1. Introduction

During short-term rainfall prediction with any prediction model, if an error structure of predicted rainfall is properly analyzed with past prediction results and effective error factors are found, the error structure would be very useful to a present rainfall prediction for giving uncertainty of the prediction values. The important factors of the error structure would be amount and distribution of bias between observed rainfall and predicted rainfall (or simply prediction error) and its spatial correlation.

In this research, error structure of a real-time rainfall prediction by a translation model is analyzed to get its statistics, error distribution and spatial correlation coefficient. And then prediction error fields are simulated as a spatially correlated random field according to the characteristics of the prediction error structure.

2. Error Structure Analysis

Translation model (Shiiba et al., 1984) is used for short-term rainfall prediction. In the translation model, the horizontal rainfall distribution, \( z(x,y,t) \) with the spatial coordinate \((x,y)\) at time \(t\) is modeled as:

\[
\frac{\partial z}{\partial t} + u \frac{\partial z}{\partial x} + v \frac{\partial z}{\partial y} = w
\]  

(1)

where, \( u \) and \( v \) are advection velocity along \( x \) and \( y \), and \( w \) is rainfall growth-decay rate. Three spatial rainfall distributions, which have 3 km and 5 minutes resolution, are used to determine \( u \) and \( v \). When forecasting rainfall fields, the growth decay rate \( w \) is always assumed to be zero in this study.

Radar rainfall event used here is observed at Miyama radar station of Kinki area on 1990/9/12 and 9/13. Figure 1 shows average rainfall intensity from the observed radar data. Because it is not that rainfall exists over all grids, average rainfall intensity calculated from only rainfall grids has larger value than the average from all grids.

After 1hr lead prediction is done by the translation model, absolute prediction error \( E_{a_i} \) on grid \( i \) is calculated from difference between predicted rainfalls \( R_{p_i} \) and observed rainfalls \( R_{o_i} \) on the grid \( i \).

\[
E_{a_i} = R_{o_i} - R_{p_i}
\]  

(2)

Figure 2 shows statistics of the prediction error. Average is distributed around zero while it varies as time pass. Large standard deviation is shown when the rainfall intensity is big. Frequency distributions of \( E_a \) show a normal distribution pattern while it is various as time pass (Fig 3). Here, the statistics and the distributions are calculated only with non-zero prediction error.
Figure 4 and Figure 5 show spatial correlation coefficients from the prediction error fields. Each coefficient is calculated by grouping each pairs of error which apart one grid for 3km, two grids for 6km, etc. The coefficient variance with distance shows that effective range of the correlation is around 10 km.

3. Prediction Error Fields Simulation

If historical error structure is reasonably modeled, prediction error field could be simulated. Let’s assume predicted error field vector \( \mathbf{Y} \) can be decomposed to a matrix \( B \) which has a spatial correlation characteristic and a random value vector \( x; \mathbf{Y} = Bx \). Here, the random values \( x \) are uncorrelated each other but follows a given probability distribution. Then a covariance of the vector \( \mathbf{Y} \) would be same as the covariance matrix of the matrix \( B \). (Tachikawa et al., 2003)

\[
E[\mathbf{Y} \mathbf{Y}^T] = E[Bxx^TB^T] = BE[xx^TB^T] = BB^T \tag{3}
\]

After getting the matrix \( B \) from the covariance matrix \( BB^T \) using numerical matrix decomposition method, the error field vector \( \mathbf{Y} \) can be simulated with the matrix \( B \) and the random value vector \( x \).

Figure 6 shows example of simulated random error field using the covariance decomposition method. The simulated error fields are successfully reflect error structure of the past prediction error.

4. Further Research

Rainfall-runoff simulation using predicted rainfall data only gives deterministic runoff results without its uncertainty. If the random rainfall error field introduced here is used as a noise layer between rainfall and runoff surface, it would be very useful to figure out uncertainties in a runoff prediction.

5. References


Keywords: Radar rainfall prediction, Error structure, Random error field