Considerations on shape identification for flow channel including a rotational body

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Abstract
In this paper, we describe the shape identification of a channel in incompressible viscous flow considering a rotational body. The purpose of this study is to obtain an optimal shape that will closely approach the target velocity in the target regions. The incompressible Navier Stokes equations are employed as the governing equations, and the adjoint variable method is applied to obtain the sensitivity for the optimal shape. The traction method is used to control the numerical oscillations of sensitivity for the shape update. Shape identification analysis is carried out using computed flow velocities in the target regions of target shape.

Keywords shape identification, adjoint variable method, finite element method, incompressible viscous fluid field, rotational body

Research Activity Group Mathematical Design

1. Introduction
In this study, an investigation is carried out into the shape optimization problem of the outer shape of a flow channel that includes a rotational body. So far, the shape optimization problem in Stokes flow is mathematically explained: the optimal shape for minimizing drag over an obstacle in Stokes flow is that of a rugby ball [1]. In shape optimization analysis, the sensitivity distribution with respect to the coordinate of the target body is oscillated on the boundary of the target body. In the present study, the traction method [2] is used to suppress the numerical oscillation of sensitivity. The traction method is applied to numerous shape optimization problems, such as the channel shape optimization problem considering the minimization of dissipation energy [3]; that in potential flow [4]; for that of a body located in a steady-state viscous flow field considering drag minimization and lift maximization [5]; for the stability of a Navier-Stokes flow field [6]. It has been confirmed that smoothing of the sensitivity distribution can be appropriately carried out and the optimal shape can be obtained numerically. Based on this knowledge, we extend here the shape optimization problem in an unsteady-state viscous flow field to that considering a rotational body [7]. Fan type and blade shape are constantly being studied with the aim of improving the performance of common mechanical devices that feature rotational bodies, such as turbojet engines, air conditioners, and heat sinks [8]; however, there appear to be few technical papers on the shape optimization problems specific to these mechanical devices. We therefore anticipate that the results of this study will assist in this regard. In some numerical experiments shown in reference [7], the identified shape remains virtually unchanged near the vertex, because a flow field is stagnant in that location, and the modified gradient does not significantly change. We therefore carry out the shape identification analysis using the taper type initial shape, and discuss its result in this paper. In the shape identification, the target shape is first given, and we examine whether the distributions of state variables such as the flow velocity for the obtained shape are specified with the target distributions for the aim of improving the performance. The shear-slip mesh update method is frequently employed in flow simulations that include a rotational body [9]. In the present study, we employ the shear-slip mesh update method to analyze unsteady-state viscous flow fields considering a rotational body. The mixed interpolation method is used in the flow analysis, and the traction method is employed in the shape identification problem. Some numerical experiments are shown by using the Freefem++ program code [10] in this paper.

2. Formulations for the shape identification problem
The performance function is first defined to obtain the shape that will optimally satisfy the objectives. In this study, we define it as shown in (1) assuming the objective of the flow velocity to be close to the target velocity in the target regions.

\[ J = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} Q_i (u_i - u_{ti})^2 d\Omega dt, \]  

(1)

where \( Q_i, u_i \) and \( u_{ti} \) indicate weight constant, computed flow velocity and target flow velocity. \( t_0, t_f \) and \( \Omega \) de-
note an initial time, terminal time and computational domain. The weight constant $Q_l$ is given as 1 in the target regions, and as 0 in other regions. The target flow velocity $u_{ti}$ is a preassigned condition in shape identification analysis, so it is necessary to give this condition appropriately to be able to obtain the optimized shape: this is because the target flow velocity $u_{ti}$ is affected by the inflow and the outflow boundary conditions, the Reynolds number and the angular velocity of the body. The incompressible Navier-Stokes equations shown in (2) and (3) are employed as the governing equations.

\begin{equation}
\dot{u}_i + u_j u_{i,j} + p,i - \frac{1}{Re} u_{i,j,j} = 0 \quad \text{in } t \in [t_0, t_f] \text{ in } \Omega, \tag{2}
\end{equation}

\begin{equation}
u_{i,i} = 0 \quad \text{in } t \in [t_0, t_f] \text{ in } \Omega. \tag{3}
\end{equation}

The initial and boundary conditions shown in (4), (5) and (6) are introduced as the constraint conditions for the performance function. The volume preservation condition shown in (7) is also considered in this study.

\begin{equation}
u_i = \hat{u}_i \quad \text{at } t = t_0 \text{ in } \Omega, \tag{4}
\end{equation}

\begin{equation}u_i = \hat{u}_i \quad \text{in } t \in [t_0, t_f] \text{ on } \Gamma_i, \Gamma_r, \text{ and } \Gamma_d, \tag{5}
\end{equation}

\begin{equation}T_i = \hat{T}_i = \left( -p\delta_{ij} + \frac{1}{Re} u_{i,j} \right) n_j \quad \text{in } t \in [t_0, t_f] \text{ on } \Gamma_d, \tag{6}
\end{equation}

\begin{equation}A_i = A_{\text{initial}} \quad \text{in } \Omega, \tag{7}
\end{equation}

where $u_i$, $p$, $Re$, $\delta_{ij}$ and $n_i$ respectively indicate a flow velocity, pressure, Reynolds number, Dirac’s delta function and direction cosine of the unit outward normal of the boundary. $\Gamma_i$, $\Gamma_2$, $\Gamma_r$ and $\Gamma_d$ represent the Dirichlet, Neumann, rotational body, and design boundaries. On the rotational boundary, the flow velocity is given as $u_i = -\omega \sin \theta$, $u_2 = \omega \cos \theta$. The parameters $\omega$, $r$ and $\theta$ respectively indicate the angular of velocity, distance from the rotational center point, and angle about the rotational center point. In addition, in (7), $m_x$ and $A_{\text{initial}}$ represent the total number of elements and the area of the whole domain except for the rotational body at the initial iteration. The boundary condition applied to $\Gamma_r$ is referred to as the adhesive boundary condition [11]. On introducing the adjoint variables, the performance function is extended to (8), which is referred to as the Lagrange function.

\begin{equation}J^* = \frac{1}{2} \int_{t_0}^{t_f} \int_\Omega Q_l(u_i - u_{i,t})^2 d\Omega dt + \int_{t_0}^{t_f} \int_\Omega u_i^* \left( \dot{u}_i + u_j u_{i,j} + p,i - \frac{1}{Re} u_{i,j,j} \right) d\Omega dt + \int_{t_0}^{t_f} \int_\Omega p^* u_{i,i} d\Omega dt, \tag{8}
\end{equation}

where $u_i^*$ and $p_i^*$ respectively indicate the adjoint variables for the flow velocity and pressure. The first variation of the Lagrange function must be zero to satisfy the stationary condition, and is written as (9).

\begin{equation}\delta J^* = \frac{\partial J^*}{\partial u_i} \delta u_i^* + \frac{\partial J^*}{\partial p} \delta p^* + \frac{\partial J^*}{\partial \delta x_i} \delta x_i, \tag{9}
\end{equation}

where $x_i$ indicates the nodal coordinates. Considering the stationary condition of the Lagrange function, the adjoint equations and conditions for adjoint variables shown in (10)–(14) are obtained.

\begin{equation}-u^*_{i,i} + u_{j,i} u_{j,i}^* - (u_{j,i}^*)_i - p^*_{i,i} - \frac{1}{Re} u_{i,j,j}^* + Q_l (u_i - u_{i,t}) = 0 \quad \text{in } t \in [t_0, t_f] \text{ in } \Omega, \tag{10}
\end{equation}

\begin{equation}u^*_{i,i} = 0 \quad \text{in } t \in [t_0, t_f] \text{ in } \Omega, \tag{11}
\end{equation}

\begin{equation}u^*_{i,i} = 0 \quad \text{at } t = t_f \text{ in } \Omega, \tag{12}
\end{equation}

\begin{equation}u^*_{i,i} = 0 \quad \text{in } t \in [t_0, t_f] \text{ on } \Gamma_i, \Gamma_r, \text{ and } \Gamma_d. \tag{13}
\end{equation}

The gradient of the Lagrange function with respect to the coordinates on the design boundary, $G_i$, is derived using (15).

\begin{equation}\int_{t_0}^{t_f} \int_\Gamma T_i^* \delta u_i d\Gamma dt = \int_{t_0}^{t_f} \int_{\Gamma_\Delta} T_i^* \delta u_i d\Gamma dt + \int_{t_0}^{t_f} \int_{\Gamma_\Delta} T_i^* u_{i,j} \delta x_j d\Gamma dt = \int_{t_0}^{t_f} \int_{\Gamma_\Delta} T_i^* \delta u_i d\Gamma dt + \int_{\Gamma_\Delta} G_i \delta x_j d\Gamma. \tag{15}
\end{equation}

$\Gamma_\Delta$ represents $\Gamma_i$, $\Gamma_2$, and $\Gamma_r$. The gradient $G_i$ is modified by the traction method, in which the gradient $G_i$ is employed as the external force for the linear elastic body, and the displacement value is used as the modified gradient $G_i^*$.

\begin{equation}K_{ij} G_{i}^{*} = G_{i}, \tag{16}
\end{equation}

where $K_{ij}$ indicates the stiffness matrix. $G_i$ is defined on the design boundary and $G_i^*$ is in the domain. The numerical procedure is shown below.

1. Select the initial surface coordinates $x_i$ on design boundary $\Gamma_d$, convergence criterion $\varepsilon$, and number of iterations $l = 0$.

2. Solve the Navier-Stokes equations, (2) and (3) under the initial and boundary conditions, (4)–(6).

3. Compute the performance function, i.e., (1). If the value of the judgment equation $|J^{l+1} - J^{l}|/|J^0|$ is lower than convergence criterion $\varepsilon$, this computation finalizes. Otherwise, go to step 4.

4. Solve the adjoint equations, (10) and (11) under the terminal and the boundary conditions, (12)–(14).

5. Compute the gradient of the Lagrange function with respect to the surface coordinates on the design boundary (15).

6. Compute the modified gradient based on the traction method, i.e., (16).
7. Update the surface coordinates on the design boundary using the shape update equation, \( x_{i}^{l+1} = x_{i}^{l} - \eta G_{i}^{l} \), where \( \eta \) indicates the step length and return to step 2.

3. Numerical experiments

Shape identification analysis considering a rotational body is carried out. The two computational models shown in Figs. 1 and 2 are employed as the initial shape, and the boundary conditions are shown in these figures. The computational conditions are shown in Table 1. The initial time \( t_{0} \) and terminal time \( t_{f} \) are given as 0.00 and 8.00. The initial condition is given as \( u_{i} = 0 \) in the whole domain. The non-dimensional angular of velocities \( \omega \) is given as 2\( \pi \). The shear-slip mesh update method is employed to carry out the flow field considering a rotational body. The first-order triangular element is employed for the variables \( p \) and \( p^* \), and the second-order triangular element is used for the variables \( u_{i} \) and \( u_{i}^* \) for the interpolation of the state and the adjoint variables in these problem. The target shape in the shape identification analysis is shown in Fig. 3, and the computed flow velocities in the target regions are used as the target flow velocity \( u_{i} \) in the shape identification analysis. In the target regions, the weighting constant \( Q_{i} \) is given as 1 in the target regions; otherwise the weighting constant \( Q_{i} \) is given as 0. In addition, the nodes in target regions are not updated at each iteration, and other nodes are updated based on the traction method. To discretize the governing and adjoint equations in space, the Galerkin-characteristics finite element method and the conventional finite element method are used in cases A and B, respectively. As the discretization in time, the backward Euler method and the Runge-Kutta method are employed in cases A and B, respectively. As the computational result, Fig. 4 shows the variation in both the performance function and area. In all cases, it can be seen that the value of the performance function gradually decreases with the area remaining constant. In comparison with the initial shape, the obtained shape and target shape are shown in Figs. 5 and 6.

From the results of numerical experiments, it can be said that the appropriate shape can be obtained such that the value of the performance function decreases to near zero. On the other hand, it is found that it is unusual for the target shape to be correctly obtained. In case A, it is found that the obtained shape is affected by the vertex of the initial shape. The distribution of the velocity vector and the pressure contour near the vertex of the initial shape for case A is shown in Fig. 7. From this result, it can be seen that the flow field is stagnant near the vertex: the modified gradient does not change significantly in that location. This is one reason why the identified shape remains virtually unchanged. It therefore appears to be a major challenge to carry out shape updating in a convection flow field near an abruptly narrowing section.

4. Conclusions

In this study, we have presented outer shape identification problems considering a rotational body in a flow field based on the adjoint variable and the finite element methods. The performance function is defined so as to obtain a shape by which the flow velocity comes close to the target velocity in the target regions. The incompressible Navier-Stokes equations have been employed as the governing equations. Introducing the adjoint variables, the Lagrange function is defined from the performance function and government equations. The gradient of the Lagrange function with respect to the coordinates on the design boundary is derived from the first variation of the Lagrange function and the stationary condition. The traction method is used to suppress numerical oscillation of the gradient of the Lagrange function. Shape identification analysis considering a rotational body is
carried out in the numerical experiments. In all cases, the value of the performance function decreases to near zero, although, it is unusual for the target shape to be appropriately obtained. Notably for case A, the shape affected by the vertex of the initial shape was obtained. The reason is that flow field is stagnant near the vertex, so the modified gradient near the vertex does not significantly change. In addition, in this study, the evaluation area of the flow velocity is limited in target regions. The source terms in the adjoint equations are given in the target regions, and it appears that the position of target regions receive large influence on the sensitivity with respect to movement value on the design boundary. For instance, if the evaluation term of dissipated energy is added to the performance function used in this study, the flow velocity can be evaluated in the whole domain. As one of the future work, it therefore appears that it is necessary to include the evaluation term of flow velocity in the whole domain in the performance function.

References