Force Redistribution Method for Compensating Actuator-Breakdown of Vibration-Isolation Tables Supported with a Redundant Number of Pneumatic Actuators

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Abstract
A force redistribution method for compensating actuator breakdown of vibration-isolation tables is studied. The vibration-isolation table is supported by eight pneumatic actuators and has a redundant number of actuators with respect to the degrees of freedom of table motion. We propose a force redistribution method that utilizes the redundancy of the actuators. In the proposed method, when some of the actuators break down, their output forces are redistributed on the unbroken actuators. We construct a detailed mathematical model in view of the behavior of the vibration-isolation table when some of the actuators break down. A type 1 digital servo system is applied to control the vibration and position of the vibration-isolation table. We perform numerical simulations in which some of the actuators break down at a certain time and the pressure of the broken actuators decreases at a constant rate. The numerical simulations examine the effectiveness of the proposed method.

Key words: Force Redistribution, Vibration-Isolation Table, Actuator Breakdown, Redundancy, Pneumatic Actuator

1. Introduction

Recently, semiconductor exposure and inspection apparatuses for precise processing and measurement have increased in number. Such apparatuses suffer from problems with vibration transmitted from the ground and from the mechanical devices mounted on the apparatuses themselves. Hence, vibration-isolation tables have been widely used to prevent vibrations from affecting these apparatuses. Additionally, active vibration controls are required to obtain higher vibration-isolation capability and are being widely studied(1)-(4). When some of the actuators supporting active vibration-isolation tables break down, the transfer characteristics from the inputs of the table system to the displacements, velocities, and accelerations change dramatically and the performance of the vibration-isolation tables deteriorates greatly. Furthermore, because the number of actuators is usually equal to the degrees of freedom (DOF) of the table’s motion, the breakdown of the actuators can, in the worst case, result in falling down of the table, which is dangerous and must be prevented. When some of the actuators break down, a vibration-isolation table supported by a redundant number of actuators with respect to the table’s DOFs can provide many benefits, because the additional actuators may help in recovering the stability of the system. Moreover, the vibration-isolation capability (arrangement...
of the pole of the closed-loop system) can be ensured using the remaining unbroken actuators.

This study targets active control of vibration-isolation tables supported by the bellows-type air spring, which is widely used for vibration-isolation tables. This study considers a vibration-isolation table supported by a redundant number of actuators and investigates a method for adequately re-distributing the forces on the unbroken actuators to compensate for the broken actuators, as well as to ensure not only the system’s stability but also the vibration-isolation capability, when some of the actuators break down. In this study, the stability of the system is evaluated in terms of the real part of poles and the vibration damping property.

2. Mathematical model

Fig. 1 shows a vibration-isolation table supported by a redundant number of pneumatic actuators with respect to the DOFs of the table’s motion. The coordinate system O−xyz is defined as an orthogonal coordinate system. The z-axis passes through the center of gravity of the table in the equilibrium state, and z represents the vertical displacement of the table. The inclination angles of the table around the x− and y−axes passing through the center of gravity of the table are represented by \( \theta_x \) and \( \theta_y \), respectively.

The pneumatic actuators are driven by pneumatic pressure supplied by pneumatic regulators controlled by stepping motors (5)–(8). The pneumatic regulators are shown in Fig. 2. Air compressed by a compressor is supplied to an actuator through a regulator, which causes the pressure in the actuator to change. This study assumes that the vibration-isolation table is supported by \( N (= 8) \) pneumatic actuators. The density of air in the \( i \)-th pneumatic actuator, \( \rho_i \), is expressed as

\[
\rho_i = \frac{m_i}{V_i}, \tag{1}
\]

where \( m_i \) and \( V_i \) represent the mass and volume of air in the \( i \)-th actuator, respectively. Using the ideal gas law, the state equation of air in the \( i \)-th actuator is expressed as

\[
p_iV_i = m_iRT_i, \tag{2}
\]

where \( p_i \), \( R \), and \( T_i \) represent pressure, gas constant, and air temperature, respectively, for the \( i \)-th actuator. From the energy conservation law and assuming the process is adiabatic, it follows that

\[
\frac{d}{dt}(m_iC_vT_i) = C_pq_{in}T_i - p_i\frac{dV_i}{dt}, \tag{3}
\]

where \( C_v \), \( C_p \), and \( q_{in} \) represent specific heat at constant volume, specific heat at constant pressure, and mass flow rate of air, respectively, in the \( i \)-th actuator. The following equation is derived from Eqs. (1)–(3) (9):

\[
\dot{\rho}_iV_i + \kappa p_iV_i = \kappa RT_iq_{in}, \tag{4}
\]
where $\kappa = C_p/C_v$ represents specific heat ratio. The mass flow rate $q_{\text{in}}$ is assumed to be proportional to the difference between the pressure $p_i$ of the $i$-th actuator and the output pressure of the $i$-th pneumatic regulator $p_{\text{di}}$, i.e.,

$$q_{\text{in}} = c_i(p_{\text{di}} - p_i),$$

(5)

where $c_i$ represents the proportional constant. The equilibrium state is defined as the state wherein the gravity acting on the table balances the force and the moment generated by all actuators. The pressure at the equilibrium state and the variation in the pressure of the $i$-th actuator are represented by $p_{\text{0i}}$ and $\delta p_i$, respectively. Note that the table displacements are not uniquely determined under the equilibrium state. The volume, air temperature, and the vertical displacement of the $i$-th actuator at the equilibrium state are represented by $V_{\text{0i}}$, $T_{\text{0i}}$ and $z_{\text{0i}}$, respectively. The variation in the volume, the initial length, and the vertical displacement of the $i$-th actuator are represented by $\delta V_i$, $L_i$, and $\delta z_i$, respectively. Then the following relations are derived:

$$p_i = p_{\text{0i}} + \delta p_i, \quad V_i = V_{\text{0i}} + \delta V_i, \quad z_i = L_i + z_{\text{0i}} + \delta z_i, \quad V_i = A_i z_i.$$  

(6)

When $\delta p_i \ll p_{\text{0i}}$ and $\delta V_i \ll V_{\text{0i}}$, we can approximate as $p_i/p_{\text{0i}} = 1$ and $V_i/V_{\text{0i}} = 1$. Thus, from Eqs. (4)–(6), the state equation for the $i$-th pneumatic actuator is obtained as

$$(\delta \dot{p}_i)V_{\text{0i}} + \kappa RT_{\text{0i}} c_i (\delta p_i) = -\kappa p_{\text{0i}} A_i \delta z_i + \kappa RT_{\text{0i}} c_i p_{\text{di}} - \kappa RT_{\text{0i}} c_i p_{\text{0i}}.$$  

(7)

From Eq. (6), the state equation can be rewritten as

$$\kappa p_{\text{0i}} A_i \delta z_i + \kappa RT_{\text{0i}} c_i p_{\text{di}} + \dot{p}_i V_{\text{0i}} = \kappa RT_{\text{0i}} c_i p_{\text{0i}}.$$  

(8)

There are short time delays in the responses from the rotations of the stepping motors $\phi_i$ to the output pressures of the pneumatic regulators $p_{\text{di}}$. The relation between $\phi_i$ and $p_{\text{di}}$ is expressed as a first order delay system,

$$p_{\phi} = \mu(H\phi - p_{\phi}),$$

(9)

where

$$\phi = [\phi_1 \ldots \phi_N]^T, \quad p_{\phi} = [p_{\phi1} \ldots p_{\phiN}]^T, \quad \mu = \text{diag}[\mu_1 \ldots \mu_N], \quad H = \text{diag}[H_1 \ldots H_N],$$

where $\mu_i$ and $H_i$ represent proportional constants. Next, the rotation angle of the stepping motor $\phi_i$ is proportional to the sum of the number of input pulses per unit time supplied to the $i$-th stepping-motor driver $u_i$. Then the following relation is obtained:

$$\phi = K_u u_i.$$  

(10)

where

$$K_u = \text{diag}[K_{u1} \ldots K_{uN}], \quad u = [u_1 \ldots u_N]^T.$$  

Here $K_{ui}$ represents the proportional constant.

The $i$-th actuator is placed at $(x_i, y_i)$ of the coordinate system $O-xy$, as shown in Fig. 1, and the force generated by the $i$-th actuator is represented by $f_i$. The relation between the forces $f_i$ and the pressures $p_i$ of the $i$-th pneumatic actuator is expressed as

$$f = A_u p,$$  

(11)

where

$$f = [f_1 \ldots f_N]^T, \quad A_u = \text{diag}[A_{u1} \ldots A_{uN}], \quad p = [p_1 \ldots p_N]^T.$$  

The force generated by the $i$-th actuator is proportional to the difference between the pressure $p_i$ of the $i$-th actuator and the output pressure of the $i$-th pneumatic regulator $p_{\text{di}}$, i.e.,

$$q_{\text{in}} = c_i(p_{\text{di}} - p_i),$$  

(5)
Next, the equations of motion for the table are expressed as
\[ M\ddot{X} = F^* - Mg, \]  
\[ F^* = G^T f, \]
where
\[ M = \text{diag}[M, J_x, J_y], \quad X = [z, \theta_z, \theta_y]^T, \quad F^* = [F^*, M^*_x, M^*_y]^T, \quad g = [g, 0, 0]^T, \]
\[ G_b = \begin{bmatrix} 1 & y_1 & -x_1 \\ \vdots & \vdots & \vdots \\ 1 & y_N & -x_N \end{bmatrix}. \]

Here \( M \) represents the mass of the table, and \( J_x \) and \( J_y \) represent the moments of inertia of the table around the \( x- \) and \( y- \) axes passing through the center of gravity, respectively. From Eqs. (8)–(13), the state equation of motion of the table is expressed as
\[ \dot{x} = A_c x + B_c u - E_c g, \]
\[ y = C_c x = X, \]
where
\[ A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & M^{-1}G^T A_b & 0 & 0 & 0 \\ 0 & W_1G_b & -W_2 & W_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu & \mu H & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad x = \begin{bmatrix} X \\ p \\ \phi \end{bmatrix} \]
\[ C_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad p_0 = \text{diag}[p_{01} \ldots p_{0N}], \quad V_0 = \text{diag}[V_{01} \ldots V_{0N}], \quad W_1 = \text{diag}[-\frac{\kappa_{p0}A_b}{v_0} \ldots -\frac{\kappa_{p0}A_b}{v_N}], \quad W_2 = \text{diag}[-\frac{\kappa_{p0}A_b}{v_0} \ldots -\frac{\kappa_{p0}A_b}{v_N}]. \]

The continuous time system (14) is transformed to a discrete time system,
\[ x(k+1) = Ax(k) + Bu(k) - Eq, \]
\[ y(k) = Cx(k), \]
where \( A = e^{A_c \Delta t}, B = \int_0^{\Delta t} e^{A_c \tau} d\tau \cdot B_c, C = C_c \) and \( E = \int_0^{\Delta t} e^{A_c \tau} d\tau \cdot E_c \). The sampling period for the measurement and controller is represented by \( \Delta t \).

3. Controller design

In this study, the vibration-isolation table is controlled by a type 1 digital servo controller. To construct the type 1 digital servo controller using the integral characteristics of the stepping motor given by Eq. (10), the state equation (14) around an equilibrium state is reconstructed as
\[ \dot{q} = A_o q + B_o \phi - E_o g, \]
\[ y = C_o q, \]
where
\[ A_o = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & M^{-1}G^T A_b & 0 & 0 & 0 \\ 0 & W_1G_b & -W_2 & W_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu & \mu H & 0 \end{bmatrix}, \quad B_o = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_o = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad q = \begin{bmatrix} X \\ p \\ \phi \end{bmatrix}, \quad C_o = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \]
The continuous time system (16) is transformed to a discrete time system,

\begin{align*}
q(k + 1) &= A_d q(k) + B_d \phi(k) - E_d g, \\
y(k) &= C_d q(k),
\end{align*}

where \( A_d = e^{A_d t} \), \( B_d = \int_0^t e^{A_d \tau} d\tau \cdot B_o \), \( C_d = C_o \) and \( E_d = \int_0^t e^{A_d \tau} d\tau \cdot E_o \). The error signal is expressed as

\[ e(k) = R_e(k) - y(k), \]

where \( R_e(k) = [z_e(k) \theta_o(k) \theta_o(k)]^T \) represents the reference signal. The backward difference of the state vector \( \Delta q(k + 1) = q(k + 1) - q(k) \) is expressed as

\[ \Delta q(k + 1) = A_d \Delta q(k) + B_d \Delta \phi(k). \]

From Eqs. (18) and (19), the expanded state equation is obtained as

\[
\begin{bmatrix}
\begin{bmatrix}
\Delta \phi(k + 1) \\
\Delta q(k + 1)
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix}
I \\
0
\end{bmatrix} & -C_d A_d \\
A_d & B_d
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\Delta \phi(k) \\
\Delta q(k)
\end{bmatrix}
\end{bmatrix} +
\begin{bmatrix}
\begin{bmatrix}
0 \\
I
\end{bmatrix}
\end{bmatrix} \Delta R_e(k) + \begin{bmatrix}
\begin{bmatrix}
0 \\
I
\end{bmatrix}
\end{bmatrix} \Delta R_i(k + 1).
\]

The relation \( \Delta R_i(k + 1) = 0 \) is satisfied when the reference signal \( R_i(k) \) does not change. Therefore, the following expression is obtained:

\[ X_0(k + 1) = \Phi X_0(k) + \Gamma \Delta \phi(k), \]

where \( X_0 = [e^T(\Delta q^T(\Delta \phi(k)))]^T \). If a closed-loop system is stabilized by applying a suitable control input, \( X_0(k) \Rightarrow 0 \) when \( k \rightarrow \infty \), i.e., \( e(k) \Rightarrow 0 \). Therefore, it becomes a stationary robust controller for parameter variation. A cost function to determine a control input is defined as

\[ J = \sum_{k=1}^{\infty} \{ X_0^T(k)QX_0 + \Delta \phi^T(k)R \Delta \phi(k) \}. \]

The optimal control input, which minimizes the cost function, can be derived as

\[ \Delta \phi(k) = F_d X_0(k) = F_e e(k) + F_d \Delta q(k), \quad F_d = \begin{bmatrix} F_e & F_d \end{bmatrix}. \]

4. Force distribution

4.1. Cost Function

When the vibration-isolation table is supported by a larger number of actuators than the DOFs of the table’s motion, the system has redundancy with respect to the actuators. In this section, a method of adequate distribution of the output forces on the actuators is presented. The forces generated by each actuator to yield \( F^*, M_x^*, M_y^* \) are obtained by the least-squares method\(^3\). For the adequate distribution of output force on the \( i \)-th actuator, a cost function is defined as

\[ S = \frac{1}{2} \alpha_F (F - F^*)^2 + \frac{1}{2} \alpha_x (M_x - M_x^*)^2 + \frac{1}{2} \alpha_y (M_y - M_y^*)^2 + \frac{1}{2} \alpha_z \sum_{i=1}^{N} (f_i - F^*)^2 + \frac{1}{2} \sum_{i=1}^{N} \gamma_i f_i^2, \]

where \( \alpha_F, \alpha_x, \alpha_y, \alpha_z, \) and \( \gamma_i \) represent the weighting coefficients for these terms, and are defined as non-negative values. The first term on the right hand side of Eq. (24) is the square of the error with respect to the resultant force in the \( z \)-axis direction. The second and third terms are squares of the errors with respect to the resultant moment about the \( x \)- and \( y \)-axes. The fourth term is the residual sum of squares of the differences between the forces generated by the actuators and the average of the forces of all actuators. The fifth term is the sum of squares
of the forces generated by the actuators. Compared with a previous work\(^{(5),(6)}\), the fifth term is introduced in the cost function. This term is introduced to consider the conditions in which the force distribution on the broken actuators is stopped and the forces are redistributed on the unbroken actuators. The force vector \(f\) that minimizes the cost function (24) can be expressed as

\[
f = WF^* ,
\]

(25)

where \(W\) represents the transformation matrix obtained by the least-squares method that minimizes the cost function (24). When the \(i\)-th actuator does not break down, the weighting coefficient \(\gamma_i\) is set to 0. When the \(i\)-th actuator breaks down, the weighting coefficient \(\gamma_i\) is adaptively updated in response to changes in the input-output characteristics of the \(i\)-th actuator.

In the proposed method, the weighting coefficients \(\gamma_i\) need to be set to appropriate values. Then, the effect of the weighting coefficients \(\gamma_i\) when the actuators break down is considered. Table 1 lists the relation between the value of the weighting coefficients \(\gamma_i\) and the forces of each actuator when the #1 actuator in Fig. 1 breaks down and the reference output force and moments of all actuators are set to \(F^* = 8\) N, \(M^*_x = 0\) N-m, and \(M^*_y = 0\) N-m. Table 2 lists the actual force and moment that exist when the actuators generate the forces listed in Table 1. Here \(\alpha_F\), \(\alpha_x\), and \(\alpha_y\) are 1.0, and \(\alpha_\gamma\) is \(10^{-3}\).

When none of the actuators breaks down, all weighting coefficients \(\gamma_i\) are set to 0, and the result is shown as the term “Normal” in Tables 1 and 2. As shown in Table 1, the force is distributed on the broken actuator when \(\gamma_1\) is small. As the weighting coefficient \(\gamma_1\) is increased, the force distributed on the broken actuators is removed and the forces are redistributed on the unbroken actuators. Therefore, the weighting coefficient \(\gamma_i\) needs to have a sufficiently large value.

4.2. Force Redistribution

When some actuators break down, the system may become unstable by distributing the forces on the broken actuators. A force redistribution method to compensate for the actuator breakdown is proposed to avoid this problem. The conventional force distribution method using a pseudo-inverse matrix\(^{(1)-(4)}\) is not suitable because it is difficult to expand such method to distribute the optimal output force on the unbroken actuators according to the performance

<table>
<thead>
<tr>
<th>(\gamma_1)</th>
<th>(f_1)</th>
<th>(f_2)</th>
<th>(f_3)</th>
<th>(f_4)</th>
<th>(f_5)</th>
<th>(f_6)</th>
<th>(f_7)</th>
<th>(f_8)</th>
</tr>
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<tbody>
<tr>
<td>0 (Normal)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>0.64833</td>
<td>1.18884</td>
<td>1.0810</td>
<td>0.97322</td>
<td>0.86541</td>
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<td>1.08103</td>
<td>1.18884</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>0.15566</td>
<td>1.45340</td>
<td>1.19455</td>
<td>0.93570</td>
<td>0.67685</td>
<td>0.93570</td>
<td>1.19455</td>
<td>1.45340</td>
</tr>
<tr>
<td>(10^{-1})</td>
<td>0.01810</td>
<td>1.52726</td>
<td>1.22624</td>
<td>0.92523</td>
<td>0.62421</td>
<td>0.92523</td>
<td>1.22624</td>
<td>1.52726</td>
</tr>
<tr>
<td>(10^0)</td>
<td>0.00184</td>
<td>1.53599</td>
<td>1.22999</td>
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</tr>
<tr>
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<td>0.00018</td>
<td>1.53688</td>
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<tr>
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<td>0.61728</td>
<td>0.92385</td>
<td>1.23041</td>
<td>1.53698</td>
</tr>
</tbody>
</table>

Table 2  Actual resultant force and moments obtained by the proposed method when \(\gamma_1\) is varied \((F^* = 8\) N, \(M^*_x = 0\) N-m, \(M^*_y = 0\) N-m)
deterioration and the changes in the input-output characteristics of the actuators caused by the breakdown. In this study, to generate the force $F^*$ and moment $M^*$ as a total system, the force distributed on the broken actuators is removed, and the forces are redistributed on the unbroken actuators by adjusting the weighting coefficient $\gamma$ in Eq. (24) according to the changes in the input–output characteristics of the actuators caused by the breakdown. From Eqs. (13) and (25), the force redistribution equation is obtained as

$$f' = W G^T \phi f.$$  \hfill (26)

Assuming that the delays from the increments in the rotation angles of the stepping motors $\Delta \phi$ to the increments in the pressures of the pneumatic actuators $\Delta p$ can be neglected in the frequency bandwidth of active control, the relation in one sample step is expressed as

$$\Delta p = K_1 \Delta \phi.$$  \hfill (27)

From Eq. (10), the relation between the input pulses per unit time to rotate the stepping motor $u_i$ and the rotation angles of the stepping motor $\phi_i$ is expressed as

$$\Delta \phi = K_v u \Delta t.$$  \hfill (28)

Representing the optimal control input obtained from the force redistribution by $u'$, the equations are expressed as

$$\Delta f' = \Delta t A_a K_w K_a u', \quad \Delta f = A_a K_w K_a \Delta \phi = \Delta t A_a K_w \Delta \phi.$$  \hfill (29)

From Eqs. (26) and (29), the optimal control input based on the force redistribution is obtained as

$$u' = \frac{1}{\Delta t} K_v^{-1} (K_a)^{-1} W G^T \phi (A_a K_w) \Delta \phi.$$  \hfill (30)

The block diagram of the closed-loop system is shown in Fig. 3.

The force distributed on the broken actuators is removed by using the optimal control input obtained from the force redistribution $u'$, and the forces are redistributed on the unbroken actuators. Therefore, the input–output characteristics are recovered, and as a result, the dynamic characteristics (arrangement of a poles) of a closed-loop system and the stability of the controller can be maintained when the actuators break down.

4.3. Features of the proposed method and relation with the conventional method

One of the features of the proposed method is an ability to distribute the outputs on the actuators at an intermediate rate by adjusting the weighting coefficient. By using the feature, the proposed method can gradually stop distribution of the outputs on some actuators while suppressing the effects on the movements of the vibration-isolation table. Those actuators can be also replaced with new actuators, and the desired actuator outputs can then be distributed on the new actuators gradually to operate the vibration-isolation table continuously while minimizing the effects on the movements of the table. Moreover, the proposed method can change
the distribution rate in real time, because the computational load for calculating the proposed transformation matrix $W$ is small. On the other hand, when using the pseudo-inverse matrix, two distribution rates—one is the rate when the output forces are distributed on all actuators, and the other is the rate when the distribution on the some actuators are stopped—must be prepared beforehand. An intermediate distribution rate can then be obtained by linear interpolation. However, it is necessary to verify whether the distribution rate is appropriate and optimal. On the other hand, the proposed distribution method agrees with a distribution method using a pseudo-inverse matrix when $\alpha_i$, $\gamma_i$ of the unbroken actuators, and $\gamma_i$ of the broken actuators in Eq. (24) are set to 0, 1, and $\infty$, respectively. Therefore, the proposed distribution method can be thought as a generalized and extended form of a distribution method using a pseudo-inverse matrix. Further, an optimal distribution rate is ensured by using the proposed distribution method.

In addition to the proposed method, there are methods in which only the unbroken actuators are used when some of the actuators break down. Such methods may involve, for example, switching to the controller that uses only the unbroken actuators. In such a case, the next two methods are possible.

a) A method in which the control gain is calculated beforehand and the controller is changed

b) A method in which the controller (for example, the optimal regulator or the linear quadratic regulator (LQR)) is reconstructed after detecting breakdown of actuators

In case a), it is necessary to calculate the control gains of all combination of actuator breakdown beforehand. The number of combinations of actuator breakdown is $2^N$, where $N$ is the number of actuators. Thus, because the number of combinations increases exponentially as the number of actuators increases, calculating the gains of all combinations beforehand is impractical in terms of memory consumption. In the proposed method, the weighting coefficient $\gamma_i$ of the $i$-th actuator that breaks down is simply set to a sufficiently large value, and the transformation matrix $W$ can be calculated without using a lot of memory. Moreover, the proposed method has a large advantage because it can cope with every combination of actuator breakdown.

In case b), it is necessary to recover the stability of the control system immediately when actuators are breaking down. However, the computational load for recalculating an optimal controller is not small, and this is not desirable. In contrast, the computational load for calculating the proposed transformation matrix $W$ is small.

In addition, the proposed force redistribution method can be utilized independently of the controller design, and it not only can work with a combination of various controllers but also has the advantage that recalculation of feedback gain is not needed. In this study, a type I digital servo system is applied as an example, and the results are shown in the next section.

5. Simulation results

In the following simulations, the vibration-isolation table is controlled by a type I digital servo controller, and some actuators are assumed to be broken down. The outputs from the broken actuators are lost at a rate of 95% per second. The parameter specifications for the simulations of the table and the actuators are shown in Table 3. The weighting matrices in Eq. (22) are set to

$$Q = \text{diag}[1.0 \times 10^{12} 1.0 \times 10^{12} 1.0 \times 10^{12} 1.0 \times 10^{12} 1.0 \times 10^{12} 1.0 \times 10^{12} \ 1.0 \times 10^5 1.0 \times 10^5 1.0 \times 10^5 \ 1.0 \times 10^{-10} 1.0 \times 10^{-10} 1.0 \times 10^{-10} 1.0 \times 10^{-10} 1.0 \times 10^{-10} \ 1.0 \times 10^{-8} 1.0 \times 10^{-8} 1.0 \times 10^{-8} 1.0 \times 10^{-8}]$$
Table 3 Parameters of the vibration-isolation table

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>530 kg</td>
</tr>
<tr>
<td>$J_x$</td>
<td>29.63 kg·m²</td>
</tr>
<tr>
<td>$J_y$</td>
<td>30.11 kg·m²</td>
</tr>
<tr>
<td>$\rho_{uo}$</td>
<td>$8.267 \times 10^4$ N/m²</td>
</tr>
<tr>
<td>$V_{uo}$</td>
<td>$3.927 \times 10^{-4}$ m³</td>
</tr>
<tr>
<td>$T_{uo}$</td>
<td>288 K</td>
</tr>
<tr>
<td>$L_i$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>$z_{0i}$</td>
<td>$5.0 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>1.4</td>
</tr>
<tr>
<td>$K_{ii}$</td>
<td>1.25 Pa/pulse</td>
</tr>
<tr>
<td>$R$</td>
<td>$287$ J/(kg·K)</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>$1.0 \times 10^2$ s⁻¹</td>
</tr>
<tr>
<td>$l_i$</td>
<td>0.184 m</td>
</tr>
<tr>
<td>$L_i$</td>
<td>0.272 m</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>$1.0 \times 10^{-8}$ s⁻¹</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>$1.0 \times 10^{-8}$ m⁻¹</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>$1.0 \times 10^{-8}$ m⁻¹</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>$1.0 \times 10^{-8}$ m⁻¹</td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>$1.0 \times 10^{-8}$ m⁻¹</td>
</tr>
</tbody>
</table>

$1.0 \times 10^{-8}$, $1.0 \times 10^{-8}$, $1.0 \times 10^{-8}$, $1.0 \times 10^{-8}$, $1.0 \times 10^{-8}$],

$R = \text{diag}[1.0, 1.0, 1.0, 1.0, 1.0, 1.0]$. 

In these simulations, some actuators are broken down at 5 s. The weighting coefficients $\gamma_i$ in the evaluation function (24) are set to 0 before the actuators are broken down. When the $i$-th actuator is broken down, the weighting coefficient $\gamma_i$ is increased to $10^3$, and force redistribution is performed.

The initial conditions for the table displacements are set to $z (= L_i + z_{0i}) = 150$ mm, $\theta_x = 0$ rad, and $\theta_y = 0$ rad. Reference signals of the servo controller are set to $z_r = 150$ mm, $\theta_xr = 0$ rad, and $\theta_yr = 0$ rad.

![Fig. 4 Results of simulation when actuator #7 is broken down at 5 s.](image1)

![Fig. 5 Time historical responses of actuator outputs when actuator #7 is broken down at 5 s.](image2)
First, a simulation in which actuator #7 is broken down is performed, and the cases with and without the force redistribution method are compared. The time historical responses of the table displacements are shown in Fig. 4, and the output pressures of the actuators are shown in Fig. 5. Without the force redistribution method, the table displacements are recovered by the integral compensation of the type 1 digital servo controller. However, vibrational responses were observed. In contrast, stable responses with less vibration were obtained with the force redistribution method.

Next, a simulation in which actuators #3, #6, and #7 are broken down is performed, and the cases with and without the force redistribution method are compared. The results are shown in Figs. 6–9. Without the force redistribution method, the time historical responses of the table displacements are shown in Fig. 6, and the output pressures of the actuators are shown in Fig. 8. With the force redistribution method, the time historical responses of the table displacements are shown in Fig. 7, and the output pressures of the actuators are shown in Fig. 9. Without the force redistribution method, it takes over 10 s until vibrational responses are no longer observed. In contrast, with the force redistribution method, stable responses with less vibration are obtained, and it takes about 1 s until the table displacements are recovered. Hence, the stability of a system is successfully recovered by the proposed force redistribution method.
method, which compensates a part that cannot be compensated with the integral compensation of the type 1 digital servo controller.

6. Conclusions

This paper has proposed a force redistribution method that utilizes actuator redundancy. First, a mathematical model was constructed for the vibration-isolation table supported by a redundant number of pneumatic actuators with respect to the DOFs of the table’s motion. Next, the vibration-isolation table was controlled by a type 1 digital servo controller, and the effectiveness of the proposed method was examined by numerical simulations in which some actuators break down. In the proposed method, the distribution of the output forces on the broken actuators is stopped, and the output forces are redistributed on the unbroken actuators by taking advantage of the redundancy of the number of actuators. Moreover, it was shown that the redistribution method ensures the stability of the system and suppresses the vibrational responses after actuator breakdown. In future, the effectiveness of the proposed method is necessary to be verified by experiments.

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References

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