Sensitivity Analysis and Optimization of Parameters in Xinanjiang Model
Taking Into Account Their Time Scale Dependency

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1. Introduction

The Xinanjiang model is the most popular rainfall-runoff model in China. There are fifteen parameters in a modified Xinanjiang model. In this study, the sensitivity of parameters in Xinanjiang model at different time scale was taken into account. We try to understand the sensitivities of each parameters and their time scale dependency, then, based on the analysis, develop an effective method to automatically calculate the parameters.

2. Time scale dependent sensitivity analysis

The Morris method is used here to analyze sensitivities of parameters at different time scales. For simplicity, consider a model which has \( k \) parameters that could take integer values in the set of \( \{0,1/(\rho-1),2/(\rho-1),\ldots,1\} \). So the impact of changing the \( i \)-th parameter for the model output, named as elementary effect, is defined as:

\[
d'_i(x) = \frac{j(x + e_i \Delta) - j(x)}{\Delta}
\]  

where \( j \) is the model output; \( \Delta \) is a predetermined multiple of \( 1/(\rho-1) \); \( x=(x_1,x_2,\ldots,x_k) \) is a selected parameters; and \( x+e_i \Delta \) is the transformed parameters, where \( e_i \) is a vector of zeros but with a unit as its \( i \)-th component.

Two sensitivity measures were calculated to evaluate the distribution of elementary effect, \( F_i \). Measure \( \mu^* \)

\[
\mu^* = \frac{1}{r} \sum_{j=1}^{r} |d'_i(x^{(j)})|
\]

estimates the overall effect of the parameter on the model output. And measure \( \sigma \)

\[
\sigma = \sqrt{\frac{1}{r} \sum_{j=1}^{r} (d'_i(x^{(j)}) - \overline{d'_i})^2}
\]

estimates the interaction that the parameter accepts from other parameters. Here \( r \) is the number of sampling from \( F_i \). The larger the measure \( \mu^* \) is, the more sensitive that parameter shows. And the larger the measure \( \sigma \) is, the stronger the effects that parameter accepts.

In this study, the Xinanjiang model is run at daily scale, and from the simulated daily discharge (output), the monthly and annual discharge are calculated. The modeling efficiencies are calculated from annual, monthly and daily discharge data respectively. They are used in the evaluation of the parameter sensitivities.

Figure 1 shows the sensitivity measures at annual, monthly and daily scale respectively. It is clearly that the parameters for data adjustment (Cp and Cep) are sensitive than all other parameters and interact with each other at annual scale (Figure 1(a)). After excluding these parameters, parameters controlling runoff generation (B, WLM, WUM) become sensitive together with several parameters controlling runoff routing (KG, CG)(Figure 1(b)). And parameters controlling runoff component separation and routing (B, SM, KI, KG, CI, CS) are relative sensitive at both monthly and daily scale (Figure 1(c) and (d)).

3. Multi-step optimization scheme

Based on the information from sensitivity analysis, a multi-step optimization scheme is set up with the help of SCEM-UA algorithm. The SCEM-UA algorithm is a global optimization method which is capable of handling high parameter dimensionality problem. The multi-step optimization scheme is described as follows:
(1) Assume a random set of parameters for initial values, and optimize parameters for data adjustment (Cp and Cep) at annual scale.

(2) Substitute the optimum values of Cp and Cep got from step (1) into the initial values while keep the other parameters the same. Then optimize the parameter group (SM, EX, KI, KG, CS, CI and CG) at daily scale.

(3) The optimum values got from step (1) and (2) are set to be initial values and optimize the parameter group (IMP, B, WM, WUM, WLM and C) at annual scale.

4. Case studies

In order to reduce the impact of measurement errors, a series of ‘perfect’ runoff data generated by Xinanjiang model directly was used as reference runoff firstly. 15 parameters were optimized step by step as describing above. All parameters converged to the true values with small errors. Figure 2(a) shows the hydrograph that calculated by using the optimum parameters.

The result of perfect data indicates that multi-step optimization scheme could search the set of true values theoretically. Then, the same series of observed runoff data were used instead of perfect data to check the capacity of this scheme. The hydrograph (Figure 2(b)) shows that the simulated daily runoff gives good fits to the observed data.

5. Conclusion

The time scale dependent sensitivity of parameters in Xinanjiang model was taken into account. Based on the sensitivity analysis, a multi-step optimization scheme is set up with the help of SCEM-UA algorithm. Theoretically this scheme is capable of searching the true values of parameters. And for the observed data, it also showed a good performance and acceptable optimum result.

References


Keywords: Time scale dependency, Sensitivity analysis, Multi-step optimization scheme