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On the Fracture of Metals*

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In Part I, a phenomenological fracture criterion of polycrystalline metals under conventional loading conditions is presented on the premise that there exist generally two modes of fracture; ductile shear fracture and brittle cleavage fracture. The two major effects on fracture; that is, triaxiality of stress and plastic strains preceding fracture are formulated mathematically in this criterion. The experimental results under combined stresses on ductile metals are well explained on the basis of this criterion. In Part II, the tensile fracture tests of mild steel cylindrical bars having circumferential hyperbolic notches are performed, and the plastic stress and strain distributions at fracture necessary for an application of this criterion to practical problems are analysed approximately. The various fracture behaviors in these tests are well explained and a view as to the notch brittleness are given.

Introduction

The fact that the mechanism of fracture has been, as yet, very little understood results from an incomplete knowledge regarding the effects of various influence factors and the stress and strain distributions at the moment of fracturing. From this standpoint, the author presented, in Part I, a phenomenological fracture criterion on polycrystalline metals in a general form so as to be applicable commonly to many kinds of materials under general polyaxial stress states, explaining the results on ductile metals. And in Part II, in order to apply this criterion to practical problems, the author carried out tensile fracture tests on mild steel cylindrical bars having hyperbolic notches and presented a viewpoint on the notch brittleness phenomenon after analysing the plastic stress and strain distributions at fracture across the notched cross section. The present paper presents an abbreviated summary of the two papers combined⁽¹⁾ described in detail in Japanese.

Part I. Fracture Criterion

1.1 Presentation of Fracture Criterion

The distinction between the so-called ductile and brittle materials is considered to be convenient and to depend on the loading condition. In addition, the fracture mechanisms of metals can be classified into a number of types. In such complex circumstances, the author selected especially the two major types of fracture of general metals;

that is, the ductile (shear) fracture and the brittle (cleavage) fracture under statical loading at room temperature.

(1) Two types of fracture

First, the shear (slip, ductile) fracture is considered to be a process of slipping apart by linking together of a number of micro-cracks due to plastic deformations which precede the fracture.

Secondly, the cleavage (tensile, brittle) fracture is considered to be due to a mechanism similar to Griffith's type fracture. However, the Griffith cracks in metals have a different nature from that of completely brittle materials. The concentrated tensile stress is assumed to be produced by the arrays of blocked dislocations which are identified with the Griffith cracks, including non-metallic inclusions or secondary metallurgical phases, etc.

In an engineering sense, the cleavage fracture was defined conveniently in such a broader sense. It is possible to see it in ductile materials under such testing conditions as under high rate of straining, triaxial stress and low temperature, etc., in which it is difficult for further plastic deformations to occur.

These two modes of fracture correspond to the two distinguishable directions of macroscopic fracture surfaces which are almost nearly normal to σ_m (maximum tensile stress) or along τ_m (maximum shearing stress). This correspondence remains to be interpreted as a subject of further study.

(2) Fracture stress surfaces

The author's conception of fracture is based on a similar hypothesis as Ludwik's.⁽²⁾ However,

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the author considered two kinds of limiting fracture stress; τ_{cr} and σ_{cr} , as Davidenkov⁽³⁾ did.

The fracture stresses have been expressed as curves in stress-strain diagrams by previous investigators and the curve for ductile fracture in Davidenkov's analysis has a negative slope with increasing plastic strain, which are different from the author's. Both types of fracture in metals are always accompanied by plastic deformations and both mechanisms are related to tensile stress, not to mention shear stress as suggested by the conceptions of dislocation.⁽⁴⁾ These two facts are designated as the effects of the entire plastic prestrains preceding fracture and the triaxiality of stress state at fracture. Therefore, these two major factors are formulated mathematically through the use of three-dimensional invariants of strain and stress, γ_n and Θ , in the fracture stress surfaces which are shown as flat surfaces in Fig. 1 and expressed as follows:

$$\tau_{cr} = a + b\Theta + c\gamma_n \quad (1.1)$$

$$\sigma_{cr} = a' + b'\Theta + c'\gamma_n \quad (1.2)$$

where $a, \dots; a', \dots$ are the material constants which are functions of temperature, strain rate and metallurgical factors, etc.

$$\left. \begin{aligned} \gamma_n &= (2/3) \{ (\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2 \}^{1/2} \\ &: \text{Octahedral shearing strain} \\ \Theta &= (\sigma_1 + \sigma_2 + \sigma_3)/3 \\ &: \text{Hydrostatic tension} \end{aligned} \right\} \quad (2)$$

Fig. 1 Fracture stress surfaces and flow curves

The fracture stresses increase linearly with increasing γ_n by work hardening⁽⁵⁾. The author supposes that a negative slope of the τ_{cr} curve by Davidenkov would be introduced to explain the decrease of fracture stress with increasing the strain as seen in the "Technical cohesive strength curve" by Kuntze⁽⁶⁾ on the notched specimens. Moreover, the fracture stresses decrease linearly with increasing Θ as demonstrated by Bridgman⁽⁷⁾

and others⁽⁸⁾⁽⁹⁾ on the effects of the triaxiality of stress.

(3) Law of plastic flow and fracture conditions

The plastic flow curve necessary for determining the intersection with the fracture stress surface may be discussed on the octahedral shear stress theory according to the "plastic deformation theory". This is applicable to ductile homogeneous metals excepting the cases of complicated loading processes. There is a need of some corrections for brittle materials such as grey cast irons. The law can be expressed as follows:

$$\left. \begin{aligned} \beta \epsilon_1 &= [\sigma_1 - (\sigma_2 + \sigma_3)/2] \\ \beta \epsilon_2 &= [\sigma_2 - (\sigma_3 + \sigma_1)/2] \\ \beta \epsilon_3 &= [\sigma_3 - (\sigma_1 + \sigma_2)/2] \end{aligned} \right\} \quad (3)$$

where $\beta = 3\tau_n/\gamma_n$, τ_n : octahedral shearing stress

$$\tau_n = (1/3) \cdot \{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \}^{1/2}$$

After analysis of the stress and strain distributions, the flow curves of τ_m and σ_m are represented schematically in Fig. 1 as the curves of OP and OP' , respectively. The location and the mode of initial fracturing will be determined depending on which intersection of τ_m or σ_m and τ_{cr} or σ_{cr} , respectively, is achieved earlier.

1.2 Application of Fracture Criterion

As an example, this criterion is applied to the ductile homogeneous materials exhibiting shear fracture in all fields under biaxial stresses. Taking into account the experimental evidences indicating that the values of τ_n at fracture are not much different under various combined stresses and the flow curve can be approximated by a linear line before fracture, the flow curve can be written as

$$\gamma_n = \alpha \tau_n + \beta \quad (4)$$

Then, Eq. (1.1) reduces to

$$\tau_{cr} = (a + c\beta) + b\Theta + c\alpha \tau_n \quad (5)$$

The three coefficients; $(a + c\beta)$, b and $c\alpha$ in Eq. (5) can be expressed in terms of three fundamental fracture stresses f_t , $-f_c$, and f_s under simple tension, compression, and torsion, respectively. For example, in the experiments under biaxial stresses carried out on thin tubular specimens under axial load and internal pressure, the ratio of the radial stress σ_3 to the tangential stress σ_2 can be taken as constant ($k = -\sigma_3/\sigma_2$); and then τ_{cr} in equation (5) can be finally expressed as

$$\begin{aligned} \tau_{cr} &= [2(2 - \sqrt{3})\xi\eta f_t + (2 - \sqrt{3})(1 - \xi)\eta\sigma_1(1 + \mu \\ &\quad - k\mu) + 2\{\xi - \eta(1 + \xi)\}|\sigma_1| \times \\ &\quad \sqrt{\mu^2(1 + k + k^2) - \mu(1 - k) + 1}] / 2\{2\xi - \sqrt{3}\eta(1 + \xi)\} \end{aligned} \quad (6)$$

where $\xi = f_c/f_t$, $\eta = f_s/f_t$, $\mu = \sigma_2/\sigma_1$, σ_1 : the axial stress. The shear fracture stresses will be calculated from the following conditions in each field of

combined stresses (A), (B) and (C) in Fig. 2.

$$\left. \begin{aligned} \tau_{cr} &= (\sigma_1 - \sigma_3)/2 = \sigma_1(1+k\mu)/2 \dots \text{for (A) field, } 0 \leq \mu \leq 1 \\ &= (\sigma_2 - \sigma_3)/2 = \sigma_1\mu(1+k)/2 \dots \text{for (B) field, } -k \leq 1/\mu \leq 1 \\ &= (\sigma_2 - \sigma_1)/2 = \sigma_1(\mu-1)/2 \dots \text{for (C) field, } -1/k \leq \mu \leq 0 \end{aligned} \right\} \dots (7)$$

As an example, those in the (A) field will be shown as follows:

$$\sigma_1/f_1 = 2(2-\sqrt{3})\xi\eta/2 \{ \xi - (1-\xi+\sqrt{3}\xi)\eta \} + \mu [2k \{ \xi - (\sqrt{3}-1+\xi)\eta \} + (2-\sqrt{3})(\xi-1)\eta] - 2 \{ \xi - \eta(1+\xi) \} \sqrt{\mu^2(1+k+k^2) - (1-k)\mu + 1} \quad \text{for (A) field} \dots (8)$$

The experimental results with FS-1Mg alloys⁽¹⁰⁾ are shown in Fig. 2.

The analytical heavy line obtained by putting $k=0.05$, $\xi=1.2$, $\eta=0.5$ agree fairly well with the experimental results. Similar agreement has been obtained with Brass⁽⁹⁾, Al alloys⁽⁹⁾, and mild steels⁽¹¹⁾. In the cases of materials having curved stress versus strain relations, the fracture stresses will be graphically obtained.

Fig. 2 Fracture stresses on FS-1 Mg alloy tubular specimens under biaxial combined stresses (E.G. Thomsen & J.E. Dorn) and analytical results.

The fracture resulting under purely triaxial stress state, for instance, in 0.61% C steel⁽⁹⁾ in a special case of $\sigma_2 = \sigma_3$ can be explained well also. There exists a relation between τ_m and τ_n ($\tau_m = 3\tau_n/2\sqrt{2}$); therefore the fracture stresses can be graphically determined by the intersecting line of the τ_{cr} surface with curved flow surface parallel to the axis of θ , showing remarkable decreases of shear fracture stress and ductility under triaxial stress state. These results cannot be explained by

other proposed criteria.

Part II. Tensile Fracture Tests of Mild Steel Round Bars Having Hyperbolic Notches and Notch Brittleness

2.1 Test Procedure

The specimens having circumferential hyperbolic notches, where the diameters of minimum cross sections were kept constant ($d_0 = 12$ mm), are shown in Fig. 3. The initial radii of curvature (ρ_0) are listed in Table 1.

Fig. 3 Tensile test specimens

Table 1

type	Initial Radius of Curvature ρ_0 (mm)	Notch Sharpness	
		initial $\lambda_0 = a_0/\rho_0$	at fracture $\lambda = a/\rho$
(0)	∞	0	0.84
(i)	6.000	1	1.40
(ii)	1.500	4	1.75
(iii)	0.667	9	2.25
(iv)	0.375	16	2.80

The composition of this material is as follows:
C 0.18%, Si 0.11%, Mn 0.43%, P 0.036%,
S 0.55%, Cu 0.28%

The scheme of the entire experimental arrangement is shown in Fig. 4. Resistance wire strain gauges were used so that exact load and diameter in

Fig. 4 Schematic diagram of testing apparatus and procedure

notch section may be obtained, and errors due to sudden fracture phenomenon may be avoided.

2.2 Tensile Fracture Test Results

(1) Loads (Stresses) and strains at fracture

From the loads (P) and diameters (d) measured as above, the average axial true stress σ_{za} and the average axial logarithmic strain ϵ_{za} are defined as follows:

$$\sigma_{za} = P/A = 4P/\pi d^2, \quad \epsilon_{za} = 2 \ln(d_0/d) \dots \dots (9)$$

The flow curves of σ_{za} versus ϵ_{za} are shown in Fig. 5. The values of P , σ_{za} and ϵ_{za} at maximum load and at fracture denoted by suffixes "m" and "r", respectively, are plotted against λ_0 in Fig. 6. The fracture stress σ_{zr} has a maximum value at $\lambda_0=9$.

(2) Behaviors of fracture surfaces, locations of initial fracturing and surface contour radii

Fig. 5 Flow curves of average axial true stress versus average axial logarithmic strains

Fig. 6 Variations of loads, stresses and strains at maximum load and at fracture with notch sharpness.

at fracture.

The figures sectioned along the longitudinal axis after fracture are shown in Photo. 1. For less sharp notches, the fracture surfaces present the cup-and-cone behaviors and the starting point of fracture is on the center; while for sharp notches, the fracture surfaces present nearly horizontal cleavage behavior, and the fractures start from the notch bottom. The boundary over which σ_{zr} begins to decrease with increasing λ_0 because of

(0) $\lambda_0=0$

(i) $\lambda_0=1$

(ii) $\lambda_0=4$

(iii) $\lambda_0=9$

(iv) $\lambda_0=16$

Photo. 1 Figures sectioned along the longitudinal axis after fracture

notch brittleness seems to correspond to the change of the location of initial fracturing from center to notch bottom. The surface contour radii at fracture were determined after correcting the configurations after fracture by taking into account the diameters at fracture, and the notch sharpnesses at fracture were compared with the initial values in Table 1.

2.3 Approximate Analysis of Stress and Strain Distributions Across the Minimum Notch Section at Fracture

The analytical results⁽¹²⁾⁽¹³⁾⁽¹⁴⁾ so far obtained for necked bars cannot be applied to the notched specimens. The equations necessary for analyzing the stress and strain distributions of notched specimens are as follows:

Fig. 7 Equilibrium of stresses in the neighbourhood of notch

(1) Equilibrium equation

An element along the trajectories of principal stresses over the minimum notch section as shown in Fig. 7 will be considered. The neglect of small quantities leads to the following equilibrium equation

$$\sigma_z = \sigma_r [1 - (R/r)] - (d\sigma_r/dr)R + \sigma_t \cdot R/r \quad \dots (10)$$

where R may be assumed as

$$R = (a/r)^n \rho \quad \dots (11)$$

(2) Compatibility equation:

$$e^{\varepsilon_r - \varepsilon_t} = 1 + (d\varepsilon_t/d\zeta) \cdot \zeta \quad \dots (12)$$

where

$$\zeta = r/a \quad \dots (12)$$

(3) Condition of volume constancy during the plastic deformation:

$$\varepsilon_r + \varepsilon_t + \varepsilon_z = 0 \quad \dots (13)$$

(4) Stress versus strain relation

The flow curve ($\tau_n' \sim \gamma_n'$, $\tau_n' = 3\tau_n/\sqrt{2}$, $\gamma_n' = \gamma_n/\sqrt{2}$) obtained by a simple tensile test according to the Davidenkov's correction after necking, is shown in Fig. 5 in a broken line.

(5) Boundary conditions

$$\left. \begin{aligned} \sigma_{za} &= 2 \int_0^1 \sigma_z \zeta d\zeta \\ (\sigma_r)_{r=a} &= 0, \quad (\sigma_r)_{r=0} = (\sigma_t)_{r=0} = \sigma_0 \\ \varepsilon_{za} &= -2\varepsilon_{t1}, \end{aligned} \right\} \quad (14)$$

where suffix "1" denotes the notch bottom.

(6) Analysis of stress and strain distributions

For the unnotched specimens, Davidenkov's results for $n=1$ were utilized. For the notched specimens, first of all, the author assumes σ_r/σ_0 and σ_t/σ_0 as in equation (15) by adding the correction terms due to the existence of notch (γ, α ; correction coefficients).

$$\left. \begin{aligned} \sigma_r/\sigma_0 &= 1 - \zeta^{n+1} + \gamma \zeta^{n+1} (1 - \zeta^2) \\ \sigma_t/\sigma_0 &= \sigma_r/\sigma_0 + \alpha \zeta^{n+1} \end{aligned} \right\} \quad \dots (15)$$

Then σ_z/σ_0 can be obtained from equation (10). If appropriate values of γ and α are assumed, σ_0 can be obtained by equation (14). Thus, after the distribution of τ_n' across the cross section is determined, that of β can be also determined by the flow curve. Next, α is corrected so that ε_t due to equation (3) may agree nearly with the following ε_t due to equation (12)

$$\varepsilon_t = \varepsilon_{t0} + \int_0^1 \{e^{2(\sigma_r - \sigma_t)/2\beta} - 1\} d\zeta/\zeta \quad \dots (16)$$

Further, γ is corrected so that the value of ε_{t1} may nearly agree with $(-\varepsilon_{za}/2)$ by equation (14). The converged distributions for σ and τ_n' thus obtained

Fig. 8 Distributions of σ_r, σ_t and σ_z across the notch section

Fig. 9 Distribution of τ_n' across the notch section

by repeating the above processes for each value of λ are shown in Figs. 8 and 9, respectively. In the above analysis, the appropriate values of n , that is, $n=3/2$ for the (i) type and $n=2$ for the (ii)~(iv) types, were conveniently assumed. The fracture behaviors including the colours presented on the fractured surfaces agree with the degrees of plastic deformations shown in Fig. 9.

2.4 Explanation of Notch Brittleness by the Fracture Criterion Proposed by the Author

The values of τ_m and σ_m analyzed as above are plotted against θ in Fig. 10 in fine and bold-face curved lines, respectively. The assumed values of τ_{cr} and σ_{cr} for each value of τ_n are also shown by fine and heavy straight lines, respectively, which are consistent with the experimental results previously obtained. The fracture will be discussed based on the intersection of the τ_m or σ_m curves and the τ_{cr} or σ_{cr} lines under the same value of τ_n .

Fig. 10 Diagram for interpretation of fracture behaviors and notch brittleness

For less sharp notches, σ_m at the center is very large under high value of θ . Thus the cleavage fracture at first occurs at the center. σ_m decreases from center to contour, while τ_m increases, causing the growth of shear fracture toward the center, which would be expected to result in the cup-and-cone type fracture surface.

For sharp notches, τ_m at the notch bottom is large because of stress concentration, the shear fracture first occurring at the notch bottom. τ_m decreases suddenly from contour to center, while σ_m increases. In addition, a concentrated tensile

stress will be promoted at the tips of cracks produced by the initial shear fracture; thus the propagation of cleavage brittle fracture to the center might be caused.

For the intermediate (iii) type, either of the above two phenomena occurred, or the fracture seemed to start from the intermediate points. The latter would result from the greatest value of σ_m in that range. The greatest value of σ_{cr} in this specimen is probably due to not having both extremes of the cleavage fracture at the center and shear fracture at the notch bottom.

There is a need for further study on the propagation of cracks to interpret the fracture behaviors; however, the above descriptions seem to agree with them, especially with the start of fracture.

Conclusions

The author does not believe that complicated fracture problems can be fully explained by the above criterion, if various influence factors such as stress gradient, anisotropy, strain rate, etc. are neglected. However, the fracture of metals can be explained to a certain extent on the whole without contradiction to the experimental evidences by the maintenance of the two major influence factors in this criterion and by the analysis of the stress and strain distributions at fracture. Furthermore, quantitative research on the fracture stress surfaces remains to be further discussed in the future after arranging the testing machines and apparatus. An application to brittle materials, e.g. grey cast irons will be discussed in Part III in the paper that follows.

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Experiment on the Lateral Buckling of a Cantilever Beam with Narrow Rectangular Cross Sections*

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This is a report of an experimental study for the lateral buckling of a narrow rectangular cantilever beam with an end load. These experiments were carried out systematically in a wide range of the dimension of beam. The test pieces used for the experiment were of thin plates of mild steel and aluminum with varying heights and spans. The results obtained have enabled us to decide the range of beam dimension applicable to the theoretical formula of the buckling load, and also enabled us to establish an experimental formula for a range in which the old formula does not hold.

1. Introduction

An experiment of the lateral buckling of a narrow rectangular beam has been made by Carrington⁽¹⁾, Dumont and Hill⁽²⁾ etc., but these experiments have been carried out respectively in a small range of beam dimensions. In case of lateral buckling of beam as in Euler's theoretical formula for column, there must be some limit in using these formulae for the practical design, and the empirical formulae are necessary for the beam, of which the range of the dimension does not conform to the theoretical formulae.

The results of the present experiment have made it clear that the ratio of the height to the width of the cross section is more than 30 in a range where the old formula holds, while the empirical formulae must be used for a beam of which the ratio mentioned above is below 30.

2. Experiment

Apparatus The apparatus for fixing the test piece is mounted on the over-hanging part of $\square$ -beam fastened to the concrete foundation with bolts. In this apparatus the test piece is settled

both laterally and vertically by the two plates. At the free end of the test piece, a pail is hung with a pin passing through a hole on the horizontal centre line of the test piece. The load is supplied by water jetting from a small nozzle and estimated by the water quantity pouring into the pail for a given test time. The capacity of the nozzle varies from 15 to 500 gr. per minute and its selection depends upon the strength of the beam to be tested. Fig. 1 shows the photograph of the apparatus of our experiment.

Fig. 1 Testing apparatus

Measurement Vertical and horizontal deflections v , u , twisted angle θ of the cross section,

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