Note

Dimensionless Film Momentum Transfer Coefficient in Relation to the Analogy among Transport Phenomena

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An Mo number, which is a dimensionless formula of the film momentum transfer coefficient m_o and reverse of the dimensionless film thickness of flow, was proposed. The Mo numbers deduced from the known theoretical and experimental equations of momentum transfer phenomena for flows in a pipe and around a sphere were compared to the Nusselt numbers, Nu , and Sherwood numbers, Sh , for the corresponding heat and mass transfers, respectively, and a good analogy was found among them. The phenomena of momentum transfers were also simply explained using m_o and the Mo number. The author expects that these findings might be useful for improving the education of fundamental chemical engineering, food chemical engineering, and biochemical engineering.

Key words: Film momentum transfer coefficient, Mo number, Nu number, Transport phenomena, Analogy

1. Introduction

The analogy among transport phenomena has been one of the most interesting subjects in chemical engineering for the author since he learned it as a student.

The fluxes of physical quantities, momentum, energy and mass, are expressed by a unified equation under the condition of constant fluid density.

Flux of physical quantity = $-D_{\text{eff}} \nabla$ (Concentration of physical quantity) (1)

where D_{eff} is a constant having the unit, cm^2/s . The constant D_{eff} is a dynamic viscosity ν for momentum transfer, a temperature diffusion coefficient α for heat transfer and a binary diffusion coefficient D_{AB} for mass transfer. With an increase in the Re number, an eddy occurs due to flow instability, bulk flows in a pipe and around an immersed body such as a sphere become turbulent, and concomitantly, boundary film layers of flow (thickness, δ) develop along the wall of a pipe and surface of the body in which the fluid has a laminar flow. Due to the same origin, that is flow instability, temperature and concentration boundary film layers with thicknesses δ_t and δ_c , respectively, develop. The film heat and mass transfer coefficients are then defined and the heat and mass transfer fluxes at the

wall or the surface of the immersed body are expressed using the coefficients according to Eq. (1). However, a momentum flux is expressed using the friction coefficient f or drag coefficient C_R . Why is the momentum flux equation not expressed using a film momentum transfer coefficient m_o ? A text book [1] introduced a film momentum transfer coefficient. But it is hardly used for describing the momentum transfer to the best of the author's knowledge.

This short paper introduces a dimensionless film momentum transfer coefficient Mo which corresponds to the Nu and Sh numbers for heat and mass transfers, respectively, and shows how this number behaves in relation to the analogy with heat and mass transfer phenomena [2].

2. Definition of a Film Momentum Transfer Coefficients m_o and its Dimensionless Form Mo Number

The film momentum transfer coefficient m_o [$\text{kg}/(\text{m}^2\text{s})$] and Mo number [-] are defined by Eqs. (2) and (3), respectively.

$$\tau_w = -\mu \left(\frac{du}{dr} \right)_w = \left(\frac{\mu}{\delta} \right) \langle u \rangle = m_o \langle u \rangle \quad (2)$$

$$Mo = m_o D / \mu = 1 / (\delta / D) \quad (3)$$

where D denotes a representative length such as the diameters of the pipe, sphere, etc., μ , a viscosity, u , the fluid

velocity, $\langle u \rangle$, the average fluid velocity, r , the distance in a radial direction and subscript w , the wall position.

If we consider the meanings of the Nu and Sh numbers, Eq. (4) follows.

$$\begin{aligned} Mo &= 1/(\delta/D) = Nu/(\delta/\delta_t) = Sh/(\delta/\delta_c) \\ &= Mo/(\delta/\delta) \end{aligned} \quad (4)$$

For film thicknesses, Eqs. (5) and (6) hold.

$$(\delta/\delta_t) \propto f(Pr) = f(\nu/\alpha) \quad (5)$$

$$(\delta/\delta_c) \propto f(Sc) = f(\nu/D_{AB}) \quad (6)$$

where Pr and Sc denote the Prandtl and Schmidt numbers, respectively.

3. Flow in the Pipe

Equation (2) is applied to the fluid flow in a pipe.

$$\tau_w = -\mu (du/dr)|_w = m_o \langle u \rangle = \frac{1}{2} \rho \langle u \rangle^2 f \quad (7)$$

where ρ is the fluid density.

The m_o value is obtained by substituting the well-known correlation between the friction coefficient f and Re number into Eq. (7), and then, the Mo number is deduced from Eq. (3).

For laminar flow:

$$Mo = 8 \quad Re < 2300 \quad (8)$$

For turbulent flow:

The Blasius formula [3] yields Eq. (9).

$$Mo = 0.0395 Re^{3/4} \quad 3 \times 10^3 < Re < 10^5 \quad (9)$$

The author first discusses the case of turbulent flow. Equation (9) is the equation to be compared with the correlation of the Nu number in turbulent flow [4].

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (10)$$

Colburn [2] proposed the j factor and explained the analogy in transport phenomena. If the above result is added, the analogy is denoted by Eq. (11)

$$j = \frac{1}{2} f \approx \frac{Mo}{Re(\nu/\nu)^3} \approx \frac{Nu}{RePr^{1/3}} \approx \frac{Sh}{ReSc^{1/3}} \quad (11)$$

Note that the term $(\nu/\nu)^{1/3}$ in the denominator of the third equation is equal to unity.

The Nu number for laminar flow is rather complex because it is affected by various factors such as the pipe length, and dependence of the physical properties on the temperature distribution in the pipe. It also depends on the definition of the heat transfer coefficient. The Nu number based on a local heat transfer coefficient is comparable to the Mo number for laminar flow (Eq. (8)). The Nu numbers for following two limiting cases have been also shown to be constant values [5].

For the fully developed laminar flow with the condition of a constant heat flux through the wall :

$$Nu = 4.364 \quad (12)$$

For the fully developed laminar flow with the condition of a constant wall temperature :

$$Nu = 3.657 \quad (13)$$

The Nu numbers, Eqs. (12) and (13), had been calculated by using a flow rate-averaged temperature as the bulk temperature, while Mo number, by using cross sectional area-averaged velocity as the driving force of momentum transfer. The Nu number was calculated from the approximate theoretical temperature profile for the former condition proposed by Bird et al. [6] using the cross sectional area-averaged temperature as the bulk temperature and was 6.0 (see Eq. (A.9) in Appendix).

4. Flow around a Sphere

The relations between the drag coefficient, C_R , and m_o and the Mo number are shown in the following equations.

$$m_o \pi d_p^2 u = C_R (\pi d_p^2 / 4) (\rho u^2 / 2) \quad (14)$$

$$Mo = m_o d_p / \mu = Re C_R / 8 \quad (15)$$

where d_p is the diameter of a sphere, ρ , the fluid density and u , the approaching velocity.

Using well-known correlations between C_R and Re ,

$$Mo = 3.0 \quad Re < 6 \quad \text{Stokes region} \quad (16)$$

$$Mo = 1.25 Re^{1/2} \quad 6 < Re < 10^5 \quad \text{Allen region} \quad (17)$$

$$Mo = 0.055 Re \quad 500 < Re < 10^5 \quad \text{Newton region} \quad (18)$$

These equations are compared to the correlation of Ranz

and Marshall [7,8].

$$Nu = 2.0 + 0.6 Re^{1/2} Pr^{1/3} \quad (19)$$

For Sh number, Pr number is replaced by Sc number.

The Mo number, Nu number and Sh number take similar formula and the analogy among the three transfer phenomena holds. However, the numerical coefficients are different. There is a clear reason for this. In the Stokes' law, the drag force arises from two kinds of drag forces, the form and friction drags and their contributions are 1 vs. 2 [9]. The form drag arises from the pressure difference between front and back sides of the sphere against the flow and it does not contribute to heat or mass transfer. Therefore, 2 out of 3 of Mo number should be considered for comparison with the Nu and Sh numbers. In the range of Allen, it is interesting that the contribution of the form drag is increased to (1/2) from (1/3) for the Stokes range.

5. Summary

Although the introduction of Mo number is no more than just an idea and is calculated based on the known theoretical and experimental works, it may help students to understand deeply the analogy among the transfer phenomena. Another merit of use of m_o and the Mo number is that the momentum transport phenomenon is simply explained. "With an increase in Re number, the film thickness decreases, therefore, m_o or Mo number increases and then, the pressure drop or drag force increases."

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Appendix

The approximate theoretical temperature profile for the fully developed laminar flow in the pipe with the condition of a constant heat flux through the wall proposed by Bird et al. [6: p.296, Eq.(9.8-31)] is shown as Eq. (A.1).

$$\Theta = -4\zeta - \xi^2 + \frac{1}{4}\xi^4 + \frac{7}{24} \quad (A.1)$$

The dimensionless variables were defined by Eq. (A.2).

$$\Theta = \frac{T - T_o}{qR/k}; \quad \xi = \frac{r}{R}; \quad \zeta = \frac{zk}{\rho C_p u_{\max} R^2} \quad (A.2)$$

where T is the temperature, T_o , the temperature at the inlet of pipe, z , the axial direction from the inlet of pipe, r , the radial direction from the center of pipe, R , the radius of pipe, q , the constant heat flux from the pipe wall, k , the thermal conductivity, C_p , the specific heat of fluid and $u_{\max}$, the maximum fluid velocity.

The denominator of Θ is rewritten as Eq. (A.3).

$$qR/k = \frac{h(T_b - T_w) R}{k} = \frac{Nu}{2}(T_b - T_w) \quad (A.3)$$

where h is heat transfer coefficient, and the subscripts b and w denote the bulk and wall positions, respectively. Note that $(T_b - T_w)$ is the constant driving force of constant heat flux. An another dimensionless temperature Θ' is introduced.

$$\Theta' = \frac{T - T_o}{T_b - T_w} = \frac{Nu}{2} \Theta \quad (A.4)$$

Then, the following relation is hold.

$$\Theta'_b - \Theta'_w = 1 = \frac{Nu}{2}(\Theta_b - \Theta_w) \quad (A.5)$$

Now, if Θ_b and Θ_w are calculated from Eq. (A.1), Nu number is calculated from Eq. (A.5).

If the cross sectional area-averaged temperature is chosen as the bulk temperature, Θ_b is calculated as Eq.(A.6).

$$\begin{aligned} \Theta_b = \frac{2\pi \int_0^R \Theta r dr}{\pi R^2} &= 2 \int_0^1 (-4\zeta - \xi^2 + \\ &+ \frac{1}{4}\xi^4 + \frac{7}{24}) \xi d\xi = -4\zeta - \frac{1}{8} \end{aligned} \quad (A.6)$$

Θ_w is calculated straight forwardly.

$$\Theta_w = \Theta(\zeta, 1) = -4\zeta - \frac{11}{24} \quad (A.7)$$

Then,

$$\Theta_b - \Theta_w = \frac{1}{3} \quad (A.8)$$

By substituting Eq. (A.8) into Eq. (A.5), Nu number is deduced as Eq. (A.9).

$$Nu = 6.0 \quad (A.9)$$

In the case of using the flow rate averaged temperature as the bulk temperature, $(\Theta_b - \Theta_w)$ becomes as Eq. (A.10).

$$\Theta_b - \Theta_w = \frac{11}{24} \quad (\text{A.10})$$

Then, Nu number is calculated to be Eq. (12) in the main text.

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