and P are the amplitudes of the subharmonic and harmonic components respectively. In this equation, only $\beta^{(0)}$ and $\varepsilon^{(1)}$ are included. Figure 5 shows influences of these nonlinear components. The component $\beta^{(0)}$ determines the inclination of the resonance curves and $\varepsilon^{(1)}$ determines the magnitude. In the case of a backward subharmonic resonance of order 1/3, $\beta^{(0)}$ and $\beta^{(4)}$ are included and $\beta^{(4)}$ determines its magnitude. From the shapes of these components shown in Fig.4, it is considered that $\varepsilon^{(1)}$ with a simple shape will appear more often than $\beta^{(4)}$ with a complicated shape. This

Fig.3 Potential energy⁽¹⁵⁾

Fig.4 Equipotential line⁽¹⁵⁾

Fig.5 Influences of nonlinear components $\beta^{(0)}$ and $\epsilon^{(1)}$ to resonance $+\omega/2$

means that a forward subharmonic resonance of order 1/2 will occur more often than a backward subharmonic resonance of order 1/3.

All the above-mentioned research studied vertical shaft systems where the effect of the gravity is negligible. Yabuno and his colleagues investigated a harmonic resonance⁽¹⁹⁾⁽²⁰⁾ and a subharmonic resonance⁽²¹⁾ in horizontal Jeffcott rotor with the effect of gravity.

Tamura and Taniguchi⁽²²⁾ investigated a rigid rotor supported by ball bearings and reported nonlinear vibrations in the axial direction.

It is known that, when several external forces work to a nonlinear system simultaneously, other oscillation with a frequency equal to their summation or difference occurs. This is called a combination tone. Yamamoto et al.⁽²³⁾⁽²⁴⁾ studied various kinds of combination tones in a rotor system.

2.2 Unstable oscillations

Unstable oscillations occur due to nonlinearity even in a symmetrical rotor system. Ishida et al.⁽²⁵⁾ reported that, when an symmetrical shaft is supported by double-row deep-groove ball bearings, the quartic nonlinearity appears appreciably and unstable oscillations occur.

Yamamoto et al.⁽²⁶⁾⁽²⁷⁾ reported that, when an elastic shaft has an initial bend, the asymmetrical nonlinearity which rotates with the shaft appears and, as a result, the following unique phenomena appears. Let us consider rotating coordinate system O- $x'y'$ where the x' -axis is located in the direction of bend. Depending on the quadrant where the unbalance exists, four kinds of resonance curves appear. Figure 6 shows two of them. Let β be the angle between the unbalance and the x' -axis, A wide unstable range appears when the unbalance is located in quadrant II ($90^\circ < \beta < 180^\circ$) but it disappears when it is located in quadrant IV ($270^\circ < \beta < 360^\circ$).

Fig.6 Resonance curves of a rotor with an initial bend⁽²⁶⁾

2.3 Internal Resonance

The Jeffcott rotor is a simple model which is widely used in the linear analysis. However, this model shows the complicated characteristics when it has nonlinear spring

characteristics. In the Jeffcott rotor, the forward natural frequency p_f and the backward natural frequency p_b have the relationship $p_f : p_b = 1 : (-1)$. When natural frequencies have some specific relationship, it is called that the system has the internal resonance. Ishida et al.⁽²⁸⁾ investigated the harmonic resonance and subharmonic resonances of orders $1/2$ and $1/3$ and observed complicated resonance curves and almost periodic motions. Figure 7 shows a resonance curve at twice the major critical speed.

Ishida et al.⁽²⁹⁾ also reported an internal resonance phenomenon at subharmonic resonance when $p_2 : p_3 = 2 : (-1)$ holds there.

Fig.7 Subharmonic resonance under the effect of internal resonance⁽²⁸⁾

2.4 Chaotic Vibrations

Chaotic Vibrations in a rotor system was first observed by Ehrich⁽³⁰⁾ in an aircraft gas turbine engine which has strong nonlinearity due to squeeze film damper bearings. He also showed theoretically that chaotic vibrations occur in the transition zone between adjacent subharmonic resonances.

Inoue and Ishida⁽³¹⁾⁽³²⁾ reported that chaotic vibration may occur even in a rotor system with weak nonlinearity due to the effect of internal resonance. Figure 8(a) shows a resonance curve in the vicinity of the major critical speed and Fig.8(b) shows a time history, spectra and a Poincare map of a chaotic motion at point A.

Fig.8 Vibration effected by an internal resonance at the major critical speed⁽³¹⁾

3. Nonstationary Oscillations

Nonstationary phenomena during passage through critical speeds have been studied since Lewis⁽³³⁾ reported his study on the Jeffcott rotor in 1932. Numerous reports on this topic have been published, but most of them are studying about linear systems. About the nonlinear systems, Mitropol'skii⁽³⁴⁾ applied the asymptotic method to the major critical speed and Evan-Iwanowski⁽³⁵⁾ and Agrawal⁽³⁶⁾ applied the same method to summed-and-differential harmonic oscillations. In Japan, Ishida et al. investigated the characteristics of nonstationary oscillations passing through the major critical speed⁽³⁷⁾, a subharmonic oscillation of order $1/2$ ⁽³⁸⁾, that of order $1/3$ ⁽³⁹⁾, and a summed-and-differential

harmonic oscillation⁽⁴⁰⁾. The complex FFT method developed by them is a very useful tool to these studies. They also compared the accuracy of the asymptotic method and the complex FFT method⁽⁴¹⁾. Figure 9 shows amplitude variation curve during passage through a subharmonic resonance for two slightly different angular acceleration λ . As shown in this figure, the maximum amplitude $r'_{\max}$ changes remarkably for a very small change of acceleration in a nonlinear system. Ishida and Inoue⁽⁴²⁾ studied the influence of the internal resonance to a nonstationary oscillation during passage through the major critical speed.

Kobayashi et al.⁽⁴³⁾ studies the nonlinear transient response of an aircraft engine due to the loss of a fan blade.

Fig.9 Amplitude variation during passage through the subharmonic resonance⁽³⁸⁾

4. Asymmetrical Shafts and Asymmetrical Rotors

Nonlinearity has influence to resonances of asymmetrical shafts with directional difference in stiffness and those of asymmetrical rotors with directional difference in moment of inertia in the following two ways. The first is the disappearance of unstable oscillations due to the nonlinearity. Figure 10 shows response curves at the major critical speed of an asymmetrical rotor. In a linear system (case of $\beta^{(0)} = 0$), unstable oscillations appear in the range AB. Yamamoto et al.⁽⁴⁴⁾⁽⁴⁵⁾ showed that the response curves incline and the unstable range disappears when the symmetrical nonlinear spring characteristics exist (case of $\beta^{(0)} = 0.01$). Ishida et al.⁽⁴⁶⁾ investigated the similar phenomena at the dynamic unstable range of an asymmetrical rotor system.

The second is the appearance of unstable vibrations at subcritical resonances. As mentioned in Chapter 2, steady-state resonances appear in nonlinear rotor systems. However, Yamamoto et al. reported that unstable oscillations appear at subharmonic resonances⁽⁴⁷⁾⁽⁴⁸⁾, combination resonances⁽⁴⁸⁾⁽⁴⁹⁾ and super-combination resonance⁽⁵⁰⁾. Ikeda

Fig.10 Resonance curves at the major critical speed of an asymmetrical rotor⁽⁴⁴⁾

et al.⁽⁵¹⁾⁽⁵²⁾ investigated similar phenomena in various nonlinear resonances due to quartic nonlinearity. Ishida et al.⁽⁵³⁾ investigated the influence of the internal resonance to the resonances at the major critical speed, twice the major critical speed and three times the major critical speed. They⁽⁵⁴⁾ also investigated the resonances and the influence of internal resonance in a horizontal asymmetrical shaft system considering the gravity.

5. Nonlinear Resonances in Continuous Rotor Systems

Concerning to continuous rotor systems, most of the research considered geometric nonlinearity which is produced by the extension of an elastic shaft. Nonlinear resonance of a continuous rotor with geometric nonlinearity was first studied by Bolotin⁽⁵⁵⁾. In Japan, Ishida et al.⁽⁵⁶⁾ studied various nonlinear resonances of a slender vertical shaft. They showed that the resonance curve bend towards the higher speed side as shaft becomes slender (See Fig.11) and two kinds of combination resonance occur. When a slender shaft is supported horizontally, the shaft has asymmetrical nonlinearity due to the gravity and the effects of internal resonance $1: (-1)$ appear. About the horizontal rotors, Ishida et al.⁽⁵⁷⁾⁽⁵⁸⁾ and Nagasaka et al.⁽⁵⁹⁾ studied harmonic resonance⁽⁵⁷⁾ and subharmonic resonance of order $1/2$ ⁽⁵⁸⁾, that of order $1/3$ ⁽⁵⁷⁾ (See Fig.12) and combination and super-combination resonance⁽⁵⁹⁾. When the shaft becomes very slender, the geometric nonlinearity becomes strong. Nagasaka et al.⁽⁶⁰⁾ reported complicated phenomena in such a system.

Nonlinear resonances of continuous asymmetrical slender shafts were also investigated. Nagasaka et al.⁽⁶¹⁾ also investigated the nonlinear phenomena at the major critical speed and the secondary critical speed of a continuous asymmetrical rotor.

Fig.11 Major critical speed⁽⁵⁶⁾

Fig.12 Combination resonance ($2p_f - p_b = \omega$)⁽⁵⁶⁾

6. Rotor with Internal Friction

In a rotor system, various frictions appear between its parts. The friction between moving parts are called internal friction or internal damping. This friction is important

Fig.13 Entrainment phenomena at a subharmonic resonance $2p_f = \omega$ ⁽⁶⁴⁾

because it destabilizes a rotor in the post critical speed range. This friction is further classified into a hysteretic damping working in the shaft material and a structural damping working between the shaft and the mounted elements. Vibrations due to the former were first studied by Kimball⁽⁶²⁾ and Newkirk⁽⁶³⁾. Generally, the latter appears more strongly and therefore more serious. Ishida and Yamamoto⁽⁶⁴⁾⁽⁶⁵⁾ investigated a system with structural damping and observed entrainment phenomena at subharmonic resonances of order 1/2 and 1/3. Figure 13 shows the response curves in the vicinity of the subharmonic resonance. A self-excited oscillation (denoted by ○) occurred with almost constant amplitude in the wide range above the major critical speed. However, this self-excited oscillation disappeared when a subharmonic resonance of order 1/2 occurred.

7. Cracked Rotor

Cracks appear in rotors due to fatigue. In order to develop a vibration diagnosis system detecting cracks, research on vibrations of cracked rotors began. The first research was reported in the 1970's by Gasch⁽⁶⁶⁾ and Henry and Okah-Avae⁽⁶⁷⁾. Since then, many studies were reported but most of them replace a cracked rotor with open-close mechanisms to a linear asymmetrical shaft with time-varying coefficient.

In Japan, Ishida et al. reported a series of research considering nonlinearity in their analysis. First, Ishida et al.⁽⁶⁸⁾ investigated the phenomena at the major critical speed of a vertical shaft. They obtained the response curves by combining the solution of an asymmetrical shaft (Open crack) and a symmetrical shaft (closed crack). Then, they studied a horizontal model shown in Fig.14. They represented the spring characteristics of a cracked rotor by a power series in the rotating coordinate system and investigated the phenomena at the harmonic resonance⁽⁶⁹⁾, the secondary critical speed⁽⁷⁰⁾, subharmonic and super-subharmonic resonances⁽⁷¹⁾ and internal resonance phenomena⁽⁷²⁾. Furthermore, Ishida et al. studied various problems, such as resonances of a cracked rotor with static nonlinearity⁽⁷³⁾, nonstationary oscillations⁽⁷⁴⁾, a 4DOF cracked rotor⁽⁷⁵⁾ and detection of a crack using a harmonic excitation⁽⁷⁶⁾. Inoue et al.⁽⁷⁷⁾⁽⁷⁸⁾ studied the modeling of a cracked shaft by the finite element method.

Fig.14 Cracked rotor and its spring characteristics⁽⁶⁹⁾

8. Rotor with Radial Clearance between Two Non-Rotating Parts

Radial clearances in rotor systems generally produce strong nonlinearity. Clearances

are classified into two groups. One is the clearance between two non-rotating parts. For example, there is such clearances between a bearing holder and a casing in a squeeze-film damper bearing. To compensate heat expansion, clearance is sometimes made between a bearing outer ring and a bearing holder. The other is clearance between a rotating part and a non-rotating part. The clearance between a rotor and a guide is one example. Research on the former is introduced in this chapter and that on the latter is introduced in Chap.9.

8.1 Steady-State Resonances

In 1955, Yamamoto⁽⁴⁾ studied the response curves and their stability at the major critical speed. A result is shown in Fig.15 in which curves for various magnitudes of clearance e_0 relative to the unbalance e are illustrated. Kobayashi⁽⁷⁹⁾⁽⁸⁰⁾ studied unbalance responses and gave guidelines for the design of a rotor with squeeze-film damper bearing. Yamauchi⁽⁸¹⁾ and Kobayashi et al.⁽⁸²⁾ developed a method for the analysis of such a nonlinear multi-degree-of-freedom system.

Fig.15 Analytical model and response curve⁽⁴⁾

8.2 Self-Excited Oscillation due to Clearance

Ishida et al.⁽⁸³⁾ reported that self-excited oscillations occur in addition to various nonlinear resonance in a system with radial clearance between a bearing holder and a housing. Figure 16 shows the experimental setup and response curves. Inagaki and Ishida⁽⁸⁴⁾ explained the mechanism of the self-excited oscillation considering energy balance.

Fig.16 Analytical model and response curve

9. Vibration due to Contact between a Rotor and a Stator

9.1 Rubbing (Self-Excited Vibrations)

Rubbing is a self-excited whirling which occurs due to the dry friction when a rotor has contact with a stator. Newkirk⁽⁸⁵⁾ reported this oscillation in 1926. In Japan, Sawaragi⁽⁸⁶⁾ studied a backward whirl in 1951. Watanabe et al.⁽⁸⁷⁾ investigated the effects of various

parameters and Wada and Ishihara⁽⁸⁸⁾ studied effects of guide material (or friction).

9.2 Steady-State Resonances

Watanabe et al.⁽⁸⁹⁾ studied subharmonic resonances which occur when a rotor has contact with a casing. In magnetic bearings, there is clearance between a rotor and a backup bearing. In this case, the inner ring of the backup bearing rotates freely. Inoue et al.⁽⁹⁰⁾ studied nonlinear resonances for various fault patterns of an active magnetic bearing.

9.3 Nonstationary Oscillations

Yanabe et al.⁽⁹¹⁾ studied nonstationary oscillations when a rotor with an annular guard passes through the critical speed with a constant acceleration. They⁽⁹²⁾ also investigated the phenomena considering the characteristics of motor torque.

10. Fluid Film Bearings

Elastic and damping properties of the oil film in journal bearings have nonlinear characteristics. Gotoda⁽⁹³⁾ considered these nonlinearities to improve the accuracy of the response curves. Saito⁽⁹⁴⁾ studied the unbalance response of flexible rotors supported by oil film damper bearing. Since oil-film has large damping, subresonances generally do not occur. However, the damping of water is small. Yamaguchi et al.⁽⁹⁵⁾ reported a subharmonic resonance and a jump phenomenon in a rotor supported by water-lubricated bearings.

11. Torsional Systems

Tsuda reported a series of papers on torsional vibrations of a power-transmission system with clearance. He studied a 1DOF nonlinear model without damper⁽⁹⁶⁾, that with damper⁽⁹⁷⁾, subharmonic vibration⁽⁹⁸⁾, a two-mass coefficient system⁽⁹⁹⁾ and ultraharmonic resonances⁽¹⁰⁰⁾. He also confirmed the approximate solution by an analogue computer⁽¹⁰¹⁾.

12. Vibration Suppression

12.1 Basic Systems

Ishida and Inoue⁽¹⁰²⁾ studied a passive vibration control of a nonlinear rotor system. Inoue et al.⁽¹⁰³⁾ investigated vibration control and unbalance estimation of a nonlinear rotor system using disturbance observer. About the suppression of a self-excited oscillation, Ishida et al.⁽¹⁰⁴⁾ studied suppression of the forward rub, backward rub and impact-type rub and Inoue et al.⁽¹⁰⁵⁾ studied suppression of the forward rub.

12.2 Discontinuous Spring Characteristics

Ishida and Liu⁽¹⁰⁶⁾ developed a new vibration suppression method utilizing

Fig.17 Spring Characteristics

Fig.18 Response curve

discontinuous spring characteristics shown in Fig.17. They succeeded to suppress a steady-state resonance as shown in Fig.18. They⁽¹⁰⁷⁾ also could suppress unstable range using the same method.

12.3 Centrifugal Pendulum and Roller Vibration Absorbers for Torsional Vibrations

In 1930's, centrifugal pendulum vibration absorbers for torsional vibration were invented by Salmon (roller type) and by Salazin and Chilton (bifilar type)⁽¹⁰⁸⁾. They were widely used in aircraft combustion engines before the advent of gas turbine engines. However, although their effects were admitted, their vibration characteristics are not known in detail. Ishida et al. studied a centrifugal pendulum vibration absorbers of pendulum type⁽¹⁰⁹⁾ and that of roller type⁽¹¹⁰⁾. Figure 19(a) shows experimental setup and Fig.19(b) shows response curves for the cases of no pendulum, two pendulums and four pendulums.

(a) Experimental set-up

(b) Effect of pendulum absorbers

Fig.19 Rotor system with four centrifugal pendulums⁽¹⁰⁹⁾

12.4 Ball Balancers and Pendulum Balancers

In 1932, Thearle⁽¹¹¹⁾ invented a ball balancer which balance the rotor automatically. Inoue and his colleagues studied automatic balancers in detail. They studies fundamental characteristics of a basic ball balancer⁽¹¹²⁾⁽¹¹³⁾ and its application to dynamic unbalance⁽¹¹⁴⁾. Jinnouchi et al. investigated the characteristics of a ball balancer with many balls⁽¹¹⁵⁾.

There are two problems in a ball balancer. One is the effect of friction. Due to the inevitable friction between balls and a wall, balls cannot reach the optimum position. Ishida and et al.⁽¹¹⁶⁾ solved this problem using multi-channel ball balancer. A result is shown in Fig.20. Ishida and Liu⁽¹¹⁷⁾ studied a rotor with a ball balancer and the discontinuous spring characteristics. The other is the occurrence of self-excited oscillation. Inoue et al.⁽¹¹⁸⁾ investigated this phenomena. Inoue et al.⁽¹¹⁹⁾ studied the suppression of vibration using both a ball balancer and the axial control of a repulsive magnetic bearing.

The role of balls can be replaced by pendulums. Kubo and Jinnouchi⁽¹²⁰⁾ studied a pendulum balancer and Jinnouchi et al.⁽¹²¹⁾ studied the stability of a rotor with this balancer.

(a) Experimental setup

(b) Response curves

Fig.20 Experiments of a rotor with and without ball balancers

13. Miscellaneous Topics

13.1 Helicopter

Pendulum absorbers are adopted to suppress vibrations of helicopter blades. Since these pendulums oscillate with large amplitude to suppress blade vibrations, the nonlinear analysis is necessary. Nagasaka et al. studied this problem with a 2DOF rigid blade model (a blade and a pendulum)⁽¹²²⁾, a 3DOF rigid blade model (a fuselage, a blade and a pendulum)⁽¹²³⁾ and a 3DOF flexible blade model (a mount, a blade and a pendulum)⁽¹²⁴⁾.

13.2 Wind Turbine

Wind turbine is a special type of rotating machinery in which long blades rotate in the vertical plane under the action of the gravitational force. Inoue et al.⁽¹²⁵⁾ studied nonlinear vibration of a rotating flexible blade of a wind turbine. Ikeda et al.⁽¹²⁶⁾ reported the mode localization phenomena in a three-blade wind turbine.

13.3 Magnetic Bearing

Magnetic force generally has nonlinear characteristics. Sugai et al.⁽¹²⁷⁾ studied the nonlinear spring characteristics of a repulsive magnetic bearing, especially, about the effect of the length and the axial displacement of magnet. Inoue et al.⁽¹²⁸⁾ discussed a vibration suppression method in a repulsive magnetic bearing. They controlled the axial position of the inner ring relative to the outer ring. Sugiura et al.⁽¹²⁹⁾ studied combination resonances of a rotor which is levitated above a high-Tc superconducting bulk.

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