Use of L-moment Diagram and Goodness-of-fit Test in Flood Frequency Analysis

Dept. of Civil Eng., University of Moratuwa, Nimal P. D. GAMAGE
Dept. of Env. Science & Human Eng., Saitama University, Takeshi FUJINO

1. Introduction
Estimation of flooding potential is often required for watersheds with insufficient or nonexistent hydrometric information. A difficulty associated with flood frequency analysis is the lack of sufficient data. Consequently, it is difficult to identify the parent distribution from which a sample is drawn. Large standard errors of estimate result because of small sample sizes. Estimation of the magnitude of floods greater than the length of the available record represents an extrapolation in which the standard error of estimate increases rapidly with increasing return periods.

The L-moments are analogous to conventional moments and are estimated by linear combinations of order statistics. However, L-moments are more convenient because they are directly interpretable as measures of the scale and of the shape of probability distributions (Hosking 1987). Hosking and Wallis (1993) extended the use of L-moments and developed statistics that can be used for regional frequency analysis to measure discordancy, regional homogeneity, and goodness-of-fit.

This study contributes to the resolution process by proposing a combination of measures and applying them to the catchments of diverse hydrologic behavior in Sri Lanka.

2. Data
Locations of the 65 sites selected after screening are shown in Figure 1. It was found that certain sites were regulated after some years. Those data were removed from this analysis. The selected data set contains a total of 1494 stream flow observations of the period 1940 to 2000. The range of record lengths at the gauging stations are from 10 to 54 years of annual maximum daily flow values with a mean record length of 23 years and a median length of 22 years. The catchment drainage areas range from 45 km² to 3071 km² with an average value of 646 km² and a median value of 345 km².

3. Goodness-of-fit Test
The aim of the goodness-of-fit is to test whether a given distribution fits the data acceptably closely. A related aim is to choose, from a number of candidate distributions, the one that gives the best fit to the data. Reasonable possibilities include the distributions; Generalized Pareto, Generalized Extreme Value, Generalized Logistic, Generalized Normal, Pearson Type III. The goodness-of-fit criterion for each of various distributions is defined in terms of L-moments and is termed the Z-statistic (Hosking and Wallis, 1993):

\[
Z_{DIST} = \frac{(\tilde{r}_4^{DIST} - \bar{r}_4 + \beta_4)}{\sigma_4}
\]

where \(DIST\) refers to a candidate distribution, \(\tilde{r}_4^{DIST}\) is the population L-kurtosis of selected distribution, \(\bar{r}_4\) is the sample L-kurtosis, \(\beta_4\) is the bias of sample L-kurtosis defined as:

\[
\beta_4 = \frac{1}{N_{sim}} \sum_{m=1}^{N_{sim}} (\tilde{r}_4^m - \bar{r}_4)
\]

and \(\sigma_4\) is the standard deviation of sample L-kurtosis defined as:

\[
\sigma_4 = \sqrt{\frac{1}{N_{sim} - 1} \sum_{m=1}^{N_{sim}} (\tilde{r}_4^m - \bar{r}_4)^2 - N_{sim}\beta_4^2}
\]

A four parameter kappa distribution is fitted to the sample L-moment ratios. The kappa distribution is used to simulate 500 regions similar to the observed region, and from these simulations \(\beta_4\) and \(\sigma_4\) is estimated. Hosking and Wallis (1997) recommend that only distributions for which \(|Z_{DIST}| \leq 1.64\) should be considered as suitable distributions for the particular site.

4. Results and Discussion
The L-moment ratios as well as their regional averages are calculated for the selected sites. The \(t_3\) versus \(t_4\) moment ratio diagram for different stations is shown in Fig. 2. Five curves and five points, representing theoretical relationships for popular distributions, are plotted in Fig. 2. These distributions are the Exponential (E), Gumbel (G), Logistic (L), Normal (N), Uniform (U), Generalized Pareto (GPA), Generalized Extreme Value (GEV), Generalized Logistic (GLO), Generalized Normal (GNO), Pearson Type III (PE3). As one might expect for such a large region, the results in Fig. 2 suggest that the region
as a whole is heterogeneous. Data points are widely scattered in Fig. 2. This conclusion is also supported by the results obtained from the regional homogeneity test (Hosking and Wallis, 1993). Values of the site discordancy measure $D_i$, the heterogeneity measure $H$, and the goodness-of-fit measure ($Z_{DIST}$) were computed for the whole region. Three sites, site numbers 2 ($D_i = 7.50$), 55 ($D_i = 4.65$), and 58 ($D_i = 3.71$), were found to be discordant with the region of 65 sites as a whole. The heterogeneity measures, $H = 11.56$, indicates a very high degree of heterogeneity in the selected sites as a region.

Logistic distributions seem to give the best at-site estimates with 27 catchments having lowest $Z_{DIST}$ while Generalized Pareto distribution gives best estimates for 15 sites, Generalized Extreme Value distribution gives best estimates for 10 sites, Generalized Normal distribution gives best estimates for 8 sites, and Pearson Type III distribution gives best estimates for 7 sites.

In conclusion, annual maximum flow data from 65 sites in Sri Lanka were analyzed using L-moments to study regional homogeneity and assess goodness-of-fit in frequency analysis. The region as a whole is heterogeneous. A comparative study of the data with respect to the goodness of fit of at-site and regional quantile estimates revealed that the application of more than one quantitative procedure for the selection of a probability distribution contributes to the credibility of the selection. The L-moment diagram appears to be suitable for an exploratory stage where inappropriate distributions could be screened out while goodness-of-fit test appears to be suitable for a confirmatory stage where descriptive performance of screened distributions is examined.

The main conclusion from the prior analysis is that the selected sites are statistically heterogeneous. For the region as a whole the goodness-of-fit measure for different distributions are; Generalized Logistic = -0.47, Generalized Extreme Value = -2.99, Generalized Normal = -4.52, Pearson Type III = -7.19, Generalized Pareto = -9.48. The Generalized Logistic distribution seems to be the best choice if the region is to be considered as a single unit.

The quantile estimates of the at-site distributions for the 65 catchments were calculated. For these catchments, the Generalized Logistic distributions seem to give the best at-site estimates with 27 catchments having lowest $Z_{DIST}$ while Generalized Pareto distribution gives best estimates for 15 sites, Generalized Extreme Value distribution gives best estimates for 10 sites, Generalized Normal distribution gives best estimates for 8 sites, and Pearson Type III distribution gives best estimates for 7 sites.

References

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