原 著

Improved Method of Analysing Mortality Statistics

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死亡統計解析の新しい方法

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Abstract

The angular transformation

$$\theta = \text{Arcsin } \sqrt{P}$$

transforms a probability p varying from 0 to 1 into an angle varying from 0 to $\pi/2$ radians and is remarkable in that the amount of information concerning θ is constant.

$\text{Arcsin } \sqrt{P}$ may be written

$$\text{Arcsin } \sqrt{P} = \sqrt{P} + \frac{1}{2} \frac{P}{3} \sqrt{P} + \frac{1 \cdot 3}{2 \cdot 4} \frac{P^2}{5} \sqrt{P} + \dots + \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \frac{P^n}{2n+1} \sqrt{P} + \dots$$

[Taylor expansion of $f(x)$ about $x=p$]

If a probability p were as small as a death rate, the second term $p^{2/3}/2 \cdot 3$ and the following terms $3p^{2/5}/2 \cdot 4 \cdot 5 + \dots$ would be negligible altogether. Thus, normal deviate for the mortality of any sample community and its standard error E_t are calculated respectively as follows:

$$t = \frac{2\Sigma[N_A(\sqrt{P_A} - \sqrt{P_a})]}{\Sigma N_A}, \text{ and } E_t = \frac{1}{\sqrt{\Sigma N_A}}$$

P : age specific death rate for age A among sample community.

P_a : age specific death rate for the same age among the standard population.

N : the number of persons A years old among sample community.

Normal deviate for the mortality of 46 prefectures has been investigated using vital statistics of Japan of 1960: these values were further comparatively studied with the crude or corrected death rates respectively. Naturally, correlation between normal deviate for the mortality and corrected death rate was far higher than that between normal deviate and crude death rate both in male and female populations.

It has been deducted by the author that normal deviate for mortality is a useful indicator for evaluating levels of health of communities.

【抄録】 訂正死亡率や0才平均余命は、いずれも年令別死亡率から導かれる。これら解析方法にみられる共通の欠陥は、比率の変動の情報量としての重みが無視されていることである。著者は、既往の諸方法に代わるべき一新法を考案した。

率Pを逆正弦変換し、かつこれを Taylor 展開して第2項以下を省略すると

$$\begin{aligned}\theta = \text{Arcsin } \sqrt{P} &= \sqrt{P} + \frac{1}{2} \frac{P}{3} \sqrt{P} + \frac{1 \cdot 3}{2 \cdot 4} \frac{P^2}{5} \sqrt{P} + \dots \\ &\dots + \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \frac{P^n}{2n+1} \sqrt{P} + \dots \approx \sqrt{P}\end{aligned}$$

標準集団における年令別死亡率を P_A 、標本集団における年令別死亡率および人口をそれぞれ P_A および N_A とする。後者が前者の任意標本である場合には

$$t = \frac{2\Sigma[N_A(\sqrt{P_A} - \sqrt{P_a})]}{\Sigma N_A}$$

は平均0、標準偏差 $1/\sqrt{\Sigma N_A}$ なる正規分布をする。よつて、この式によつて算出された t をもつて、地域の死亡正規偏差とすることができる。

男女別に、昭和35年全国人口を標準集団として、46都道府県について死亡正規偏差を算出し、これを粗死亡率および訂正死亡率と比較検討し、もつて、死亡正規偏差が地域の健康水準を表わすのに有用な示標であることを示した。

The amount of information of observed rate is related not only to the sample size but also to the value of the rate itself^(1,2). Nevertheless, in computing adjusted death rates, adjustment is made only for the age distribution of the population, the disparity of the invariance of rate according to its value being given no consideration^(3,4).

The author has worked out a way to obviate the above-mentioned defect of the corrected death rates.

The Computing of Normal Deviate

The angular transformation

$$\theta = \text{Arcsin } \sqrt{p}$$

transforms a probability p varying from 0 to 1 into an angle varying from 0 to $\pi/2$ radians, and is remarkable in that the amount of information concerning θ is constant.

$\text{Arcsin } \sqrt{P}$ may be written

$$\text{Arcsin } \sqrt{P} = \sqrt{P} + \frac{1}{2} \frac{P}{3} \sqrt{P} + \frac{1 \cdot 3}{2 \cdot 4} \frac{P^2}{5} \sqrt{P} + \dots$$

$$+ \dots + \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \frac{P^n}{2n+1} \sqrt{P} + \dots$$

[Taylor-series expansion of $f(x)$ about $x=p$]

If a probability p were as small as a death rate, the second term $p^{3/2}/2 \cdot 3$ and the following terms $3p^{5/2}/2 \cdot 4 \cdot 5 + \dots$ would be negligible altogether. Thus, normal deviate t for the mortality of any sample community and its standard error Et are calculated respectively as follows:

$$t = \frac{2\Sigma[N_A(\sqrt{P_A} - \sqrt{P_a})]}{\Sigma N_A},$$

$$\text{and } Et = \frac{1}{\sqrt{\Sigma N_A}}.$$

P_A : age specific death rate for age A among sample community.

P_a : age specific death rate for the same age among the standard population.

N_A : the number of persons A years old among sample community.

Table 1. Normal Deviate Computation for a Sample Community
Female, Iwata Prefecture, 1960

(1) Age	(2) Population N_A	(3) Death Rate P_A	(4) Death Rate for Stundard Population P_a	(5) $\sqrt{P_A}$	(6) $\sqrt{P_a}$
Total	742400				
0— 4	75400	0.010000	0.007399	0.10000	0.08602
5— 9	85300	0.000914	0.000757	0.03023	0.02751
10—14	85000	0.000529	0.000423	0.02300	0.02827
15—19	62500	0.000976	0.000799	0.03124	0.02827
20—24	61400	0.001596	0.001362	0.03995	0.03691
25—29	64000	0.001688	0.001546	0.04109	0.03932
30—34	61000	0.001902	0.001807	0.04361	0.04251
35—39	48600	0.002305	0.002250	0.04801	0.04743
40—44	41200	0.003471	0.002961	0.05892	0.05442
45—49	35700	0.005014	0.004492	0.07081	0.06702
50—54	34400	0.006192	0.006673	0.07869	0.08169
55—59	27500	0.010327	0.009907	0.10162	0.09953
60—64	20300	0.021133	0.015825	0.14537	0.12580
65—69	16700	0.029401	0.026706	0.17147	0.16342
70—74	12500	0.052400	0.046520	0.22891	0.21568
75—79	6800	0.112353	0.082208	0.33519	0.28672
80 & over	4100	0.216341	0.154651	0.46512	0.39326
$\Sigma (N_A \sqrt{P_A}) = 47421.43100$					
$\Sigma (N_A \sqrt{P_a}) = 43751.64300$					
Normal Deviate for the Prefecture = $\frac{(47421.431 - 43751.643) \times 2}{742400} = +0.00989$					

An example of normal deviate computations is shown in Table 1.

Materials and Results

The study was based on the vital statistics in Japan in 1960 and the report of one percent sampling survey of census in the same year. Total male or female population was regarded as a standard population.

Table 2 and 3 give normal deviate for mortality of male and female respectively in 46 prefectures. The tables show also crude and corrected death rates, being referred to the report from the Division of

Health and Welfare Statistics.⁵⁾

Discussion

1. Assuming that P_A and P_a are age specific death rate for age A respectively among standard population and among its random sample, and that N_A is the number of persons A years old among sample community, the distribution of $\text{Arcsin } \sqrt{P_A}$ would be a normal distribution with a mean $\text{Arcsin } \sqrt{P_a}$ and variance $1/(4N_A)$.

Then,

$$2N_A (\text{Arcsin } \sqrt{P_A} - \text{Arcsin } \sqrt{P_a}) \\ \approx 2N_A (\sqrt{P_A} - \sqrt{P_a})$$

would have a standardized normal distribu-

Table 2. Crude Death Rates, Corrected Death Rates and Normal Deviate for Mortality; For Individual Prefectures Male, 1960

No.	Prefecture	Population		Crude Death Rate		Corrected Death Rate		Normal Deviate for Mortality	
		Number	Proportion	per 1000	Ranking	per 1000	Ranking	Value	Ranking
	All Japan	45819500	1000	8.23		8.23		0	
1	Hokkaido	2561500	56	6.98	4	8.13	16	+0.0009	24
2	Aomori	683800	15	8.45	11	10.09	45	+0.0134	46
3	Iwate	703700	15	8.78	17	8.80	36	+0.0069	42
4	Miyagi	835500	18	7.87	6	8.88	38	-0.0003	20
5	Akita	658400	14	8.92	19	10.63	46	+0.0088	45
6	Yamagata	633200	14	9.35	31	9.20	44	+0.0050	39
7	Fukushima	986000	22	9.13	24	9.02	43	+0.0063	40
8	Ibaraki	1004900	22	9.24	26	8.46	28	+0.0010	25
9	Tochigi	733400	16	9.14	25	8.97	41	+0.0039	36
10	Gunma	768100	17	8.90	18	8.91	39	+0.0006	22
11	Saitama	1190400	26	8.54	13	8.79	35	+0.0030	33
12	Chiba	1132900	25	9.00	21	8.34	27	+0.0001	21
13	Tokyo	4967100	108	5.59	1	7.65	4	-0.0106	1
14	Kanagawa	1727800	38	6.51	2	8.15	18	-0.0039	9
15	Niigata	1170800	26	8.68	15	8.23	23	-0.0026	11
16	Toyama	492800	11	9.10	22	8.75	34	+0.0039	37
17	Ishikawa	465800	10	9.59	37	8.66	31	+0.0016	30
18	Fukui	362200	8	9.58	36	8.17	19	-0.0040	8
19	Yamanashi	382000	8	8.97	20	7.69	6	-0.0044	7
20	Nagano	956500	21	9.26	27	7.84	8	-0.0082	3
21	Gifu	790700	17	8.70	16	7.67	5	-0.0067	5
22	Sizuoka	1379400	30	7.97	7	7.73	7	+0.0017	31
23	Aichi	2075700	45	7.39	5	7.59	2	-0.0066	6
24	Mie	712600	16	9.37	32	8.04	15	-0.0036	10
25	Shiga	403100	9	9.81	39	8.32	26	-0.0008	16
26	Kyoto	970200	21	8.12	9	7.60	3	-0.0081	4
27	Osaka	2756700	60	6.97	3	8.67	32	-0.0003	19
28	Hyogo	1912000	42	8.22	10	8.19	21	-0.0005	17
29	Nara	381600	8	9.60	38	8.97	42	+0.0014	28
30	Wakayama	483400	11	9.38	33	7.94	10	-0.0017	12
31	Tottori	288800	6	9.82	41	8.04	14	-0.0015	14
32	Shimane	430400	9	10.37	46	7.93	9	-0.0015	13
33	Okayama	819000	18	9.81	40	7.47	1	-0.0086	2
34	Hiroshima	1052200	23	9.34	29	8.15	17	+0.0010	26
35	Yamaguchi	780200	17	9.41	34	8.22	22	+0.0038	35
36	Tokushima	402200	9	10.15	44	8.67	33	+0.0016	29
37	Kagawa	434900	9	9.92	42	8.01	13	-0.0004	18
38	Ehime	714500	16	9.52	35	7.94	11	+0.0012	27
39	Kochi	414600	9	10.34	45	8.24	24	-0.0014	15
40	Fukuoka	1933600	42	8.01	8	8.49	29	+0.0049	38
41	Saga	448300	10	9.27	28	8.87	37	+0.0071	44
42	Nagasaki	860300	19	8.65	14	8.54	30	+0.0064	41
43	Kumamoto	884800	19	9.34	30	8.19	20	+0.0033	34
44	Oita	586200	13	9.98	43	8.94	40	+0.0071	43
45	Miyazaki	552500	12	8.49	12	8.27	25	+0.0008	23
46	Kagoshima	934800	20	9.11	23	7.97	12	+0.0028	32

tion with mean 0 and variance $1/(4N_A)$. Then, normal deviate t for the mortality of a random sample community is calculated as follows:

$$t = \frac{2\sum [N_A(\sqrt{P_A} - \sqrt{p_A})]}{\sum N_A}$$

The distribution of t obtained here is a normal distribution with mean 0 and variance $1/\sum N_A$.

Thus, normal deviate t for the mortality of any sample community expressed in the

Table 3. Crude Death Rates, Corrected Death Rates and Normal Deviate for Mortality; For Individual Prefectures Female, 1960

No.	Prefecture	Population		Crude Death Rate		Corrected Death Rate		Normal Deviate for Mortality	
		Number	Proportion	per 1000	Ranking	per 1000	Ranking	Value	Ranking
	All Japan	47527700	1000	6.92		6.92		0	
1	Hokkaido	2493000	52	5.51	3	6.94	16	+0.0008	28
2	Aomori	728100	15	6.66	10	7.55	41	+0.0059	40
3	Iwate	742400	16	7.26	15	8.58	46	+0.0099	46
4	Miyagi	889800	19	6.29	5	7.01	23	-0.0028	7
5	Akita	696700	15	6.66	9	7.58	43	+0.0051	38
6	Yamagata	704900	15	7.37	16	7.39	38	-0.0009	18
7	Fukushima	1057700	22	7.55	20	7.90	45	+0.0065	42
8	Ibaraki	1048100	22	8.09	32	7.12	30	+0.0015	31
9	Tochigi	787700	17	7.45	18	7.00	22	+0.0028	35
10	Gunma	820500	17	7.41	17	7.04	25	+0.0002	26
11	Saitama	1250700	26	7.19	13	6.87	13	+0.0017	32
12	Chiba	1182400	25	7.69	25	7.03	24	+0.0000	24
13	Tokyo	4658600	99	4.73	1	6.34	2	-0.0087	1
14	Kanagawa	1688300	36	5.42	2	6.62	7	-0.0061	2
15	Niigata	1270900	27	7.56	21	6.98	20	-0.0017	12
16	Toyama	522500	11	7.81	27	7.42	39	+0.0012	30
17	Ishikawa	511500	11	8.34	40	7.00	21	+0.0004	27
18	Fukui	390400	8	8.37	41	7.21	33	-0.0026	9
19	Yamanashi	408300	9	7.67	24	6.52	5	-0.0012	14
20	Nagano	1020500	21	8.14	35	7.12	29	-0.0030	6
21	Gifu	853900	18	7.23	37	6.94	18	-0.0003	22
22	Sizuoka	1403100	30	6.52	8	6.40	3	-0.0043	4
23	Aichi	2143100	45	6.34	6	7.16	31	-0.0027	8
24	Mie	761100	16	8.16	36	6.94	17	+0.0002	25
25	Shiga	446200	9	8.96	45	7.11	27	-0.0005	21
26	Kyoto	1020800	21	7.22	14	6.67	9	-0.0048	3
27	Osaka	2739600	58	5.84	4	6.87	14	-0.0006	20
28	Hyogo	1995500	42	6.83	11	6.61	6	-0.0011	15
29	Nara	410100	9	8.24	38	7.57	42	+0.0031	36
30	Wakayama	520500	11	8.03	31	6.65	8	-0.0015	13
31	Tottori	310300	7	8.51	42	7.19	32	-0.0009	17
32	Shimane	459600	10	8.66	43	6.89	15	-0.0007	19
33	Okayama	868400	18	8.13	33	6.77	10	-0.0019	10
34	Hiroshima	1124900	24	7.73	26	6.80	11	-0.0010	16
35	Yamaguchi	822900	17	7.64	22	7.28	35	-0.0001	23
36	Tokushima	437900	9	8.69	44	7.36	36	+0.0062	41
37	Kagawa	484800	10	7.99	30	6.48	4	-0.0036	5
38	Ehime	780400	16	7.64	23	6.34	1	-0.0020	11
39	Kochi	447400	9	9.03	46	7.26	34	+0.0018	33
40	Fukuoka	2033800	43	6.40	7	6.95	19	+0.0011	29
41	Saga	493800	10	7.90	28	7.65	44	+0.0084	45
42	Nagasaki	887600	19	7.51	19	7.54	40	+0.0082	44
43	Kumamoto	967600	20	7.98	29	7.11	28	+0.0055	39
44	Oita	640000	13	8.28	39	7.37	37	+0.0073	43
45	Miyazaki	582100	12	7.02	12	6.83	12	+0.0020	34
46	Kagoshima	1019300	21	8.13	34	7.08	26	+0.0049	37

above-mentioned equation could be used as an indicator of the mortality.

This indicator t has standard error $1/\sqrt{\Sigma N_A}$.

The indicator which the author reported earlier^{6,7)} was the divided value of t by its

standard error $1/\sqrt{\Sigma N_A}$. The author, from now on, uses t in the sense indicated in the present paper.

2. Normal deviate for the mortality of 46 prefectures has been studied further comparatively with the crude or corrected death

Fig. Correlation between two indicators

Upper left: between death rate and corrected death rate in male.

Upper right: the same as the above in female.

Center left: between crude death rate and normal deviate for mortality in male.

Center right: the same as the above in female.

Lower left: corrected death rate and normal deviate for mortality in male.

Lower right: the same as the above in female.

rates respectively. (Fig.)

In computing correlation coefficient between two indicators, a weight was attached to each indicator in proportion to the popu-

lation of the prefecture. Naturally, correlation between normal deviate for the mortality and corrected death rate was far higher than that between normal deviate and

crude death rate both in male and female populations.

Although, corrected death rate was more closely related to normal deviate for mortality than to crude death rate, correlation coefficient between normal deviate for mortality and corrected death rate was just only +0.7840 in the male and +0.8161 in the female. In other words, common factors influence both corrected death rate and normal deviate for mortality in the ratio of 0.8 to total factors, and uncommon factors influence separately each indicator in the ratio of $\sqrt{1-0.8^2}=0.6$.

This difference between two indicators is mainly due to whether the disparity of the invariance of rates according to its value is taken into account or not.

In computing corrected death rate, adjustment is made only for the age distribution of the population, the disparity of the invariance of rates according to its value being given no consideration. For example, excess number of deaths 10 in the case of expected number of deaths 100 is equally treated as the excess deaths 10 in the case of expected number of deaths 10. This computation does not seem to be reasonable from the standpoint of the theory of information.

3. The comparison of certain cause-specific corrected death rate of some two populations is made usually for elucidating which population is more affected by the cause of the disease. In this epidemiological survey, normal deviate analysis would be considered to be more effective than the method using corrected death rate.

4. Normal deviate for the mortality is based on the structure of observed sample

population which is not always similar to the structure of standard population. In addition, though standard error of the corrected death rate is almost impossible to calculate, that of normal deviate for observed mortality is very easily obtained. From this point of view, normal deviate for mortality has advantage over corrected death rate.

Summary and Conclusion

Improved method has been worked out to obviate the defect of the corrected death rates.

Applying angular transformation

$$\begin{aligned}\theta &= \text{Arcsin } \sqrt{P} = \sqrt{P} + \frac{1}{2} \frac{P}{3} \sqrt{P} \\ &+ \frac{1 \cdot 3 P^2}{2 \cdot 4 \cdot 5} \sqrt{P} + \dots \\ &+ \frac{1 \cdot 3 \dots (2n-1)}{2 \cdot 4 \dots 2n} \frac{P^n}{2n+1} \sqrt{P} + \dots \approx \sqrt{P}\end{aligned}$$

normal deviate t for the mortality of any sample community and its standard error E_t are respectively calculated as follows:

$$t = \frac{2\mathcal{E}[N_A(\sqrt{P_A} - \sqrt{P_a})]}{\mathcal{E}N_A}, \quad \text{and } E_t = \frac{1}{\sqrt{\mathcal{E}N_A}}$$

P_A : age specific death rate for age A among sample community.

P_a : age specific death rate for the same age among the standard population.

N_A : the number of persons A years old among sample community.

It has been deducted by the author that normal deviate for mortality is a useful indicator for evaluating levels of health of communities.

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