Credit risk valuation model for real estate non-recourse loan

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Abstract
In this paper we propose a practical cost-effective model to estimate the credit risk of a large portfolio of real estate non-recourse loans. It uses information that is as easy to get and update as possible, such as real estate investment indices and macroeconomic indices. Empirical characteristics of real estates can be taken into account, such as serial correlations, cross-sectional correlations within individual properties, lagged effects of macroeconomic factors.

Keywords credit risk, non-recourse loan, real estate, LTV, macroeconomic factor

Research Activity Group Mathematical Finance

1. Introduction
A real estate non-recourse loan (NRL) is a loan that is secured by a pledge of real properties as collateral. For a financial institute that manages a lot of real estate NRLs, it is necessary to estimate probable losses from the defaults or the changes of mark-to-market loan values.

As for a NRL, if the borrower defaults, the lender can seize the collaterals but the recovery is limited to the value of them. Thus, the value of a NRL depends rather on the value of mortgage properties than financial or credit conditions of the lender. To estimate the risk of real estate NRLs, it is natural to employ structural approaches, in which a NRL’s default is triggered by loan-to-value ratio (LTV) exceeding some threshold. This kind of approaches was first introduced by Merton [1] to model the defaults of corporate bonds, and has been employed for modeling defaults of commercial mortgage backed securities (CMBS), which are securities backed by NRLs. Liu et. al. [2] and Kau et. al. [3] and Shiu et. al. [4] employed the first passage time model to estimate default probabilities of CMBS on the basis of LTV. They described the dynamics of a property value as a simple lognormal process. However, considering the application to stress testing, it is more desirable to use a model that can reflect some macroeconomic scenarios.

In the appraisal practice, the value of a real property is computed by adding up the discount values of the net cash flows from it. Kanzaki and Sasaki [5] proposed a dynamic model combined with widely used appraisal method: capitalization rate (cap-rate) model. “Cap-rate of a standard property” was statistically estimated by the use of a large database of individual trade information. They also developed a time series model of the cap-rate process lagged with macroeconomic factors. Although this type of model is a useful tool to valuate appraised value of each collateral property on credit administrations, it will cost a lot to develop or purchase a large database and continuously update and maintain it.

In the risk management practice in financial institutions, which hold a large number of real estate NRLs, it is often needed to fast estimate rough distributions of losses or the effects of macroeconomic scenarios on the basis of a relatively simple model. We developed a new practical cost-effective model which uses information that is as easy to get and update as possible, such as real estate investment indices, published by the Association for Real Estate Securitization Japan (ARES), and macroeconomic indices by government organizations, and so on. It can take empirical characteristics of real estates into account, such as serial correlations, cross-sectional correlations within individual properties, lagged effects of macroeconomic factors. In our model, we have set the following assumptions: (1) All of the characteristics of each individual property are involved in the appraised value at the loan origination. (2) A real estate NRL defaults as soon as the appraised value of the collaterals falls short of the remaining principal, i.e. first passage time model. (3) After the default, there exists the liquidity risk that the sales value of the collaterals differs from the appraised value.

An outline for this paper is as follows. Section 2 presents the model formulations, and Section 3 provides a description of the data and estimates the model for real estates in Japan. In Section 4, we show numerical examples of the risk measurement of virtual NRLs with our model. A conclusion and future extension are provided in Section 5.

2. Model
In this section, we introduce a credit risk valuation model for NRL, which enables us to calculate the losses at default time. First, we model the time series of real
estate investment indices, which determine the approximate level of property values. Then, we model appraised value of collateral properties of target NRL. In our real estate investment indices modeling, we consider serial correlations of the indices. Also, we incorporate the macroeconomic factors, which precede the property values, in the model to capture the turning points of the indices. Real estate investment indices are calculated by property types, such as office-type, residential-type, retail-type, etc. We describe the time series of the indices with vector auto regressive (VAR) model:

\[
\begin{align*}
&\Delta \log(Z_i^1) \\
&\Delta \log(Z_i^2) \\
&\vdots \\
&\Delta \log(Z_i^C)
\end{align*}
\]

\[
= \begin{pmatrix}
\alpha_1 + \beta_1 \Delta \log(Z_{i-1}^1) + \gamma_1 \Delta M_{t-1-t} + \epsilon_1 \\
\alpha_2 + \beta_2 \Delta \log(Z_{i-1}^2) + \gamma_2 \Delta M_{t-1-t} + \epsilon_2 \\
\vdots \\
\alpha_C + \beta_C \Delta \log(Z_{i-1}^C) + \gamma_C \Delta M_{t-1-t} + \epsilon_C
\end{pmatrix},
\]

where \(Z_i^c\) denotes the index value of property type \(c \in \{1, 2, \ldots, C\}\) at time \(t\) and \(\Delta \log(Z_i^c) := \log(Z_i^c) - \log(Z_{i-1}^c)\). Coefficients \(\alpha, \beta, \gamma\) and \(\epsilon\) are the parameters to be estimated. Random variable \(M_t\) stands for a macroeconomic variable and \(\epsilon \in \{0, 1, 2, \ldots\}\) is the time lag. Random variables \((\epsilon_1, \epsilon_2, \ldots, \epsilon_C)\) denote random noises which have normal distribution with mean 0, namely \((\epsilon_1, \epsilon_2, \ldots, \epsilon_C) \sim N(0, \Sigma)\).

With real estate investment indices \(Z\), the appraised value of collateral property \(V_i^c\) is modeled as follows:

\[
\log(V_i^c) = \log(V_{i-1}^c) + \sqrt{R_{C(i)} \Delta \log(Z_i^c)} + \sqrt{1 - R_{C(i)} \epsilon_{i,t}},
\]

where, \(i \in \{1, 2, \ldots, I\}\) denotes the label of each property. \(C(i)\) denotes the property type to which the collateral property \(i\) belongs. \(R_{C(i)}\) is the square of correlation coefficient. Random variable \(\epsilon_{i,t}\) represents error term and \(\epsilon_{i,t} \sim N(0, \sigma_{\epsilon(i)})\). Let us denote the principal of NRL at time \(t\) as \(P_{NRL,t}\). Let \(\tau\) denote the time of default which is defined as the first time that the underlying process, the sum of all collateral values \(V_{NRL,t} = \sum_i V_i^c\), crosses the barrier \(P_{NRL,t}(1 + A)\):

\[
\tau = \inf\{t|V_{NRL,t} < P_{NRL,t}(1 + A)\}.
\]

Here, the additional constant term \(A > 0\) is introduced for sound risk management. In usual case, we set \(A = 0\).

We obtain sales values of collaterals by adjusting liquidity risk on appraised values:

\[
V_{SELL, NRL}^c = \sum_i (1 - \ell_i) \times V_i^c.
\]

Here, the random variable \(\ell_i \sim N(0, \sigma_{\ell})\) represents the liquidity risk, which means the risk of difference between sales value and appraised value. If \((1 - \ell_i) > 0\) (i.e. \(\ell_i < 0\)), the sales value of property \(i\) is higher than the appraised value. On the other hand, if \((1 - \ell_i) < 0\) (i.e. \(\ell_i > 0\)), the sales value of property \(i\) is lower than the appraised value. The constant \(\sigma_{\ell}\) denotes the standard deviation of the spread rates, (sales value – appraised value)/appraised value. Then, we obtain the loss given default (LGD) as

\[
LGD = \max\{P_{NRL,t} - V_{SELL, NRL}^c, 0\}.
\]

3. Model estimation

In this section, we show the model estimation procedure by the use of sample data in Japan.

3.1 Data

We use the sample data of Tokyo stock exchange J-REIT index and ARES Japan Property Index (AJPI). AJPI is a real estate investment performance index, and generally shows the investment return in a certain investment period. In particular, we use the capital return of AJPI, with property types of office-type, residential-type and retail-type, for real estate investment indices \(Z\). Also, we use appraised value samples of properties which constitute J-REITs. The sample period is from March 2002 to March 2010. We set the time unit of the time series models as 6 month in order to keep the sample series independent. For leading indicator, we lined up some candidate macroeconomic variables as follows: Tokyo stock exchange REIT index, business sentiment diffusion index of real estate sector, diffusion index on lending attitude of financial institute against real estate sector, the number of new housing starts, money stock, spread between long and short interest rates. The candidates of time lag are 0, 6, 9, 12, 15, 18, 24 months. We selected the appropriate variable and time lag by following two steps. First, we plotted the time series of AJPI and the variables and choose the time lag under which the turning points of AJPI and the variables are close. Then, we calculated the correlation coefficient of AJPI and variables and choose the set of variable and time lag from both point of view. As a result, we selected business sentiment diffusion index of real estate with 6 month lag, diffusion index on lending attitude of financial institute against real estate with 12 month lag and Tokyo stock exchange REIT index with 9 month lag. Figs. 1, 2 and 3 are the time-series plots of AJPI and these figures show the turning points of AJPI and the variables are close.

3.2 Estimation result

We estimate the real estate index time series model with multiple linear regression analysis. Table 1 shows the estimation result of AJPI time series model with business sentiment diffusion index with 6 month lag. Table 1 shows the coefficient of determination \(R^2\) exceed 50% for each property type. In addition, as the estimated coefficient of leading indicator is positive with the business sentiment diffusion index, the turning point of business sentiment diffusion index captures the turning point of estimated AJPI values. The estimated coefficients with leading indicators of lending attitude of financial institute against real estate sector and Tokyo stock exchange REIT index were not significant, thus we do not focus on them hereafter.

The estimates of covariance matrix \(\Sigma\) for business sen-
Table 1. Estimation Result of AJPI time series model with business sentiment diffusion index with 6 month lag.

<table>
<thead>
<tr>
<th></th>
<th>coefficient</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office</td>
<td>α</td>
<td>0.001</td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>0.620</td>
<td>3.994</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>0.002</td>
<td>2.885</td>
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<tr>
<td>Adjusted R²</td>
<td></td>
<td></td>
<td>0.818</td>
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Retail

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<th>p-value</th>
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</thead>
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<tr>
<td></td>
<td>α</td>
<td>–0.001</td>
<td>–1.149</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>0.429</td>
<td>1.467</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>0.002</td>
<td>1.513</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td></td>
<td></td>
<td>0.576</td>
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Residential

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<th>p-value</th>
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<tr>
<td></td>
<td>α</td>
<td>–0.006</td>
<td>–1.044</td>
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<tr>
<td></td>
<td>β</td>
<td>0.324</td>
<td>0.877</td>
</tr>
<tr>
<td></td>
<td>γ</td>
<td>0.002</td>
<td>1.845</td>
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<tr>
<td>Adjusted R²</td>
<td></td>
<td></td>
<td>0.628</td>
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</table>

Table 2. MLE of appraised value model.

<table>
<thead>
<tr>
<th></th>
<th>Office</th>
<th>Retail</th>
<th>Residential</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_c</td>
<td>0.857</td>
<td>0.965</td>
<td>0.987</td>
</tr>
<tr>
<td>δ_c</td>
<td>0.146</td>
<td>0.279</td>
<td>0.385</td>
</tr>
<tr>
<td>LL_c</td>
<td>6157.7</td>
<td>1766.9</td>
<td>9456.2</td>
</tr>
</tbody>
</table>

where

\[ L_i = \sum_{s=2}^{n(i)} \left\{ -\frac{1}{2} \ln \left( 2\pi S_c^2 \right) + \frac{\left( \Delta \ln V_i,T(s) - \mu_{c,s} \right)^2}{2S_c^2} \right\}, \]

\[ \mu_{c,s} = \sqrt{R_c} \left( \alpha_c + \beta_c \Delta \ln Z_c^T(s-1) + \gamma_c \Delta M_{s-1} \right), \]

\[ \Delta M_{s-1} = M_{T(s-1)} - M_{T(s-1)-1}, \]

\[ S^2_c = R_c \sigma_c^2 + (1 - R_c) \delta_c^2. \]

Here, \( \sigma_c \) is the root of the diagonal element of \( \Sigma \). Table 2 shows the estimated parameters of appraised value model. Finally, we estimate liquidity risk parameter \( \sigma \) by the use of traded records of properties which constitute J-REITs after 2008 and obtained \( \sigma \) = 13.68%.

4. Numerical examples of credit risk valuation of NRLs

In this section, we show numerical examples of calculating losses at the default time of sample NRLs. We obtain loss distribution of a NRL with Monte Carlo simulation with the estimated parameters in Section 3. In particular, we utilize business sentiment diffusion index for the leading indicator. In order to generate scenarios of business sentiment diffusion index, we model time series of business sentiment diffusion index with vector regression model:

\[ \Delta M_t = \alpha_0 + \alpha_1 \Delta M_{t-1} + \epsilon, \epsilon \sim N(0, \sigma). \]

We obtain estimated parameters \( \alpha_0 = -0.791, \alpha_1 = 0.427, \sigma = 7.22 \) with the least squares method. In order to consider the high volatility in the real estate bubble

To estimate appraised value model, we use maximum likelihood estimation method with sample data \( V_i,T(s) (s = 0, 1, \cdots, n(i)) \) of property \( i \). Likelihood function \( LL_c \) is obtained as follows:

\[ LL_c(R_c, \delta_c) = \sum_{i:C(i)=c} L_i, \]
period in 1980s, we adjust the volatility as follows:

$$\Sigma = \left( \frac{\sigma_{LT}}{\sigma_{ST}} \right)^2 \times \hat{\Sigma}, \quad \sigma = \left( \frac{\sigma_{LT}}{\sigma_{ST}} \right)^2 \times \hat{\sigma}_{C(i)}.$$  

Here, $\hat{\Sigma}$ and $\hat{\sigma}$ are the estimated volatilities in Section 3. $\sigma_{LT} = 12.85\%$ is the volatility of MU-CBex capital return from 1970 to 2009. $\sigma_{ST} = 6.58\%$ is the volatility of MU-CBex capital return from 2002 to 2009.

We set two sample NRLs of single Tranche with no amortization. The statement of NRLs are described in Table 3. LTV of both NRLs, which is defined by $V_{NRL}/P_{NRL,t}$, are almost in the same level. We set the property types of all collaterals are office-type. The evaluation date is March 31, 2010 and the time horizon is 1 year. The number of simulation scenarios is 50,000. For simplicity, we assume that the default does not occurs before the time horizon.

Table 3 shows the risk measures of sample NRL. Here, PD is the default probability of the NRL. EL is the expected value of standardized LGD and the standardized LGD is obtained as $\frac{LGD}{P_{NRL,t}}$. 99.9% VaR is the 99.9%-percentile of the distribution of standardized LGD. While LTV of both NRLs are in almost same level, the loss of NRL 2 is much smaller than that of NRL 1, by the dispersion effect of collateral.

5. Concluding remarks

We proposed a credit risk valuation model for non-recourse loans in this paper. Introducing a waterfall structure model, our model can be applied to the risk managements of a tranched CMBS. Moreover, it will be possible to estimate credit risks of loans for REITs, with some categorizations by maturities and simplification of loan portfolio within one REIT. Although we have adopted LTV as a default trigger this time, it will be extended to “a double trigger model” by adding net cash flow dynamics and determining the other default condition with “debt service coverage ratio”.

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Note

The views and opinions expressed here are those of the authors and do not reflect the views of the authors employer.

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