Nonlinear Analysis of Tremor and Applications

Horia-Nicolai Teodorescu†,* and Abraham Kandel**

1. Introduction

It is widely recognized that, despite significant progresses in neurology, posturography, electromyography, and investigation techniques, "tremor is commonly encountered in medical practice, but can be difficult to diagnose and manage.1)" The purpose of this paper is to synthesize the results obtained by us regarding the characterization of the tremor by means of classic and nonlinear dynamics techniques and to introduce new results and technique refinements. We also indicate how the results may be integrated in knowledge-based systems for automatic diagnosis or for rehabilitation systems.

Many researchers have recently analyzed tremor as related to various diseases.1-16) However, the techniques to measure and analyze tremor remain rather undeveloped. In a series of papers,17-24) we have proposed new sensors, new methods to analyze the tremor, we introduced new parameters to describe the tremor movements in limbs, and new software to perform the analysis. Moreover, we developed new applications related to rehabilitation and tremor control, and we analyzed chaos features in head tremor and mandibular tremor, and "tremored voice."

2. The Tremor Measurement and Set-ups

Tremor signal can be acquired using various techniques. Among the previously reported techniques used in tremor measurement are the capacitive sensors, inductive sensors, 1-D or triaxial5,6) accelerometric sensors, resistive sensor,25) mechanical sensors,26) pressure-sensitive digitizing tablets6) and optical sensors. Recently, image-processing methods were reported as tentative methods to determine the hand movements. Force and surface electromyogram (EMG) signals are frequently recorded and analyzed (e.g., Ref. 14). The sensors used to measure the tremor should be carefully selected. Any sensor can be used to perform the chaos analysis, provided its linearity is high and its noise is low enough. Especially the noise can destroy the nonlinear information in the signal and lead to incorrect results.

For the nonlinear analysis, the sampling frequency has to be much higher than the bandwidth of the noise, and the acquisition should be of high precision (at least 12 bits); else, the sampling and quantization noise generate unreliable results.

We used a specially designed sensor and measuring system, based on a resonant sensor and on the commercial "Tremor analyzer"TM system, developed by T & T™ Ltd., according to our design.17) The system measures the movement of the whole hand and can be adapted to measure the leg, jaw, and head tremor. Moreover, accelerometric sensors were used to measure...
individual finger tremor and the analysis was complemented by electromyography.

3. Signal Processing and Basic Results

Signal processing is done in two steps. First, the signal is pre-processed to remove the main perturbations. Then, the signal is processed in view of the analysis.

3.1 Pre-processing
The tremor is a complex movement, including components derived from respiration movements and other movements unrelated to tremor. The respiration contributes low frequency components to the movements of the hand. These components should be removed, taking into account that respiration has a basic frequency of about 0.1 to 0.3 Hz for adult subjects, with a bandwidth in the range 0.05 to 3 Hz. A high-pass filtering, at about 2 Hz, is recommended. Acquiring the respiration signal may help in removing this component by tuning an LP filter to remove the respiration component. Adaptive signal canceling may also be adopted. Similar means may be used to remove the movements induced by the blood flow. On the other hand, there is a random component in the movement; its bandwidth is difficult to determine in the present stage. We estimate this bandwidth at frequencies higher than 10 to 40 Hz
t.

In our experiments, the signal is low-pass pre-filtered at and high-pass filtered at 50 Hz and 0.1 Hz respectively, with analogue filters. Then, the signal is sampled with a sampling frequency of 100 Hz and digitized on 12 bits. Power spectra are computed using a 512 point fast Fourier transform, performed on each time window. Subjects were asked to keep fixed the right arm in a position that makes an angle of about 20° with the horizontal (under the horizontal). The subjects are seating. For all the duration of the experiments (one to three minutes) subjects had to maintain the same position of the hand. Some fatigue is expected to influence the last part of the determinations. In another set of experiments, the subjects were required to hold in their hand an object (weighting about 1 kg), such that muscular effort and fatigue is induced. Data (signal windows) exhibiting a too high variation of the parameters have been rejected.

![Figure 1](image1.png)

**Fig. 1** a) Example of tremor signal. 2400 samples. Relative units. b) Graph of the smoothed signal (2400 samples)

![Figure 2](image2.png)

**Fig. 2** a) Difference signal. b) Difference signal after smoothing (2400 samples)
3.2 Signal Processing and Classic Signal Analysis

An example of original data (after preprocessing) is shown in Fig. 1a). Although the signal is already suitable for analysis (and was used as such in our previous researches), we found that separation into two components, one of high frequencies and the other one of low frequencies, allows us finer analysis and derivation of significant new signal parameters (features). The first component is obtained by low-pass filtering (smoothing) of the signal. The smoothing has been done by the moving average method. The smoothed signal is shown in Fig. 1b).

We derived a second signal by making the difference between successive samples, \( x[n] - x[n-1] \), where \( x[n] \) represents the \( n \)-th sample of the signal. The result for the signal in Fig. 1a) is shown in Fig. 2a). After smoothing the difference signal, one obtains the signal shown in Fig. 2b).

Notice that a pattern may be suspected in the smoothed difference signal and its analysis will be exemplified in the subsequent sections.

Fourier analysis

The original, smoothed and smoothed difference signals are first analyzed by means of Fourier (frequency) analysis—a method well established in tremor analysis. The results of the spectral analysis applied to the original and to the smoothed difference signals are shown in Fig. 3.

In case of the linear representation of the spectrum, the low frequency component (the main frequency in the tremor spectrum) dominates and masks the other components. The use of logarithmic-linear coordinates shows more information, including the higher frequency components. The second graph (Fig. 3b)) is plot in log-linear coordinates, to evidence the significant frequency peaks (the "main frequencies" of the tremor movement). Notice the presence of non-harmonic components. These may be interpreted as an indication that two independent processes simultaneously develop. The second and the third frequency peaks are better evidenced in the spectrum of the smoothed difference signal (Fig. 3c)). Moreover, notice the existence of a large bandwidth spectrum, indicating the possibility of a nonlinear dynamics and noise.

The "main frequency" of the tremor signal is an important parameter, most researchers using it. This value may be defined, in terms of the Fourier spectrum, as the frequency corresponding to the first peak in the graph. As a remark, we found in several cases that,

![Fig. 3](image-url)

![Fig. 4](image-url)

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under muscular effort, the tremor signal spectrum has significantly changed: the main frequency decreases and the movement becomes more regular, with lower contents in higher frequencies. This is supporting the conclusion that the use of muscular effort tests is essential for both diagnosis and rehabilitation tests.

**Statistical analysis**

Another classic analysis method is the statistical analysis of the signal. The empirical probability distribution of the smoothed and smoothed difference signals are shown in Fig. 4.

The probability distribution of the smoothed signal (Fig. 4 a)) is far from the Gauss (normal) distribution: it is asymmetrical and multi-modal. The probability distribution of the smoothed difference signal (Fig. 4 b)) is much closer to the Gauss distribution; hence, it is (almost white) noise-like. Therefore, we may suspect that several components exist in the signal. This is one more reason to separate the two components (smoothed and difference) by appropriate filtering.

**Correlation function**

Another tools that we have found suited for the analysis is the correlation function. The significance of the correlation function relates to its main properties, summarized below. The correlation function of a purely random (white noise-type) signal is zero, except the origin; moreover, for non-white random processes, the correlation function decreases to zero very fast. On the other hand, the correlation function of a periodic function is a periodic function, with the same period. Consequently, the correlation function allows us to differentiate between periodic and noise-like signals, moreover to evidence and separate periodic components. Moreover, the correlation function evidences the time duration the signal samples correlate (and thus can be predicted). This duration is evidenced by the decay of the function that defines a “time constant” (see Fig. 5 a)). This time constant can be use as a characteristic parameter of the tremor signal.

In case of a Fig. 5 b), notice that a quasi-periodicity in the smoothed difference signal is revealed, by the correlation function, in the smoothed difference signal. The period is about 6 samples, corresponding to about 16 Hz. The dynamics of this underlying process will be analyzed in the subsequent sections.

4. **Nonlinear (Chaos) Analysis**

There is an extensive literature to support the hypothesis that many biologic processes, including neural processes, exhibit chaotic dynamics. Chaotic behaviors are characterized by low predictability of the dynamic evolution and by high sensitivity to initial conditions (see, for instance, the textbook27; any other textbook on chaos may be used as reference for the basic concepts utilized in this paper).

**Phase diagrams**

The first step in chaos analysis is the plotting of the phase diagram. This diagram typically represents the derivative of the signal versus the signal itself. If the signal is periodical, the diagram represents a closed curve. If the signal is chaotic, it represents a non-closed curve that shows a “strange attractor” (generally, a
somewhat regular shape). That is specific to the nonlinear regime. If the signal is pure noise, the diagram represents a uniformly and randomly filled region of the space. Versions of the diagram represent the signal with a time lag, versus the non-delayed signal (see Fig. 6 a).

The representations with different time lags (e.g., 1, 50 and 100) of the original signal show that such diagrams (phase diagrams) are not always relevant. Indeed, apart the demonstration—by the shape of the phase diagram—that the signal is not periodical, yet not a pure noise, the information in the phase diagram is irrelevant if the attractor is not already known. Moreover, the noise, or high frequency components not suitably separated, can make the diagram useless. Therefore, numerical parameters are required for a precise and reliable characterization of the nonlinear regime. The phase diagrams of the signal components, namely of the smoothed and smoothed difference signals (Fig. 6 b) and 6 c), show better defined attractors and may be used to recognize specific tremor patterns.

**Lyapunov maximal exponent**

A positive Lyapunov exponent is known to be the primary indicator of a chaotic dynamics. It relieves the degree of divergence of the “trajectories,” that is, the initial condition sensitivity and unpredictability of the signal. The low values of the Lyapunov exponent show a “weakly-chaotic” regime (weakly divergent trajectories).

The maximal Lyapunov exponent (and the correlation dimension, reported below) has been computed for time series of 1000 to 2500 samples from the recordings. The Lyapunov exponent was computed considering an embedding dimension of 3 to 6, the number of points (samples) was \( n = 1 \) to 3, and the radius of coverage was \( 10^{-3} - 4 \). Selected experimental results are shown in Table 1 for exemplification. The values of the Lyapunov exponent, for different trials, are in the range 0.08 to 0.7, also depending on the type of signal (original, smoothed, difference, smoothed difference signal). The results do not show significant differences in the Lyapunov exponents for young and healthy subjects, when contrasted to the group of about 45 year-old people we have tested.

The uncertainty in the value for the smoothed signal, for \( D = 3 \), shows that a higher imbedding dimension is needed in this case (\( D = 4 \)). Notice that the Lyapunov exponents for all the signals are positive. This is an indication that the signals include chaotic components. Moreover, this indicates that both the lower frequency components and the higher frequency components of the tremor signal are complex signals, possibly representing two different nonlinear superposed processes. Further medical investigations are required to determine if this hypothesis is true and what are the physiological bases of the two components: their underlying processes may be related to two different neuro-muscular processes that combine to produce

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**Table 1** Lyapunov exponent values for the signals 
(\( D = 3 \) and \( D = 4 \), \( D = 3 \))

<table>
<thead>
<tr>
<th>Signal</th>
<th>Exponent ( (D = 3) )</th>
<th>Exponent ( (D = 4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original signal</td>
<td>0.39 ± 0.03</td>
<td>0.27 ± 0.03</td>
</tr>
<tr>
<td>Smoothed signal</td>
<td>0.20 ± 0.3</td>
<td>0.19 ± 0.03</td>
</tr>
<tr>
<td>Difference signal</td>
<td>0.71 ± 0.03</td>
<td>0.41 ± 0.03</td>
</tr>
<tr>
<td>Smoothed difference signal</td>
<td>0.38 ± 0.03</td>
<td>0.35 ± 0.03</td>
</tr>
</tbody>
</table>
Correlation dimension ($D$)

Another parameter frequently used to characterize chaotic behaviors is the correlation dimension. The correlation dimension resulted from the signal is in the range $[0.5; 5]$, for the original signal. Most recordings exhibit a $D$ value in the range $[2.5; 3]$. The range of $D$ is somewhat higher for the signal acquired under conditions without muscular effort than under muscular effort. In Table 2, the values of the correlation dimension are presented, for various subjects, under no induced muscular effort and under effort condition. High differences in the correlation dimension of the signal have been found for several young subjects.

Notice the high intra-subject variability, possibly dependent on the state of the subject. Some subjects are more "self-consistent," e.g. the values for subject #1 are close, even under effort conditions. Figure 7 presents the results for two subjects, for different experimental time series, acquired during several days.

The correlation dimension has been found to vary during the time series, when measured with a moving window. Figure 8 exemplifies this variation.

The findings show that the spreading of the values somewhat decreases under effort, for some subjects (see Fig. 9). It is too early to derive definite diagnostic-related conclusions, but it can be stated that the results are indicating that the correlation dimension is a parameter to be considered in characterizing the tremor.

### Table 2  Correlation dimension for several subjects. Extreme values are separated by a vertical bar (after Ref.21).

<table>
<thead>
<tr>
<th>Subject #</th>
<th>Correlation dimension $D$</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>S#1</td>
<td>3.20±0.16</td>
<td>no effort</td>
</tr>
<tr>
<td>S#1</td>
<td>2.97±0.15</td>
<td>under effort</td>
</tr>
<tr>
<td>S#2</td>
<td>2.58±0.24 $</td>
<td>$ 3.36±0.40</td>
</tr>
<tr>
<td>S#2</td>
<td>2.02±0.16 $</td>
<td>$ 2.83±0.30</td>
</tr>
<tr>
<td>S#3</td>
<td>1.64±0.44</td>
<td>no effort</td>
</tr>
<tr>
<td>S#3</td>
<td>2.93±0.31</td>
<td>under effort</td>
</tr>
<tr>
<td>S#4</td>
<td>2.78±0.34 $</td>
<td>$ 3.53±0.48</td>
</tr>
<tr>
<td>S#5*</td>
<td>1.65±0.30 $</td>
<td>$ 2.84±0.18</td>
</tr>
<tr>
<td>S#6</td>
<td>2.33±0.24</td>
<td>no effort</td>
</tr>
</tbody>
</table>

* Alcoholic subject

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![Correlation Dimension](image1)

**Fig. 7** Graph of the correlation dimension for different experiments, without effort, for two subjects

![Variation of the Correlation Dimension](image2)

**Fig. 8** Variation of the correlation dimension in a moving window, for one subject

![Correlation Dimension for Free Condition and Under Effort](image3)

**Fig. 9** Correlation dimension for free condition and under effort tremor
Table 3  Correlation dimension for the signals (embedding dimension $D=4$ and $D=5$)

<table>
<thead>
<tr>
<th>Signal</th>
<th>Correlation dimension $(D=4)$</th>
<th>Correlation dimension $(D=5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original signal</td>
<td>$3.7 \pm 0.14$</td>
<td>$4.1 \pm 0.23$</td>
</tr>
<tr>
<td>Smoothed signal</td>
<td>$2.1 \pm 0.16$</td>
<td>$2.2 \pm 0.22$</td>
</tr>
<tr>
<td>Difference signal</td>
<td>$3.9 \pm 0.17$</td>
<td>$4.5 \pm 0.12$</td>
</tr>
<tr>
<td>Smoothed difference signal</td>
<td>$2.8 \pm 0.2$</td>
<td>$3.1 \pm 0.27$</td>
</tr>
</tbody>
</table>

The correlation dimension is significantly different for the original signal, the smoothed signal and the difference signal (see Table 3). All signals, including the smoothed and the smoothed difference signal have correlation coefficients values that are generally higher than 2.

Both the computations of the Lyapunov maximal exponent and of the correlation dimension are largely affected by the computation conditions, namely by the embedding dimension. We found that an appropriate choice is $D=3$ or $D=4$ for the smoothed tremor signals. Low values ($D<3$) are erroneous because the dimension of the embedding space is too low to correctly represent the tremor signal (these values are found to be in the range 1.5 to 3). Higher values of $D (D>5)$ are difficult to manipulate, because the computation requires long time series (of the order of magnitude $10^6$ or higher). Such time series can not be obtained because of the fatigue of the subjects after about 1 minute; consequently, longer time series are not stationary and can not be interpreted as a representative process. Because the non-smoothed time series may exhibit noise and high embedding dimensions, it is recommended to use filtered original and difference signals.

5. Tremor Characterization for Fuzzy Knowledge Bases in Diagnosis and Rehabilitation

Automatic diagnosis and the use in rehabilitation systems are two major applications of tremor analysis. For these purposes, we have proposed\textsuperscript{19–22} the use of a fuzzy knowledge base, briefly presented below. The knowledge base includes rules referring to both the classic and chaotic features of the signals, because the classic parameters alone can not consistently represent the whole tremor process. When the nature of the tremor involves different types of processes (as discussed in the previous sections), the analysis should include information on every type of processes and the component analysis may become a hierarchical process.

The realization of a helpful feedback in tremor-related rehabilitation is a major objective of the tremor analysis. The feedback has to be simple enough for easy understanding and learning. It should make use of easy to grasp visual and audible information. In this purpose, the information has to be appropriately compressed.

With the purpose of creating a feedback to help patients to become aware of the characteristics of the tremor of their limbs, moreover to help them controlling the tremor, we proposed\textsuperscript{21,22} a representation of the tremor by images and sounds. This representation is synthetic and is based on the linguistic and fuzzy valuation of the main parameters of the tremor. The use of linguistic and fuzzy valuations is justified by its simplicity (easy to understand by the patient), moreover by a good information compression.
To reduce the number of represented parameters, yet preserving the essential information, we performed a fusing of the numerical parameters derived from the analysis. The evaluation of the amplitude is modified by “aggravating factors,” namely the frequency of the tremor, and the tremor irregularity. Examples of the membership functions used for two parameters are shown in Figs. 10 and 11.

The rules in the rule base used to globally characterize the tremor are in the generic form:

If (the tremor) Amplitude A is in the range $A_1$ to $A_3$
and Main Frequency is in the range $F_1$ to $F_4$
and $F_{\text{High}}/F_{\text{Low}}$ is in the range $DF_1$ to $DF_3$
and Correlation Dimension is in the range $D_1$ to $D_3$
and Lyapunov exponent is higher than 0.2
then Tremor amplitude-frequency (fused) parameter $AF$ is in the range $x$ of $AF_1-AF_5$
moreover The irregularity (fused) parameter $\text{IRREG}$ is $z$ in the range $\text{IRR}_1-\text{IRR}_3$.

The rules establish the relations between premises and consequences (the appropriate linguistic degrees, derived by empirical methods). The parameters $AF$ and $\text{IRREG}$ are synthetic fuzzy features of the movements. The $F_{\text{High}}/F_{\text{Low}}$ parameter is the ratio of the energy of the signal in the high frequencies band (>6 Hz) vs. the energy in the low frequencies band (<6 Hz). The rule base is under evaluation.

The results of the inference described above are defuzzified and used in the feedback used for rehabilitation purposes, based on images on the display. There are various feedback facilities under current evaluation with the system. In one of them, the feedback is realized by showing on the computer screen a simplified version of the screen used in the full-research program. In another version, a screen showing a ball imitating the vertical tremor of the hand and having the diameter and color representing two other parameters, moreover showing an analog indicator is used for the feedback purpose. The analog indicator shows either an average of the tremor amplitude, or an evaluation of the tremor amplitude performed according to the rule base.

6. Conclusions

We have described a complete methodology for tremor analysis, including classic and chaotic parameters. We have evidenced that tremor may include a significant nonlinear, chaotic component, which has not been previously dealt with in the literature, moreover, that this component may be of interest in diagnosis and rehabilitation. Some conclusions derived from our work are:

- The nonlinear analysis is an alternative method that supplements the classic methods based on Fourier analysis and on the analysis of the time-amplitude (waveform) analysis.
- The Lyapunov exponent, beyond its essential role in assessing the existence of chaos in tremor, can help diagnosis (characterizes the type of tremor).
- The phase diagram may be useful in the visual recognition of the type of tremor movements; however it is not a precise method. The phase diagram interpretation is prone to errors; moreover time lags have to be appropriately chosen.
- The time series used in assessing the nonlinear features should be long enough. Indeed, the dimension of the embedding space is not known in advance and moreover it is sometimes high (higher than 3); therefore, the time series should include at least 1000 samples, corresponding to at least 10 “periods” of the signal. The values of the correlation dimension and of the maximal Lyapunov exponent are very sensitive to the length of the time series.
- The nonlinear analysis should be done after a preprocessing of the tremor signal; it is essential to perform a filtering of the signal, to remove the noise, moreover it is recommended to separate the low and the high frequency components of the signal.
- The choice of the sensor used to measure the movement is essential: the sensor characteristics (linearity, precision and frequency characteristic) can dramatically change the nonlinear analysis results, while the main results of the linear analysis may essentially remain unchanged.
- The nonlinear analysis can be complemented by a fuzzy logic-based interpretation of the results, in the frame of a fuzzy knowledge-based system.
- The nonlinear analysis, complemented by an appropriate feedback, may be used in a complex system for the purpose of rehabilitation.

The improvement of the efficacy of computer-based tremor analysis, with the emphasis on the chaotic part
of the movement, and improvements in the feedback provided to patients for rehabilitation purposes are seen as potential developments of the present research.

Acknowledgments This and related previous researches have been performed with the support of: the Grant 7RUPJ-48689 from Swiss Research Founds (FNS), Switzerland, and a Grant from Techniques & Technologies (T & T), Ltd. (T & T Ltd. has provided several measuring set-ups and software, and largely covered the research expenses).

References

19) Teodorescu, HN: Grant Proposal, FNS, Switzerland, June 1996 (Grant approved: 7RUPJ-48689). Methods were also proposed and preliminary results presented in: Teodorescu, HN: Preliminary Research Report, and Final Report, Grant 7RUPJ-48689, FNS/EPFL, Lausanne, Switzerland (unpublished; can be obtained in an abstract form from EPFL-C3i)
23) Teodorescu HN: Research Report, Grant 7RUPJ-48689, National Swiss Funds, Switzerland, 1998
Annex 1

A dedicated software package and an apparatus for multi-channel acquisition of the tremor signal have been designed. A sample screen generated by this software is presented in Fig. A1.

Fig. A1 Sample screen of the program Tremor Analyzer™. Reproduced with the permission of T & T Ltd.