A note on empirical analysis for general wrong-way risk and stressed CVA

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Abstract

We introduce a model to evaluate credit value adjustment (CVA) for large derivative portfolios considering general wrong-way risk. First, we showed the empirical evidence that suggests the existence of wrong-way risks for interest rate swaps and foreign exchange forwards. Next, we formulate a model to calculate CVA considering the correlations between the probability of default of the counterparty, the interest rate curve, and the foreign exchange rate. Finally, we show numerical examples to estimate the effect of wrong-way risk, which can be related to the CVA stress testing.

Keywords counterparty credit risk, wrong-way risk, stressed CVA

Research Activity Group Mathematical Finance

1. Introduction

Financial institutions and other industrial companies trade over the counter (OTC) derivatives for the purpose of the risk management and the pursuit of profits. In these derivative contracts, there exist probabilities to lose latent profits because of the counterparty’s default. These risks are called counterparty credit risks (CCR). Recently, financial institutions are required to measure and report the amount of CCR, and it has become standard to price derivatives adjusting the expected loss caused by CCR, which is so called the credit value adjustment (CVA). In calculating CVA, the counterparty’s probability of default (PD) possibly increases with the derivative’s exposure. This effect is called “wrong-way risk (WWR)”, and it can seriously increase the CVA.

There are two types of WWR, “specific WWR” and “general WWR”. Specific WWR means the risk that the contract itself directly affects the credit quality of the counterparty, for instance the contracts with mono-line companies to compensate the loss of securitized products. On the other hand, general WWR means the effects that macro-economic variables or market risk factors simultaneously affect the counterparty’s PD and the derivative exposures. In this paper, we focus on the general WWR which has much larger impact on derivative portfolios held by Japanese financial institutions. In Basel III, it is required for financial institutions to identify, monitor, and control the exposures which can increase the general WWR, and to consider general WWR in the CVA stress testing. For the existing model to calculate CVA with WWR, [1-3] is known. [1, 2] suggests an approach to denote the dependence of exposures and credit variables with a copula model. [3] modeled the default intensity as a function of an exposure variable and analyzed the effect on CVA. The structure of this paper is as follows. In section 2, we first formulate the extended Hull-White [3] model to consider the WWR factors of interest rate and foreign exchange (FX) rate at the same time. In section 3, we show the results of empirical analysis, whether there actually existed wrong-way risks for interest rate and FX derivatives in the past which have the majority of OTC derivatives held by Japanese financial institutions. In section 4, we calculate the CVAs for a portfolio of these derivatives considering the WWR effect. Conclusions and future extensions are shown in the final section.

2. Model

In this section, we introduce a model to evaluate CVA for large derivative portfolios considering general wrong-way risk. We consider a filtered complete probability space \(\{\Omega, \mathcal{F}, (\mathcal{F}_t), Q\}\), where \(Q\) is a risk neutral measure. We employ LIBOR market model with the principal component expression for interest rate fluctuations:

\[
\frac{dF_{1t}^m}{F_{1t}^m} = \mu^1 dt + \sigma^1 dW_{1t}^m,
\]

\[
\frac{dF_{2t}^m}{F_{2t}^m} = \mu^2 dt + \sigma^2 dW_{2t}^m,
\]

\[
\vdots
\]

\[
\frac{dF_{Mt}^m}{F_{Mt}^m} = \mu^M dt + \sigma^M dW_{Mt}^m.
\]

Here, \(F_m (m = 1, 2, \ldots, M)\) is the \(m\)-th principal component of forward LIBORS, \(\{W_{mt}^m\} (m = 1, 2, \ldots, M)\) are the independent Brownian motions.

For considering foreign currency risk, we employ FX

In particular, we consider a default intensity model which is LIBOR.

to capture the correlation between dollar-yen rate and the first principal component of forward LIBOR is enough.

The correlation between the FX rate and the first principal component of forward JPY LIBOR and certificated that the first principal component of forward LIBOR is enough to capture the correlation between dollar-yen rate and LIBOR.

Next, we introduce the default intensity model in order to capture default risk of the counterparty. In particular, we consider a default intensity model which is an extension of the model of Hull-White [3] to multiple risk factors:

Here, $a_i$ is a time dependent deterministic function, $b_n$ are the coefficient for risk factor $X_n$. In particular, we employ LIBOR and FX rates for risk factors:

CVA is the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of a counterparty’s default [4]. Unilateral CVA, which considers only the default risk of a counterparty and does not consider the default risk of oneself, is obtained as follows:

3. Empirical analysis on existence of generalized wrong-way risk

In this section, we show the result of empirical analysis on the existence of general wrong-way risk. Now, we introduce some cases that the general wrong-way risk appears. First, in global risk-off phases, rewind of carry trades, the appreciation of yen against the dollar and the increase of credit risk tends to occur at the same time. As a result, the WWR tends to appear in such cases. Fig. 1 shows the time-series of dollar-yen exchange rate and credit default swap (CDS) spread of Citigroup Inc. (Citi), which is one of the largest bank in the world. Fig. 2 shows the time-series of correlation between dollar-yen exchange rate and 5-year CDS spread of Citi.

![Fig. 1. Time-series of dollar-yen exchange rate (right axis) and 5-year CDS spread of Citi (left axis). From 2007 to 2009, and after 2012, we can recognize negative correlation between dollar-yen rate and CDS spread. Also, the time-series of CDS spread implies that there are structural changes in CDS market around summer in 2007, when the “Paribas shock” happened.](image1)

![Fig. 2. Correlation between time-series of dollar-yen exchange rate and 5-year CDS spread of Citi. From 2007 to 2009, we can recognize negative correlation between dollar-yen exchange rate and 5-year CDS spread of Citi.](image2)
Next, the appreciation of default intensity is expressed on observation time $s$ averaged default intensity rate and FX rate. Now, we mention the details. First, of CDS spreads, the principal components of interest with multiple regression analysis using realized samples coefficients of the risk factors were significantly zero or factors and FX factors, and checked whether the co-intensity model, which has risk drivers of interest rate the existence of general WWRs. We estimated a default the receive side of interest rate swaps. After mid of 2007, we can recognize the negative correlation between the first principal component of forward LIBOR and CDS spread. This implies the existence of WWR for the receive side of interest rate swaps.

Next, we show the results of an empirical analysis on the existence of general WWRs. We estimated a default intensity model, which has risk drivers of interest rate factors and FX factors, and checked whether the coefficients of the risk factors were significantly zero or not. We simply estimated the coefficients of risk factors with multiple regression analysis using realized samples of CDS spreads, the principal components of interest rate and FX rate. Now, we mention the details. First, averaged default intensity $h_T(t)$ with risk horizon $T$ is approximately obtained by CDS spread $s_T(t)$ of horizon $T$ on observation time $t$ as follows:

$$h_T(t) = \frac{s_T(t)T}{1 - R}.$$  

Next, the appreciation of default intensity is expressed as follows

$$\Delta \log h_T(t) = b_1 \Delta S_t + b_2 \Delta F_t^{-1}.$$  

We use both expression for the regression analysis. We executed the multiple regression with moving-window. The explained variable is the 5-year CDS spread and explanatory variables are the weekly appreciations of the first principal component of the interest rates and the FX rate. The sample period is from 1/3/2001 – 6/26/2013 and the each window span is 3 years. 

Fig. 5 shows the time-series of the estimated coefficients of FX factor associated with the default intensity of Citi. The estimated coefficients are significantly negative in 11/ 2007-11/2008 and this implies the existence of wrong-way risk in the period. Fig. 6 shows the time series of the estimated coefficients of the first interest rate factor associated with the default intensity of Citi. Estimated coefficients of interest rate factor are negative in the all sample period and the p-value is under 0.001 after 2008. This implies there are wrong-way risks after Lehman shock.

We have done the same analysis for several financial institutions, and obtained similar results. Thus, we recognized there were wrong-way risks in the economic stress periods like the recent financial crises. These results imply that, in CVA stress testing, we need to consider the influence of WWR on CVA valuations in order to avoid underestimation of risks.
of existence of underlying asset correlation and $\rho = 0$ for the case of no correlation between underlings. We calculated CVA by Monte-Carlo simulation with 10,000 sample paths.

Table 2 is the result of CVA valuation. We recognize that CVA tends to be large in the existence of underlying asset correlation and WWR. This implies, if we do not consider WWR on calculating CVA, we probably underestimate a counterparty’s credit risk in economic stress periods.

5. Concluding remarks

In this paper, we test the existence of WWR for interest rate and FX derivatives. We confirm that, WWR is observed with statistical significance for short positions of USD/JPY FX forward contracts and also for fixed rate receivers of interest rate derivatives. Appreciations of the yen against the US dollar, the downward yield curve shifts and widening of credit spreads tend to progress simultaneously in risk-off market environments. We also calculate CVA for a virtual derivative portfolio on a day in such risk-off periods. We confirm that if we do not take WWR or the correlations between market risk factors into account, we could considerably underestimate CVAs. For practical use of the model, we should increase the number of currencies of interest rate and FX pairs to the actual extent, and model the dependence on interest rate and FX derivatives. We confirm that, WWR increase the number of currencies of interest rate and FX pairs to the actual extent, and model the dependence on interest rate and FX derivatives. We confirm that, WWR

Table 2. Result of calculating CVA (the ratio of CVA to principal in basis points).

<table>
<thead>
<tr>
<th>WWR</th>
<th>correlation</th>
<th>no correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.89</td>
<td>5.61</td>
</tr>
<tr>
<td>no WWR</td>
<td>6.24</td>
<td>7.70</td>
</tr>
</tbody>
</table>

4. An numerical example on calculating stressed CVA

In this section, we give a numerical example on evaluation of CVA. In particular, we focus on the CVA in the economic stress period, where the wrong-way risk exists. We calculate and compare the CVA in both the case of considering and not considering general wrong-way risk.

We choose 10/8/2008 for the valuation date and it is suitable for analyzing the stress period after Lehman shock. The sample data for model estimation are the JPY LIBOR, dollar-yen exchange rate, CDS spread of Citibank and the yield curve of USD at 1–10 years in 1/3/2000–10/8/2008. We set the volatility of LIBOR market model historically. Also, we set the drift of LIBOR market model by the no arbitrage condition. We assumed the LIBOR market model has 60 tenors of variables and we explained them from the first to 6th principal components. There, we confirmed that the first to 6th principal components explains over 90% of the variation of LIBOR. We estimated the volatility of FX rate model historically with the time-series samples of FX rate. We used the differences of JPY LIBOR and USD LIBOR as the drift coefficient of FX rate model. The coefficients in the default intensity model is estimated by the multiple regression, we mentioned previously. In addition, we employed the step function for the baseline default intensity $a_0$: $a_0 = a_k (T_k-1 \leq t_i < T_k)$.

We calibrated $a_0$ by the survival probability obtained from CDS spreads as follows:

$$
\exp\left(\frac{-s_k T_k}{1-R}\right) = \frac{1}{M} \sum_{j=1}^{M} \left[ \sum_{i=1}^{n(k)} \Delta t \exp(a_k + b_1 S_{ij} + b_2 F_{ij}^1) \right].
$$

Here, $\{T_k\}$ are the maturities of CDS and $t_{n(k)} = T_k$.

Table 1 shows the constitution of the target portfolio. The exposure we calculate is the netting exposure.

In calculating exposure without considering wrong-way risk, we set the coefficients of interest rate factor and FX factor in the default intensity model equal to 0. We set the correlation parameter $\rho = 0.5$ for the case

$$
\rho = \frac{\text{Cov}(a, b)}{\sigma_a \sigma_b}
$$

Table 1. Constitution of the target portfolio, which are constituted by USD-JPY FX forward and interest rate swap (IRS) on JPY LIBOR.

<table>
<thead>
<tr>
<th>Derivative Type</th>
<th>Maturity</th>
<th>Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX forward, USD-JPY, Short</td>
<td>1Y</td>
<td>1 (million Yen)</td>
</tr>
<tr>
<td>FX forward, USD-JPY, Short</td>
<td>2Y</td>
<td>1 (million Yen)</td>
</tr>
<tr>
<td>FX forward, USD-JPY, Short</td>
<td>10Y</td>
<td>1 (million Yen)</td>
</tr>
<tr>
<td>IRS, JPY LIBOR, Receive</td>
<td>1Y</td>
<td>10 (million Yen)</td>
</tr>
<tr>
<td>IRS, JPY LIBOR, Receive</td>
<td>2Y</td>
<td>10 (million Yen)</td>
</tr>
<tr>
<td>IRS, JPY LIBOR, Receive</td>
<td>3Y</td>
<td>10 (million Yen)</td>
</tr>
</tbody>
</table>

References