Multiple Magnetic Suspension Systems* (1st report: Basic Concepts and Theorems)

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Abstract

The paper proposes multiple magnetic suspension systems. The concepts are presented by using their basic models. They are classified according to multiple structure, power amplifier and coil connection. According to the first item, they can be divided into series and parallel. This paper mainly treats the latter. The controllability and the observability of each parallel suspension system are discussed based on its mathematical model. The concept of self-sensing magnetic suspension is stretched to the multiple systems. The feasibility of parallel magnetic suspension is demonstrated by simulation

Key words: Magnetic Bearing, Controller, Mechatronics, Motion Control, Observer, Controllability, Observability, Magnetic Suspension

1. Introduction

A typical magnetic suspension system using the attractive force of an electromagnet has an object to be suspended (floator) and at least one electromagnet as an actuator. Multiple electromagnets are used for the control of the multi-degree-of-freedom motions of the floator; each electromagnet is excited by one power amplifier. In this paper, “multiple magnetic suspension system” is proposed in which multiple floators are suspended with a single electromagnet or with a single power amplifier. The basic concepts and fundamental theories of multiple magnetic suspension are also presented.

2. Basic Models and Theorems of Magnetic Suspension System

2.1 Basic model

Figure 1 shows a basic model of magnetic suspension system using the attractive force of an electromagnet. The floator is assumed to be ferromagnetic. The floator is assumed to move only in the vertical direction translationally. When x presents the displacement of the mass from the equilibrium position where the attractive force generated by the electromagnet with a bias current balances the gravitational force, the equation of motion is given by

$$m\ddot{x}(t) = k_s x(t) + k_i i(t), \quad (1)$$

where

m : mass of the floator, k_s : gap-force coefficient of the electromagnet, k_i : current-force coefficient of the electromagnet, i : control current flowing through the coil.

The electrical circuit equation associated with the electromagnet becomes

$$L \frac{di}{dt} + Ri + k_b \dot{x} = v(t), \quad (2)$$

where L : inductance of coil, R : resistance of coil, k_b : velocity-voltage coefficient, v :

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control voltage applied to the coil.

The magnetic suspension systems are classified into two types according to power amplifier. One uses a power amplifier whose output is current so that the coil current is treated as a control input and the control system is designed based on only Eq.(1). This type is called as *current-controlled* magnetic suspension system. The state equation is given by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c \mathbf{x}_c(t) + \mathbf{b}_c i(t), \quad (3)$$

where

$$\mathbf{x}_c = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad (4)$$

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 \\ a_{21} & 0 \end{bmatrix}, \quad (5)$$

$$\mathbf{b}_c = \begin{bmatrix} 0 \\ b \end{bmatrix}, \quad (6)$$

$$a_{21} = \frac{k_s}{m}, \quad (7)$$

$$b = \frac{k_i}{m}. \quad (8)$$

The other type uses a power amplifier whose output is voltage so that the voltage applied to the coil is treated as a control input while the coil current is treated as a state variable; the control system is designed based on Eqs.(1) and (2). This type is called as *voltage-controlled* magnetic suspension system, and its state equation is given by

$$\dot{\mathbf{x}}_v(t) = \mathbf{A}_v \mathbf{x}_v(t) + \mathbf{b}_v v(t), \quad (9)$$

where

$$\mathbf{x}_v = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}, \quad (10)$$

$$\mathbf{A}_v = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & 0 & a_{23} \\ 0 & -a_{32} & -a_{33} \end{bmatrix}, \quad (11)$$

$$\mathbf{b}_v = \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix}, \quad (12)$$

$$a_{23} = \frac{k_i}{m} (=b), \quad (13)$$

$$a_{32} = \frac{k_b}{L}, \quad (14)$$

$$a_{33} = \frac{R}{L}, \quad (15)$$

$$b_0 = \frac{1}{L}. \quad (16)$$

Fig.1. Basic model of magnetic suspension system

In the following, the parameters defined by Eqs.(7), (8) and (13) to (16) are assumed to be *nonzero*.

2.2 Basic Theorems

As to the controllability of the magnetic suspension systems, the following two Theorems are proved easily.

Theorem 1. *The current-controlled magnetic suspension system whose state equation is given by Eq.(3) is controllable, that is, the controllability matrix $\mathbf{M}_C^c$ satisfies*

$$\text{rank } \mathbf{M}_C^c = \text{rank}[\mathbf{b}_c \quad \mathbf{A}_c \mathbf{b}_c] = 2. \quad (17)$$

Theorem 2. *The voltage-controlled magnetic suspension system whose state equation is given by Eq.(9) is controllable, that is, the controllability matrix $\mathbf{M}_C^v$ satisfies*

$$\text{rank } \mathbf{M}_C^v = \text{rank}[\mathbf{b}_v \quad \mathbf{A}_v \mathbf{b}_v \quad \mathbf{A}_v^2 \mathbf{b}_v] = 3. \quad (18)$$

When the current is treated as an output in the voltage-controlled magnetic suspension system, the output equation is given by

$$y(t) = \mathbf{c}_v \mathbf{x}_v(t), \quad (19)$$

where

$$\mathbf{c}_v = [0 \quad 0 \quad 1]. \quad (20)$$

Theorem 3. *The voltage-controlled magnetic suspension system whose state and output equations are given by Eqs.(9) and (19) is observable, that is, the observability matrix $\mathbf{M}_O^v$ satisfies*

$$\text{rank } \mathbf{M}_O^v = \text{rank} \begin{bmatrix} \mathbf{c}_v \\ \mathbf{c}_v \mathbf{A}_v \\ \mathbf{c}_v \mathbf{A}_v^2 \end{bmatrix} = 3. \quad (21)$$

The theoretical basis of self-sensing magnetic suspension ⁽¹⁾⁻⁽³⁾ is given by Theorem 2 and Theorem 3.

3. Series Multiple Magnetic Suspension System

3.1 Basic concept

The basic concept of *series* multiple magnetic suspension is illustrated by Fig.2 in which three floators are suspended with a single electromagnet. In the following, a series-type multiple magnetic suspension system with n floators is referred to as n -series

Fig.2 Series multiple magnetic suspension system with three floators

Fig.3 Series multiple magnetic suspension system three floators, two of which are made of permanent magnets

magnetic suspension system. The floator 1 is suspended by the electromagnet and magnetized. Then, the floator 2 is also magnetized through the floator 1. In the same manner, all the floators are magnetized. Therefore, they can be suspended by the single electromagnet. However, the attractive forces between the floators obtained by the magnetization are frequently insufficient to suspend the successive floator(s). To obtain sufficient attractive force, permanent magnetic (pre-magnetized) floators are used instead of ferromagnetic floators as shown by Fig.3. Yamamoto *et al.* has proposed *indirect suspension*, which corresponds to 2-series magnetic suspension, and achieved stable noncontact suspension ⁽⁴⁾.

3.2 Controllability

It can be shown analytically that both current-controlled and voltage-controlled n -series magnetic suspension systems are controllable. The proofs are presented in Ref.(5).

3.3 Observability

It can be shown analytically that both current-controlled and voltage-controlled n -series magnetic suspension systems are observable when one of the floator's displacements is detected. It is obvious that the systems are observable when multiple displacements are detected.

The voltage-controlled n -series magnetic suspension system is also observable even if only the current of the electromagnet is detected ⁽⁵⁾. Therefore, the theoretical feasibility of self-sensing suspension in series multiple magnetic suspension systems can be shown based on the controllability and observability ⁽⁵⁾.

4. Parallel Multiple Magnetic Suspension System

4.1 Basic concept

A schematic drawing of a parallel multiple magnetic suspension system is shown by Fig.4 in which three floators and electromagnets controlled with a single power amplifier. Based on this principle, the number of floators and electromagnets controlled by a single electromagnet can be set two or more arbitrarily. When the number is n and the connection of the coils is *series* as shown in Fig.4, we call it as n -parallel magnetic suspension system with *series* connection.

Another connection of the coils is *parallel* as shown in Fig.5. The parallel connection is technically reasonable in voltage-controlled parallel magnetic suspension systems. Such system is referred to as n -parallel magnetic suspension system with *parallel* connection.

As to the current-controlled type, only series-connection systems are treated in the following mainly because multiple currents cannot be controlled independently in parallel-connection systems.

Fig.4 Parallel multiple magnetic suspension system with three floators and series-connected coils

4.2 Controllability

In the following, matrix, vector, variable and parameter related to the k th subsystem consisting of a floator and the corresponding electromagnet is denoted by a parenthetic superscript k , for example, $A^{(k)}$, $x^{(k)}$, $\dot{x}^{(k)}$ and $a^{(k)}$. When the matrices or parameters are identical, the parenthetic superscripts are neglected as a_{21} .

In discussing controllability, the following condition is critical, especially in series-connected systems.

$$a_{21}^{(i)} \neq a_{21}^{(j)} \quad (i \neq j). \quad (22)$$

This implies that the open-loop *mechanical* dynamics are different among the subsystems.

In this report, mathematical proofs will be given for $n = 2$ solely because of the limited space. They can be generalized for any multiplicity ($n \geq 3$).

Current-controlled parallel multiple magnetic suspension with series connection

The state equation is given by

$$\dot{x}_{cpt2}(t) = A_{cpt2} x_{cpt2}(t) + b_{cpt2} i(t), \quad (23)$$

where

$$x_{cpt2} = \begin{bmatrix} x_c^{(1)} \\ x_c^{(2)} \end{bmatrix}, \quad (24)$$

$$A_{cpt2} = \begin{bmatrix} A_c^{(1)} & \mathbf{0} \\ \mathbf{0} & A_c^{(2)} \end{bmatrix}, \quad (25)$$

$$b_{cpt2} = \begin{bmatrix} b_c^{(1)} \\ b_c^{(2)} \end{bmatrix}. \quad (26)$$

The following theorem will be proved.

Theorem 4 *The current-controlled parallel multiple magnetic suspension system whose state equation is given by Eq. (23) is controllable when the condition (22) is satisfied.*

Proof. The controllability matrix M_C^{cpt2} made of (A_{cpt2}, b_{cpt2}) becomes

Fig.5 Parallel multiple magnetic suspension system with three floators and parallel-connected coils

$$\mathbf{M}_C^{cpt2} = \begin{bmatrix} 0 & b^{(1)} & 0 & a_{21}^{(1)}b^{(1)} \\ b^{(1)} & 0 & a_{21}^{(1)}b^{(1)} & 0 \\ 0 & b^{(2)} & 0 & a_{21}^{(2)}b^{(2)} \\ b^{(2)} & 0 & a_{21}^{(2)}b^{(2)} & 0 \end{bmatrix}. \quad (27)$$

Calculating the determinant gives

$$\det \mathbf{M}_C^{cpt2} = \{(a_{21}^{(1)} - a_{21}^{(2)})b^{(1)}b^{(2)}\}^2. \quad (28)$$

When the condition (22) holds, therefore,

$$\text{rank} \mathbf{M}_C^{cpt2} = 4. \quad (29)$$

It shows that the system described by Eq.(23) is controllable.

Q.E.D.

Voltage-controlled parallel magnetic suspension with series connection

The state equation is given by

$$\dot{\mathbf{x}}_{vpt2}(t) = \mathbf{A}_{vpt2}\mathbf{x}_{vpt2}(t) + \mathbf{b}_{vpt2}v(t), \quad (30)$$

where

$$\mathbf{x}_{vpt2} = \begin{bmatrix} \mathbf{x}_c^{(1)} \\ \mathbf{x}_c^{(2)} \\ i \end{bmatrix}, \quad (31)$$

$$\mathbf{A}_{vpt2} = \begin{bmatrix} \mathbf{A}_c^{(1)} & \mathbf{0} & \mathbf{b}_c^{(1)} \\ \mathbf{0} & \mathbf{A}_c^{(2)} & \mathbf{b}_c^{(2)} \\ 0 & -\tilde{a}_{32}^{(1)} & 0 & -\tilde{a}_{32}^{(2)} & -\tilde{a}_{33} \end{bmatrix}, \quad (32)$$

$$\mathbf{b}_{vpt2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \tilde{b}_0 \end{bmatrix}, \quad (33)$$

$$\tilde{a}_{32}^{(k)} = k_b^{(k)} / \sum_{j=1}^2 L^{(j)} \quad (k=1, 2), \quad (34)$$

$$\tilde{a}_{33} = \sum_{j=1}^2 R^{(j)} / \sum_{j=1}^2 L^{(j)}, \quad (35)$$

$$\tilde{b}_0 = 1 / \sum_{j=1}^2 L^{(j)}. \quad (36)$$

The following theorem will be proved.

Theorem 5 *The voltage-controlled parallel multiple magnetic suspension system whose state equation is given by Eq.(30) is controllable when the condition (22) is satisfied.*

Proof. Calculating the determinant of the controllability matrix $\mathbf{M}_C^{vpt2}$ made of $(\mathbf{A}_{vpt2}, \mathbf{b}_{vpt2})$ becomes

$$\det \mathbf{M}_C^{vpt2} = \begin{vmatrix} 0 & 0 & \tilde{b}_0 b^{(1)} & 0 & \tilde{b}_0 a_{21}^{(1)} b^{(1)} \\ 0 & \tilde{b}_0 b^{(1)} & 0 & \tilde{b}_0 a_{21}^{(1)} b^{(1)} & 0 \\ 0 & 0 & \tilde{b}_0 b^{(2)} & 0 & \tilde{b}_0 a_{21}^{(2)} b^{(2)} \\ 0 & \tilde{b}_0 b^{(2)} & 0 & \tilde{b}_0 a_{21}^{(2)} b^{(2)} & 0 \\ \tilde{b}_0 & 0 & 0 & 0 & 0 \end{vmatrix} = -\tilde{b}_0^5 \det \mathbf{M}_C^{cpt2}. \quad (37)$$

When the condition (22) holds, therefore,

$$\text{rank} \mathbf{M}_C^{vpt2} = 5. \quad (38)$$

It proves that the system described by Eq.(30) is controllable.

Q.E.D.

Voltage-controlled parallel multiple magnetic suspension with parallel connection

The state equation is given by

$$\dot{\mathbf{x}}_{vpp2}(t) = \mathbf{A}_{vpp2} \mathbf{x}_{vpp2}(t) + \mathbf{b}_{vpp2} v(t), \quad (39)$$

where

$$\mathbf{x}_{vpp2} = \begin{bmatrix} \mathbf{x}_v^{(1)} \\ \mathbf{x}_v^{(2)} \end{bmatrix}, \quad (40)$$

$$\mathbf{A}_{vpp2} = \begin{bmatrix} \mathbf{A}_v^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_v^{(2)} \end{bmatrix}, \quad (41)$$

$$\mathbf{b}_{vpp2} = \begin{bmatrix} \mathbf{b}_v^{(1)} \\ \mathbf{b}_v^{(2)} \end{bmatrix}. \quad (42)$$

The controllability matrix $\mathbf{M}_C^{vpp2}$ made of $(\mathbf{A}_{vpp2}, \mathbf{b}_{vpp2})$ is obtained as

$$\begin{aligned} \mathbf{M}_C^{vpp2} &= \begin{bmatrix} \mathbf{M}_C^{v(1)} & (\mathbf{A}_v^{(1)})^3 \mathbf{M}_C^{v(1)} \\ \mathbf{M}_C^{v(2)} & (\mathbf{A}_v^{(2)})^3 \mathbf{M}_C^{v(2)} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{M}_C^{v(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_C^{v(2)} \end{bmatrix} \begin{bmatrix} \mathbf{I} & (\mathbf{M}_C^{v(1)-1} \mathbf{A}_v^{(1)} \mathbf{M}_C^{v(1)})^3 \\ \mathbf{I} & (\mathbf{M}_C^{v(2)-1} \mathbf{A}_v^{(2)} \mathbf{M}_C^{v(2)})^3 \end{bmatrix}. \end{aligned} \quad (43)$$

Calculating the determinant leads to

$$\det \mathbf{M}_C^{vpp2} = -|\mathbf{M}_C^{v(1)}| \cdot |\mathbf{M}_C^{v(2)}| \cdot |(\mathbf{M}_C^{v(1)-1} \mathbf{A}_v^{(1)} \mathbf{M}_C^{v(1)})^3 - (\mathbf{M}_C^{v(2)-1} \mathbf{A}_v^{(2)} \mathbf{M}_C^{v(2)})^3|. \quad (44)$$

Therefore, the necessary and sufficient condition for the controllability is given by

$$|(\mathbf{M}_C^{v(1)-1} \mathbf{A}_v^{(1)} \mathbf{M}_C^{v(1)})^3 - (\mathbf{M}_C^{v(2)-1} \mathbf{A}_v^{(2)} \mathbf{M}_C^{v(2)})^3| \neq 0. \quad (45)$$

To represent the condition (45) in a simpler form, we introduce a new state vector as

$$\hat{\mathbf{x}}_{vpp2} = \mathbf{T}_C^{-1} \mathbf{x}_{vpp2} = \begin{bmatrix} \mathbf{T}_C^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_C^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{x}_v^{(1)} \\ \mathbf{x}_v^{(2)} \end{bmatrix}, \quad (46)$$

where $\mathbf{T}_C^{(k)}$ is a proper matrix which transforms $(\mathbf{A}_v^{(k)}, \mathbf{b}_v^{(k)})$ in a controllable canonical form. Using this state vector, we rewrite the state equation (39) as

$$\dot{\hat{\mathbf{x}}}_{vpp2}(t) = \hat{\mathbf{A}}_{vpp2} \hat{\mathbf{x}}_{vpp2}(t) + \hat{\mathbf{b}}_{vpp2} v(t), \quad (47)$$

where

$$\hat{\mathbf{x}}_{vpp2} = \begin{bmatrix} \hat{\mathbf{x}}_v^{(1)} \\ \hat{\mathbf{x}}_v^{(2)} \end{bmatrix}, \quad (48)$$

$$\hat{\mathbf{A}}_{vpp2} = \begin{bmatrix} \hat{\mathbf{A}}_v^{(1)} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{A}}_v^{(2)} \end{bmatrix}, \quad (49)$$

$$\hat{\mathbf{b}}_{vpp2} = \begin{bmatrix} \hat{\mathbf{b}}_v^{(1)} \\ \hat{\mathbf{b}}_v^{(2)} \end{bmatrix}, \quad (50)$$

$$\hat{\mathbf{A}}_v^{(k)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{21}^{(k)} a_{33}^{(k)} & a_{32}^{(k)} a_{23}^{(k)} - a_{21}^{(k)} & -a_{33}^{(k)} \end{bmatrix}, \quad (51)$$

$$\hat{\mathbf{b}}_v^{(k)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (52)$$

From Cayley-Hamilton's Theorem, we get

$$\hat{\mathbf{A}}_v^3 = -a_{33} \hat{\mathbf{A}}_v^2 + (a_{32} a_{23} - a_{21}) \hat{\mathbf{A}}_v + a_{21} a_{33} \mathbf{I}. \quad (53)$$

Using Eq.(53), we can show

$$\hat{M}_C^v^{-1} \hat{A}_v \hat{M}_C^v = \hat{A}_v^T. \quad (54)$$

Therefore, Eq.(45) becomes

$$\left| (\hat{A}_v^{(1)})^3 - (\hat{A}_v^{(2)})^3 \right| \neq 0. \quad (55)$$

To investigate the condition (55) in more detail, we assume in the following that

$$a_{21}^{(k)} = a_{23}^{(k)} a_{32}^{(k)} \quad (56)$$

This condition is rather realistic because it holds when the magnetic circuit is ideal (*no leakage flux, no saturation and no hysteresis*). The calculation of $(\hat{A}_v^{(k)})^3$ using Eq.(56) leads to

$$\hat{A}_v^3 = \begin{bmatrix} a_{21}a_{33} & 0 & -a_{33} \\ -a_{21}a_{33}^2 & a_{21}a_{33} & a_{33}^2 \\ a_{21}a_{33}^3 & -a_{21}a_{33}^2 & a_{21}a_{33} - a_{33}^3 \end{bmatrix}. \quad (57)$$

Therefore, the condition (55) becomes

$$\begin{aligned} & \left| (\hat{A}_v^{(1)})^3 - (\hat{A}_v^{(2)})^3 \right| \\ &= (a_{21}^{(1)}a_{33}^{(1)} - a_{21}^{(2)}a_{33}^{(2)})^3 - (a_{21}^{(1)} - a_{21}^{(2)})^2 (a_{33}^{(1)} - a_{33}^{(2)}) (a_{33}^{(1)}a_{33}^{(2)})^2 \neq 0 \end{aligned} \quad (58)$$

Equation (58) indicates that the suspension system (39) is controllable in the following two cases:

[case 1]

$$a_{21}^{(i)} \neq a_{21}^{(j)} \quad (i \neq j) \quad (59)$$

$$a_{33}^{(i)} = a_{33}^{(j)} (\equiv \bar{a}_{33}) \quad (i \neq j) \quad (60)$$

$$\therefore \left| (\hat{A}_v^{(i)})^3 - (\hat{A}_v^{(j)})^3 \right| = \bar{a}_{33}^3 (a_{21}^{(i)} - a_{21}^{(j)})^3 \neq 0 \quad (61)$$

[case 2]

$$a_{21}^{(i)} = a_{21}^{(j)} (\equiv \bar{a}_{21}) \quad (i \neq j) \quad (62)$$

$$a_{33}^{(i)} \neq a_{33}^{(j)} \quad (i \neq j) \quad (63)$$

$$\therefore \left| (\hat{A}_v^{(1)})^3 - (\hat{A}_v^{(2)})^3 \right| = \bar{a}_{21}^3 (a_{33}^{(1)} - a_{33}^{(2)})^3 \neq 0 \quad (64)$$

These discussions are summarized as follows.

Theorem 6 *The necessary and sufficient condition for the system described by Eq. (39) to be controllable is given by Eq.(55). When Eq.(56) is satisfied, it is given by Eq.(58).*

When the condition (56) holds, two sufficient conditions for the controllability are given by the following two corollaries:

Corollary 1. *The system (39) is controllable when Eqs.(59) and (60) are satisfied.*

Corollary 2. *The system (39) is controllable when Eqs.(62) and (63) are satisfied.*

4.3 Observability

We discuss observability to investigate the feasibility of self-sensing suspension based on the voltage induced in the coil ⁽⁶⁾.

Current-controlled parallel multiple magnetic suspension with series connection

The self-sensing suspension using the voltage induced in the coil was based on the controllability and observability of the voltage-controlled magnetic suspension system in which only the coil current is detected ⁽¹⁾. However, even in the current-controlled

suspension system, the physical voltage-current relationship described by Eq.(2) holds. It indicates the feasibility of the observer-based self-sensing suspension in the current-controlled magnetic suspension system ^{(7),(8)}. The displacement and velocity of the floator can be estimated by an observer from the detected voltage across the coil.

In the current-controlled parallel magnetic suspension, the dynamics of each subsystem is described by

$$\dot{\tilde{\mathbf{x}}}_v^{(k)}(t) = \mathbf{A}_v^{(k)} \tilde{\mathbf{x}}_v^{(k)}(t) + \mathbf{b}_v^{(k)} v^{(k)}(t), \quad (65)$$

where

$$\tilde{\mathbf{x}}_v^{(k)} = \begin{bmatrix} x^{(k)} \\ \dot{x}^{(k)} \\ i \end{bmatrix}. \quad (66)$$

The current i is the control input and common among the subsystems. Theorem 3 indicates that the displacement and velocity of each floator can be estimated based on the detected voltage $v^{(k)}$ across the coil (see Fig.6).

Voltage-controlled parallel multiple magnetic suspension with series connection

When only the current is directly detected, the output equation is given by

$$y(t) = \mathbf{C}_{vpt2} \mathbf{x}_{vpt2}(t), \quad (67)$$

where

$$\mathbf{C}_{vpt2} = [0 \ 0 \ 0 \ 0 \ 1]. \quad (68)$$

The following theorem can be proved.

Theorem 7 *The voltage-controlled parallel multiple magnetic suspension system whose state and output equations are given by Eqs.(30) and (67) is observable when Eq.(22) holds.*

Proof. Calculating the determinant of the controllability matrix $\mathbf{M}_O^{vpt2}$ made of $(\mathbf{c}_{vpt2}, \mathbf{A}_{vpt2})$ becomes

$$\begin{aligned} \det \mathbf{M}_O^{vpt2} &= \begin{vmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & a_{32}^{(1)} & 0 & a_{32}^{(2)} & 0 \\ a_{32}^{(1)} a_{21}^{(1)} & 0 & a_{32}^{(2)} a_{21}^{(2)} & 0 & 0 \\ 0 & a_{32}^{(1)} a_{21}^{(1)} & 0 & a_{32}^{(2)} a_{21}^{(2)} & 0 \\ a_{32}^{(1)} (a_{21}^{(1)})^2 & 0 & a_{32}^{(2)} (a_{21}^{(2)})^2 & 0 & 0 \end{vmatrix} \\ &= (a_{21}^{(1)} - a_{21}^{(2)})^2 a_{21}^{(1)} a_{21}^{(2)} (a_{32}^{(1)} a_{32}^{(2)})^2. \end{aligned} \quad (69)$$

When the condition (22) holds, therefore,

$$\text{rank} \mathbf{M}_O^{vpt2} = 5. \quad (70)$$

It shows that the system described by Eqs.(30) and (67) is observable.

Q.E.D.

Fig.6 Detection of the voltage across the coil for observer

Voltage-controlled parallel multiple magnetic suspension with parallel connection

The dynamics of each subsystem is given by

$$\dot{\mathbf{x}}_v^{(k)}(t) = \mathbf{A}_v^{(k)} \mathbf{x}_v^{(k)}(t) + \mathbf{b}_v^{(k)} v(t), \quad (71)$$

where

$$\mathbf{x}_v^{(k)} = \begin{bmatrix} x^{(k)} \\ \dot{x}^{(k)} \\ i^{(k)} \end{bmatrix}. \quad (72)$$

The voltage v is the control input and common among the subsystems. In contrast, the current $i^{(k)}$ is a state variable of each subsystem. When only the current is directly detected in each subsystem, the output equation becomes

$$y^{(k)}(t) = \mathbf{C}_v^{(k)} \mathbf{x}_v^{(k)}(t), \quad (73)$$

where

$$\mathbf{C}_v^{(k)} = [0 \quad 0 \quad 1]. \quad (74)$$

The following theorem can be proved.

Theorem 8 *The voltage-controlled magnetic suspension subsystem whose state and output equations are given by Eqs.(71) and (73) is observable.*

Proof. It is obvious from Theorem 3.

Q.E.D.

5. Simulation

To verify the feasibility of parallel magnetic suspension, several numerical simulations were carried out. One of them is presented here, which treats a *current-controlled 2-parallel system with series connection*. The state equation corresponding to Eq.(23) is given by

$$\dot{\mathbf{x}}_{cpt2}(t) = \mathbf{A}_{cpt2} \mathbf{x}_{cpt2}(t) + \mathbf{b}_{cpt2} i(t), \quad (75)$$

where

$$\mathbf{A}_{cpt2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1.7 & 0 \end{bmatrix}, \quad \mathbf{b}_{cpt2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

The value of each parameter is normalized ⁽⁹⁾. This system is controllable because the condition (22) is satisfied ($a_{21}^{(1)} = 1, a_{21}^{(2)} = 1.7$). Therefore, it can assign the closed-loop poles by state feedback. When the closed-loop system

$$\dot{\mathbf{x}}_{cpt2}(t) = (\mathbf{A}_{cpt2} - \mathbf{b}_{cpt2} \mathbf{K}_{cpt2}) \mathbf{x}_{cpt2}(t), \quad (76)$$

is designed to have the poles at

$$-2.79 \pm 2.53j, -1.17 \pm 0.150j, \quad (77)$$

the feedback gain matrix $\mathbf{K}_{cpt2}$ is determined as

$$\mathbf{K}_{cpt2} = [-70.6 \quad -69.9 \quad 102 \quad 77.8]. \quad (78)$$

Figure 7 shows a transient response of the closed-loop system with an initial condition of

$$\mathbf{x}_{cpt2}(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \quad (79)$$

It demonstrates that all the floaters return to the desired positions ($x^{(k)} = 0$) and the closed-loop system is stable.

6. Conclusion

Multiple magnetic suspension systems were proposed and classified based on the multiple structure (*series* or *parallel*), the output of the power amplifier (*current* or *voltage*) and the connection of the coils (*series* or *parallel*). The controllability and observability of

Fig.7 Response of a current-controlled parallel multiple magnetic suspension with two floators

parallel magnetic suspension systems were investigated based on a mathematical model of each classified system. The mathematical proofs were presented for the systems with two floators. It is to be mentioned that most of the theorems and proofs can be generalized for the systems with any number of floators. Meanwhile, the controllability and observability of series magnetic suspension systems have been discussed in Ref.(5).

The feasibility of 2-parallel (double parallel) magnetic suspension has been already demonstrated experimentally⁽¹⁰⁾.

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