Computing of multipole moments from incomplete boundary data for Magnetoencephalography inverse problem

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Abstract
In this paper, we present a method for reconstructing the dipole sources inside the human brain from radial Magnetoencephalography data measured on the part of the boundary which encloses the source. Combining the proposed method with the direct method provides a good initial solution for an optimization-based algorithm. The method is verified with the numerical simulations, phantom experiments, and a somatosensory evoked field data analysis.

Keywords inverse problem, Magnetoencephalography, multipole moment

Research Activity Group Algorithms for Matrix / Eigenvalue Problems and their Applications

1. Introduction
Magnetoencephalography (MEG) is a non-invasive brain monitoring tool that records the magnetic field outside the head generated by the neural current in the brain. Here one must solve an inverse problem to reconstruct the current source from the measured magnetic field. The conventional methods for the inverse problem assume that the current source can be represented by a relatively small number of equivalent current dipoles (ECDs). Although the usual algorithm for this source model is the non-linear least-squares method that minimizes the squared error of the data and the forward solution, it has a problem that an initial parameter estimate close to the true one is required without which the algorithm often converges to a local minimum. To address this issue, several researchers have proposed a direct method [1–4] which reconstructs the source parameters directly and algebraically from the data. From the efficiency of the algorithm, it is expected to be used for real-time monitoring of the brain activity. Also from a practical point of view, it can provide a good initial solution for the iterative algorithm.

However, the problem of the direct method is that it requires the data on the boundary which encloses the source. The practical MEG system has no sensors in front of the face and beneath the neck. The lack of data at those parts is the cause of error in computing the weighted integral of the boundary data.

The aim of this paper is to develop a method to reconstruct the source parameters from the data on the part of the boundary. First, using the multipole expansion of the radial component of the magnetic field, the multipole moments are estimated from the data only on the upper hemisphere. We regularize the linear equation proposed by Taulu et al. [5] by using the singular value decomposition. Then, by combining this method with our direct method proposed in [4], the source parameters are estimated from incomplete boundary measurements, which can be used as a good initial solution for an optimization-based algorithm.

The rest of this paper is organized as follows. In Section 2, our direct method is summarized. The method using data only on the upper hemisphere is proposed in Section 3, which is verified by numerical simulations, phantom experiments, and a real data analysis in Sections 4, 5, and 6, respectively.

2. Direct method
Assume that the head can be modeled by the three concentric spheres \( \Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \) representing the brain, skull, and scalp, respectively. The sensors measuring the radial component of the magnetic field are placed on the upper hemisphere \( \Gamma \) with radius \( R \) centered at the origin. Generally, the solution to the inverse problem in which the neural current in the brain is reconstructed from the measured MEG data is not unique. To guarantee the uniqueness, we assume that the neural current is expressed by equivalent current dipoles (ECDs) \( J_p = \sum_{k=1}^{K} p_k \delta(r - r_k) \) where \( r_k \in \Omega_1 \). The radial component of the magnetic field at \( r \in \Gamma \) is then given by the Biot-Savart law

\[
B_r(r) = \frac{\mu_0}{4\pi} \sum_{k=1}^{K} (n \times p_k) \cdot \nabla' \frac{1}{|r - r'|} |r' = r_k|,
\]

where \( \mu_0 \) is the permeability assumed to be constant in the whole space and \( n = r/|r| \) is the outward unit...
normal to $\Gamma$. Our inverse problem is to reconstruct the number $N$, positions $r_k$, and the moments $p_k$ of the ECDs from measurements of $B_r$ on $\Gamma$.

In contrast to the conventional method with the non-linear least-squares method, we proposed a direct method which reconstructs the source parameters directly and algebraically from MEG data [4]. The method is based on the multipole expansion of the radial MEG given by

$$B_r = \mu_0 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{l+1}{2l+1} M_{lm} \hat{Y}_{lm}^*(\theta, \phi),$$

(1)

where $\hat{Y}_{lm}^*(\theta, \phi)$ are the normalized spherical harmonic functions. It is shown that the multipole moments $M_{lm}$ where $l=m$ are expressed in terms of the source parameters. On the other hand, they are expressed by the boundary data. As a result, we have the algebraic equations relating the source parameters to the data:

$$\sum_{k=1}^{N} q_k S_k^{\alpha} = \alpha_m,$$

(2)

where $S_k \equiv x_k + iy_k$ is the $k$th source position projected on the $xy$-plane, $q_k \equiv r_k \times p_k$ is the magnetic moment of the $k$th ECD, $S_k \equiv [q_k]_x + i[q_k]_y$, where $[\cdot ]_x$ represents the $x$-component of the vector $\cdot$, and

$$\alpha_m = \frac{2m + 3}{(m+1)\mu_0} \int_S B_r(x + iy)^{m+1}dS,$$

(3)

where $S$ is a sphere which is centered at the origin and encloses $\Omega$. It is also shown that (2) is reduced to a generalized eigenvalue problem [6] so that the source parameters can be reconstructed algebraically from the boundary data. Although this method is theoretically simple, a problem is that we need $B_r$ on the whole $S$ which encloses $\Omega$. In the practical situation the sensors cannot be placed in front of the face and in the middle of the neck. Hence, lack of data on the part of $S$ becomes a factor of errors in computing $\alpha_m$.

3. Computation of $M_{mm}$ from data on the upper hemisphere

Truncate (1) up to order $L$:

$$B_r \simeq \sum_{l=0}^{L} \sum_{m=-l}^{l} X_{lm} \hat{Y}_{lm}^*(\theta, \phi),$$

(4)

where $X_{lm} = \mu_0 \cdot [(l+1)/(2l+1)] \cdot (M_{lm}/r^{l+2})$. Then the linear equations relating the multipole moments to radial MEG on $\Gamma$ are obtained [5]: $d = Gx$, where $d = (B_{r1}, B_{r2}, \ldots, B_{rN})^T \in \mathbb{R}^N$ is the data measured on $\Gamma$, $x = (X_{00}, X_{1-1}, X_{10}, \ldots, X_{L,L})^T \in \mathbb{C}^{(L+1)^2-1}$ consists of the unknown multipole moments, and

$$G = \begin{pmatrix}
\hat{Y}_{-L+1}^*(\theta, \phi) & \ldots & \hat{Y}_{-1}^*(\theta, \phi) & \ldots & \hat{Y}_{L+L}^*(\theta, \phi) \\
\hat{Y}_{L-1}^*(\theta, \phi) & \ldots & \hat{Y}_{L}^*(\theta, \phi) & \ldots & \hat{Y}_{L+L}^*(\theta, \phi) \\
\vdots & & \vdots & & \vdots \\
\hat{Y}_{L-1}^*(\theta, \phi) & \ldots & \hat{Y}_{L}^*(\theta, \phi) & \ldots & \hat{Y}_{L+L}^*(\theta, \phi)
\end{pmatrix}.$$

where $(\theta, \phi)$, represent the spherical coordinates of the $i$th sensor. We choose $L$ such that $L$ is maximum under the condition that the linear system becomes over-determined, that is, $N > (L+1)^2 - 1$.

In order to obtain $x$ while suppressing the effect of noise contained in $d$, we use the truncated singular value decomposition of $G$ denoted by $G_{T}^+ \cdot d$. From the components of $l = m$ in $x$ with the direct method in Section 2, we can identify the source parameters projected on the $xy$-plane. Using them as the initial solution, the $z$-coordinates and the moments of the source are determined by the non-linear least-squares method. Then, we compute the Goodness of Fit (GoF) defined by

$$\text{GoF} \% = 100 \left[ 1 - \frac{\sum_{i=1}^{N} (B_{\text{data}[i]} - B_{\text{th}[i]})^2}{\sum_{i=1}^{N} B_{\text{data}[i]}^2} \right],$$

(5)

where $B_{\text{data}[i]}$ and $B_{\text{th}[i]}$ are the data and the forward solution at the $i$th sensor position, respectively. We repeat these computations by changing $T$ and choose $T$ such that GoF becomes maximum.

Practically, the sensors called ‘gradiometers’ are often used which measure the difference of magnetic field measured on $d$ at point $r$ on $\Gamma$ and $r + b$, where $b$ is called the baseline distance, in order to cancel the noise which originates from the source far apart. In this case, we only need to change $X_{lm}$ from $X_{lm} = \mu_0 \cdot [(l+1)/(2l+1)] \cdot (M_{lm}/r^{l+2})$ to $X_{lm} = \mu_0 \cdot [(l+1)/(2l+1)] \cdot (1/r^{l+2} - 1/(r+b)^{l+2}) \cdot M_{lm}$. Thus even in this case, from $x = G_{T}^+ d$, we can obtain $M_{lm}$ where $l = m$ that are used in the direct method.

4. Numerical simulations

First, we verified our method numerically. A single dipole $(K = 1)$ was set at $r_1 = (40,40,40)$ mm with the moment $p_1 = (0,0,10)$ nAm. $N = 183$ gradiometers with $b = 50$ mm were uniformly distributed on the upper hemisphere with $R = 120$ mm using the spherical t-design [7]. For this number of the sensors $N$, the truncation order was $L = 12$. Gaussian noise was added whose standard deviation was 10% of the root-meansquares of the theoretical data.

Fig. 1 shows the relative localization error (the error divided by $R$) and GoF when changing the truncation order $T$. We observed that choosing $T = 118$ gave the maximum GoF (98.1%) and the minimum relative localization error (0.18%). Hence from GoF we can determine the optimal truncation order $T$. The relative magnetic moment error was 1.8%.

Fig. 2 shows the estimated $|q_2/q_1|$ when assuming that there were $K' = 2$ dipoles. We observed that $|q_2|$ became much smaller than $|q_1|$ for most $T$. In fact, when $T = 118$ that was the truncation order used for reconstruction, $|q_2|/|q_1| = 0.043$, showing that $|q_2|$ can be neglected compared to $|q_1|$ and hence $K = 1$. 

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5. Phantom experiments

Next we examined our method using the phantom head. We used 168 gradiometers as shown in Fig. 3 where the radius $R = 129$ mm. $L$ was set to be 11. A single current source ($K=1$) was moved on the plane $z_1 = 16$ mm and $z_1 = 42$ mm where $\sqrt{x_1^2 + y_1^2} = 62.5$ mm and $\phi_0 = \tan^{-1} \frac{y_1}{x_1} = 45 \times i$ ($i = 0, 1, \ldots, 7$) degrees. The reconstruction results are shown in Figs. 4 and 5. The mean estimation error was 3.5 mm and 2.4 mm, respectively. Fig. 6 shows an example of the relationship between the relative localization error and GoF when $z = 42$ mm and $\phi = 180^\circ$. It is observed that the truncation order $T = 78$ that maximizes GoF coincides with the order that minimizes the localization error. This coincidence was observed in all the source positions.

6. Real data analysis

We analyzed a somatosensory evoked field (SEF) data where a right hand index finger was electrically stimulated. 205 gradiometers were used. $L$ was set to be 13. In the time series shown in Fig. 7, we used the peak at 101 msec for reconstruction (the peak at 0 msec is an artifact originating from the electrical stimulation). Fig. 8 shows the contour maps of radial MEG. Fig. 9 shows the localization result. The maximum GoF was 74% when $T = 51$ with which reconstruction was conducted. In this case, two dipoles are estimated in the right and left somatosensory cortices. Figs. 10 and 11 shows $|q_2/q_1|$ and $|q_3/q_2|$, respectively, when assuming that there were $K' = 3$ dipoles. One finds that $|q_3/q_2|$ often becomes small when changing $T$ while $|q_2|$ is comparable to $|q_1|$ for wide range of $T$. In fact, when $T = 51$ which was used for reconstruction, $|q_2/q_1| = 0.83$ and $|q_3/q_2| = 0.0002$ from which we can reasonably judge that $K = 2$.

7. Conclusion

In this paper, we developed a method for computing the multipole coefficients of the radial magnetic field created by the dipole source from radial MEG data on the
Fig. 6. Relative localization error (top) and GoF (bottom) when $\varphi_1 = 180^\circ$ and $z_1 = 42$ mm. The truncation order $T = 78$ maximizes GoF and minimized the relative localization error.

Fig. 7. Time series data.

Fig. 8. Contour map of radial MEG. Red and blue colors show the outward and inward magnetic field.

Fig. 9. Localization result at 101 msec. Two dipoles are estimated in the left and right somatosensory cortices.

Fig. 10. $|q_2/q_1|$ when assuming that there were $K' = 2$ dipoles. For most $T$, $|q_2|$ is comparable to $|q_1|$. When $T = 51$ which is the truncation order used in reconstruction, $|q_2/q_1| = 0.83$.

Fig. 11. $|q_3/q_2|$ when assuming that there were $K' = 3$ dipoles. For most $T$, $|q_3|$ becomes much smaller than $|q_2|$. When $T = 51$ which is the truncation order used in reconstruction, $|q_3/q_2| = 0.0002$.

upper hemisphere, that were used in the direct inversion method for reconstructing the dipole parameters. The method was verified with the numerical simulations, phantom experiments, and somatosensory evoked field (SEF) data analysis. Although it was suggested that $K$ could be estimated from the ratio of the source strength assuming larger number of dipoles than the true one, the rigorous analysis for the threshold is required. Generalization of our method to the case when the data is given not on the upper hemisphere but on an arbitrary open surface which does not enclose the source is straightforward; its verification with simulations as well as phantom/real data analyses is also required.

References