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On Gears with Cylindrical Surfaces*

By Akira YOKOTA **

In this paper, cylindrical surfaces, which mesh with each other at one point at a constant angular velocity ratio, are investigated. In the first place, the possibility of this case is investigated, and it is clarified that this is possible when one of the pair is convex and its partner with line contact exists. Then curvatures of these surfaces are made clear. And interference of these gears is studied basing on this result. When one of the pair is given, the partner's generator direction is limited by asymptotic direction of surface which contacts with the given cylindrical surface along the line. Lastly, when one of the pair and the partner's generator direction are given, the theory that the partner's profile is calculated is clarified. These results are the foundation of cutting gears by milling or shaping.

1. Introduction

To transmit a revolving motion with a constant angular velocity ratio between two axes, skew gears have been investigated. But such gears are very difficult to manufacture. And this is one of the reasons why skew gears are not suitable for practice. Therefore, spiral gears are used instead, though they contact at one point and have little tooth bearing. And it is a matter of common knowledge to-day that point contact gears are more suitable for practice, considering the error of manufacture and mounting. But the foundation of this knowledge is that the machining method is simple. Therefore the author tries to study the gears easy to make. Taking the shaping and milling into consideration, the surface most easy to make is a cylindrical surface. So, he investigates gears with cylindrical surfaces with constant angular velocity ratio. Symbols without subscript are vector descriptions.

2. Preliminary study

In the first place, it is necessary to explain the line contact gears of which I, one of the pair, is of cylindrical surface. This is a special case of the general theory which has been clarified by Mr. M. TANIMURA.⁽¹⁾

Now, two gear surfaces are rotating about the axes a_1, a_2 , Fig. 1. These are I & II, rotating at an angular velocity ω_1 & ω_2 respectively. ω_1 & ω_2

Fig. 1

are vector amounts and these amplitudes are w_1, w_2 . Relative velocity of I to II is ω , which equals to $\omega_2 - \omega_1$. The position of P is denoted by radius vector $\mathbf{r}$. The velocity of I at $P\{\mathbf{r}\}$ is $\mathbf{v}_1$, velocity of II at same point is $\mathbf{v}_2$, and the relative velocity is $\mathbf{v}, \mathbf{v}_2 - \mathbf{v}_1$. These amounts at the origin of radius vector are $\bar{\omega}_1, \bar{\omega}_2, \bar{\omega}_3$, then

$$\left. \begin{aligned} \mathbf{v}_1 &= \omega_1 \times \mathbf{r} + \bar{\omega}_1 \\ \mathbf{v}_2 &= \omega_2 \times \mathbf{r} + \bar{\omega}_2 \\ \mathbf{v} &= \mathbf{v}_2 - \mathbf{v}_1 = \omega \times \mathbf{r} + \bar{\omega} \end{aligned} \right\} \dots\dots\dots (2.1)$$

Fig. 2 shows the aspect as I, cylindrical surface, is given, and the unit direction vector of the generator of the cylindrical surface g_1 is described. $\mathbf{r}(t, r)$ is a point on contact line. $\mathbf{r}$ describes an orbit line as t varies, and a contact line as r varies. Put the unit vector normal to this surface at point $\mathbf{r}(t, r)$ $\mathbf{n}$, and the radius of

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** Assistant Professor,
Yamagata Univ, Faculty of Eng.
Yonezawa, Yamagata Pref.

Fig. 2

curvature of this cylindrical surface at this point ρ_1 . n has a direction satisfying $n \cdot \xi_{rr} < 0$, and the profile is always convex. In other words, normal of profile is drawn to I outwards. Now, put the unit vector normal to profile at time t at point $\xi + d_1\xi$, $n + d_1n$, where subscript 1 means to concern I for that notation. For I is a cylindrical surface, next relation is kept between $d_1\xi$ & d_1n ,

$$g_1 \times n \cdot d_1\xi = \rho_1 g_1 \times n \cdot d_1n$$

Then

$$\rho_1 = [g_1 \cdot n \cdot d_1\xi] / [g_1 \cdot n \cdot d_1n] \quad (2.2)$$

and

$$\rho_1 d_1n = -d_1\xi \cdot (g_1 \times E)^2 \quad (2.3)$$

Now, consider d_1w which is a variation of w to $d_1\xi$. As the static space is the field of vector w , so next relation is held not only on the profile surface, but also all directions. And if we take $d_1\xi$ as the direction of ξ_r that is the direction of tangent of simultaneous contact line, next relation (2.6) which is differenciaded from (2.5) is held,⁽²⁾

$$dw = -d\xi \cdot (E \times \omega) \quad (2.4)$$

$$w \cdot n = 0 \quad (2.5)$$

$$w \cdot d_1n + d_1w \cdot n = 0 \quad (2.6)$$

then

$$-w \cdot \xi_r \cdot (g_1 \times E)^2 / \rho_1 - n \cdot \xi_r \cdot (E \times \omega) = 0$$

$$\therefore \xi_r = a \left(-\frac{1}{\rho_1} w \cdot (g_1 \times E)^2 + n \times \omega \right) \times n \quad (2.7)$$

a is the coefficient concerning the amount of ξ_r ,

$$a = \frac{ds}{dr} \left\{ \left(-\frac{1}{\rho_1} w \cdot (g_1 \times E)^2 + n \times \omega \right)^2 \right\}^{-\frac{1}{2}}$$

where s is the length of contact line. Next, we consider ξ_t .

ξ_t satisfies⁽¹⁾

$$\xi_t = \mu_2 v_1 + \mu_1 v_2, \quad \mu_1 + \mu_2 = 1 \quad (2.8)$$

The position which will be a contact point after time dt is $\xi + d_1\xi$,

$$d_1\xi = (\xi_t - v_1) dt = \mu_1 w dt \quad (2.9)$$

The variation of normal to this point is

$$d_1n = -d_1\xi \cdot \frac{(g_1 \times E)^2}{\rho_1} dt = -\frac{\mu_1}{\rho_1} w \cdot (g_1 \times E)^2 dt$$

The variation of the normal vector after time dt at point $\xi + d_1\xi$ is described as Dn , then

$$Dn = d_1n + \omega_1 \times n dt \quad (2.10)$$

Substituting d_1n above to this relation,

$$Dn = -\frac{\mu_1}{\rho_1} w \cdot (g_1 \times E)^2 dt + \omega_1 \times n dt \quad (2.11)$$

For Dw , variation of w at this point,

$$Dw = -\xi_t \cdot (E \times \omega) dt = \omega \times (\mu_1 w + v_1) dt \quad (2.12)$$

These Dw & Dn satisfy the condition of contact, so

$$Dw \cdot n + w \cdot Dn = 0$$

with (2.11), (2.12)

$$\omega \times (\mu_1 w + v_1) \cdot n + w \cdot \left(-\frac{\mu_1}{\rho_1} w \cdot (g_1 \times E)^2 + \omega_1 \times n \right) = 0$$

In this relation, put

$$q = w \times \omega_1 + \omega \times v_1$$

then

$$\mu_1 = -q \cdot n / \left\{ \frac{1}{\rho_1} (w \times g_1)^2 + [\omega w n] \right\} \quad (2.13)$$

(2.7), (2.9) & (2.13) give the contact line of profiles and its locus.

Next, consider the profile II. Putting slide lines and contact lines parametric lines, the first fundamental form of this profile surface is shown as

$$E_2 dt^2 + 2 F_2 dt dr + G_2 dr^2$$

And the first fundamental quantities are

$$E_2 = \mu_2^2 w^2, \quad F_2 = -\mu_2 w \cdot \xi_r, \quad G_2 = \xi_r^2 \quad (2.15)$$

and put the second fundamental form

$$L_2 dt^2 + 2 M_2 dt dr + N_2 dr^2$$

then

$$\left. \begin{aligned} L_2 &= \mu_2^2 [n \cdot \omega w] - \mu_2 n \cdot q \\ M_2 &= -\mu_2 [n \cdot \omega \xi_r] \\ N_2 &= n \cdot \xi_{rr} \end{aligned} \right\} \quad (2.16)$$

And I is a cylindrical surface, so its total curvature is zero. Then among the secondary fundamental quantities of I,

$$L_1 N_1 - M_1^2 = 0$$

And the fundamental quantities are⁽¹⁾

$$\left. \begin{aligned} L_1 &= \mu_1^2 [n \cdot \omega w] + \mu_1 n \cdot q \\ M_1 &= \mu_1 [n \cdot \omega \xi_r] \\ N_1 &= n \cdot \xi_{rr} \end{aligned} \right\}$$

$$\text{so, } N_1 = \frac{M_1^2}{L_1} = \frac{\mu_1^2 [n \cdot \omega w_r]^2}{\mu_1^2 [n \cdot \omega w] + \mu_1 n \cdot q}$$

Referring to (2.7) & (2.13), (2.16) is

$$\left. \begin{aligned} L_2 &= \mu_2 [n \cdot \omega w] + \mu_1 \mu_2 (w \times g_1)^2 / \rho_1 \\ M_2 &= -\mu_2 a [\omega \times q_1 q_1 w] / \rho_1 \\ N_2 &= -a^2 [n \cdot \omega q_1]^2 / \rho_1 \end{aligned} \right\} \quad (2.17)$$

The total curvatures of I & II are

$$K_1 = P + Q / \mu_1, \quad K_2 = P - Q / \mu_2$$

where

$$\begin{aligned} P &= \{ [n \cdot \omega w] n \cdot \xi_{rr} - [n \cdot \omega \xi_r]^2 \} / W^2 \\ Q &= n \cdot \xi_{rr} n \cdot q / W^2 \end{aligned}$$

$$W = \sqrt{(w \times g_r)^2}$$

Now $K_1 = 0$, so $P = -Q/\mu_1$ then,

$$K_2 = -Q/\mu_1 - Q/\mu_2 = -Q/\mu_1\mu_2 \\ = -\frac{n \cdot g_{rr} n \cdot q}{W^2 \mu_1 \mu_2} \quad (2.18)$$

And n was determined outwards to I, that is, $n \cdot g_{rr} < 0$. When there is no interference between the profiles, $n \cdot q/\mu_1\mu_2 < 0$ is held, so

$$K_2 < 0 \quad (2.19)$$

is always kept.

The meshing of two cylindrical surfaces with each other is possible, when a quadratic surface osculating both surfaces is a hyperboloid. That is to say, it is necessary that surface II contacting with I along line has a hyperbolic point at this point. With (2.19), if the surface which contacts with I along line exists, the surface II has always a hyperbolic point on the contact line. Therefore, when the surface which contacts with I along line exists, the cylindrical surface which contacts with I at a point exists at the same time.

3. Two cylindrical surfaces in meshing with

Now, as Fig. 3, the cylindrical surface, whose generator's direction is g_2 , its principal

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curvature radius is ρ_2 , is fixed on II and contacts with I at $X(t)$. And direction of g_2 satisfies

$$g_1 \cdot g_2 > 0 \quad (3.1)$$

I and II contact with each other at $X + X'dt$ at time $t+dt$. The locus of $X(t)$ is defined as Meeting Line. And contact point is defined as

Touching Point, the locus of Touching Point on surface as Touching Line. They are like the contact line, contact point, orbit line, sliding line of line contact gear, but essentially different, so we should distinguish as above. Meeting Line is the locus of Touching Point in the static space. The point on I which will be a touching point after time dt , which is on the touching line, is described as $X + d_1X$, the point on II as $X + d_2X$. And touching point at $t+dt$ is shown as $X + DX$. $X + DX$ is on the surface $g(t,r)$, explained in the above section. So

$$DX = g_1 dt + g_2 dr \quad (3.2)$$

These two surfaces touch with each other at $X + DX$ at time $t+dt$, these contact points are $X + d_1X$ of I, $X + d_2X$ of II at time t . Common unit normal vector of tooth surface at $X + DX$ is $n + Dn$. At time t , normals at $X + d_1X$ to I, $X + d_2X$ to II are $n + d_1n$, $n + d_2n$. Normals are outwards to I, so

$$\left. \begin{aligned} d_1n &= -\frac{1}{\rho_1} d_1X \cdot (g_1 \times E)^2, & (\rho_1 > 0) \\ d_2n &= -\frac{1}{\rho_2} d_2X \cdot (g_2 \times E)^2, & (\rho_2 < 0) \end{aligned} \right\} \quad (3.3)$$

and

$$\left. \begin{aligned} Dn &= d_1n + \omega_1 \times ndt \\ Dn &= d_2n + \omega_2 \times ndt \end{aligned} \right\} \quad (3.4)$$

Now, put

$$\left. \begin{aligned} d_1n &= \xi g_1 \times ndt \\ d_2n &= \eta g_2 \times ndt \end{aligned} \right\} \quad (3.5)$$

then

$$Dn = (\xi g_1 \times n + \omega_1 \times n) dt = (\eta g_2 \times n + \omega_2 \times n) dt$$

so

$$(\xi g_1 - \eta g_2 - \omega) \times n = 0$$

We can put

$$\xi g_1 - \eta g_2 - \omega = -\zeta n$$

so

$$\omega = \xi g_1 - \eta g_2 + \zeta n \quad (3.6)$$

ξ, η, ζ are coefficients, when ω is described by $g_1 g_2 n$, so $\xi \eta \zeta$ should be determined as follows:

$$\omega = \omega \cdot g_1^* g_1 + \omega \cdot g_2^* g_2 + \omega \cdot n^* n \quad (3.7)$$

where g_1^*, g_2^*, n^* are the reciprocal vector to $g_1 g_2 n$, so

$$\left. \begin{aligned} \xi &= \omega \cdot g_1^* = \frac{[\omega \cdot g_1]}{[n \cdot g_1]} \\ \eta &= -\omega \cdot g_2^* = -\frac{[\omega \cdot g_2]}{[n \cdot g_2]} \\ \zeta &= \omega \cdot n^* = \frac{[\omega \cdot n]}{[n \cdot n]} \end{aligned} \right\} \quad (3.8)$$

From these relations,

$$\left. \begin{aligned} d_1n &= \omega \cdot g_1^* g_1 \times n \\ d_2n &= -\omega \cdot g_2^* g_2 \times n \end{aligned} \right\} \quad (3.9)$$

Substituting this relation to (3.4)

$$\left. \begin{aligned} Dn &= (\omega \cdot g_1^* g_1 + \omega_1) \times ndt \\ &= (-\omega \cdot g_2^* g_2 + \omega_2) \times ndt \end{aligned} \right\} \quad (3.10)$$

On the other hand, the relative velocity $w + Dw$

at $\mathbf{x} + D\mathbf{x}$ satisfies

$$D\mathbf{w} = \boldsymbol{\omega} \times D\mathbf{x} \quad (3.11)$$

And

$$D\mathbf{x} = \mathbf{x}_t dt + \mathbf{x}_r dr$$

so, put

$$dr/dt = \lambda \quad (3.12)$$

then

$$D\mathbf{x} = (\mathbf{x}_t + \lambda \mathbf{x}_r) dt = (\mu_1 \mathbf{v}_2 + \mu_2 \mathbf{v}_1 + \lambda \mathbf{x}_r) dt \quad (3.13)$$

These relations and

$$D(\mathbf{w} \cdot \mathbf{n}) = \mathbf{w} \cdot D\mathbf{n} + \mathbf{n} \cdot D\mathbf{w} = 0 \quad (3.14)$$

give next relation

$$[\mathbf{w} \cdot \mathbf{g}_1^* \mathbf{g}_1 + \boldsymbol{\omega}_1 \cdot \mathbf{n}] + [\mathbf{n} \cdot \mathbf{w} \mathbf{x}_t] + \lambda [\mathbf{n} \cdot \mathbf{w} \mathbf{x}_r] = 0$$

so

$$\lambda = \frac{[\mathbf{w} \cdot \mathbf{g}_1^* \mathbf{g}_1 \mathbf{n}] + [\mathbf{w} \cdot \boldsymbol{\omega}_1 \mathbf{n}] + [\mathbf{n} \cdot \mathbf{w} \mathbf{x}_t]}{[\mathbf{n} \cdot \mathbf{w} \mathbf{x}_r]} \quad (3.15)$$

with (2.7)

$$a\lambda = \frac{\rho_1 [\mathbf{w} \times \mathbf{g}_1 \boldsymbol{\omega} \mathbf{g}_2]}{[\mathbf{n} \cdot \mathbf{w} \mathbf{g}_1] [\boldsymbol{\omega} \times \mathbf{g}_1 \mathbf{g}_1 \mathbf{g}_2]} + \frac{\mu_1 (\mathbf{w} \times \mathbf{g}_1)^2}{[\boldsymbol{\omega} \times \mathbf{g}_1 \mathbf{g}_1 \mathbf{w}]} \quad (3.16)$$

From (3.16) & (3.13), we get a differential equation of meeting line. Touching lines according to this meeting line are

$$d_1 \mathbf{x} = D\mathbf{x} - \mathbf{v}_1 dt = (\mu_1 \mathbf{w} + \lambda \mathbf{x}_r) dt$$

$$d_2 \mathbf{x} = D\mathbf{x} - \mathbf{v}_2 dt = (-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r) dt$$

On the variation of normal $d_2 \mathbf{n}$ to $d_2 \mathbf{x}$, ρ_2 is the radius of curvature of II, with (3.3),

$$d_2 \mathbf{n} = -\frac{1}{\rho_2} d_2 \mathbf{x} \cdot (\mathbf{g}_2 \times \mathbf{E})^2$$

And from (3.5)

$$d_2 \mathbf{n} = \eta \mathbf{g}_2 \times \mathbf{n} dt = -\boldsymbol{\omega} \cdot \mathbf{g}_2^* \mathbf{g}_2 \times \mathbf{n} dt$$

From these equations

$$\frac{1}{\rho_2} (-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r) \cdot (\mathbf{g}_2 \times \mathbf{E})^2 = \boldsymbol{\omega} \cdot \mathbf{g}_2^* \mathbf{g}_2 \times \mathbf{n}$$

$$\text{then } \frac{1}{\rho_2} (-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r) \cdot (\mathbf{g}_2 + \mathbf{E})^2 \cdot \mathbf{g}_2 \times \mathbf{n} = \boldsymbol{\omega} \cdot \mathbf{g}_2^*$$

so

$$\rho_2 = \frac{[-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r \cdot \mathbf{g}_2 \mathbf{n}]}{\boldsymbol{\omega} \cdot \mathbf{g}_2^*} = \frac{[\mathbf{g}_2 \mathbf{g}_2 \times \mathbf{g}_1 - \mu_2 \mathbf{w} + \lambda \mathbf{x}_r]}{[\mathbf{n} \cdot \mathbf{g}_1 \boldsymbol{\omega}]} \quad (3.17)$$

(3.17) gives the required radius of curvature of II.

4. Interference

With the radius of curvature ρ_2 of II, we can discuss the interference between these gear surfaces. And when $\mathbf{g}_1 \mathbf{n}_1 \rho_1$ are given, we can find the allowable range of existing of $\mathbf{g}_2$.

Two cylindrical surfaces must contact convexly to each other at a point. From (3.17), when $\mathbf{g}_1 \mathbf{n}_1 \rho_1$ are given, ρ_2 varies with $\mathbf{g}_2$. So it is a critical point for interference at $\rho_2 = 0$ or ∞ . $\rho_2 = \infty$ means $[\mathbf{n} \cdot \mathbf{g}_1 \boldsymbol{\omega}] = 0$, and this is independent of $\mathbf{g}_2$, so we discuss the interference except this case.

When $\rho_2 = 0$,

$$[\mathbf{g}_2 \mathbf{g}_2 \times \mathbf{g}_1 - \mu_2 \mathbf{w} + \lambda \mathbf{x}_r] = 0$$

with

$$(-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r) \cdot \mathbf{n} = 0, \quad \mathbf{g}_2 \cdot \mathbf{n} = 0$$

$$(-\mu_2 \mathbf{w} + \lambda \mathbf{x}_r) \times \mathbf{g}_2 = 0 \quad (4.1)$$

Now, put

$$\mathbf{g}_2 = \mathbf{u} \mathbf{w} + \mathbf{v} \mathbf{x}_r \quad (4.2)$$

from (4.1)

$$-\mu_2 \mathbf{v} \mathbf{w} \times \mathbf{x}_r + \lambda \mathbf{u} \mathbf{x}_r \times \mathbf{w} = 0$$

$$\mu_2 \mathbf{v} + \lambda \mathbf{u} = 0 \quad (4.3)$$

With (3.16)

$$\left\{ \mu_2 [\mathbf{n} \cdot \boldsymbol{\omega} \mathbf{w}] + \frac{\mu_2 \mu_1}{\rho_1} (\mathbf{w} \times \mathbf{g}_1)^2 \right\} \frac{\rho_1}{\mu_2} [\mathbf{n} \cdot \mathbf{w} \mathbf{g}_1] \mathbf{u}^2 - 2 \left(-\mu_2 \frac{a}{\rho_1} [\boldsymbol{\omega} \times \mathbf{g}_1 \mathbf{g}_1 \mathbf{w}] \right) \frac{\rho_1}{a} [\mathbf{n} \cdot \mathbf{w} \mathbf{g}_1] \mathbf{u} \mathbf{v} + \left(-\frac{a^2}{\rho_1} [\mathbf{n} \cdot \boldsymbol{\omega} \mathbf{g}_1]^2 \right) [\mathbf{n} \cdot \mathbf{w} \mathbf{g}_1] \frac{\rho_1}{a^2} \mu_2 \mathbf{v}^2 = 0$$

With (2.17),

$$L_2 a^2 \mathbf{u}^2 - 2 M_2 a \mu_2 \mathbf{u} \mathbf{v} + N_2 \mu_2^2 \mathbf{v}^2 = 0$$

Put

$$d\mathbf{x} = \mathbf{x}_{2t} dt + \mathbf{x}_r dr \quad (4.4)$$

then

$$\mathbf{x}_{2t} dt = -\mu_2 \mathbf{w} dt,$$

and with

$$\alpha \mathbf{u} = -\mu_2 dt, \quad \alpha \mathbf{v} = a dr,$$

$\mathbf{x}_2$ and $d\mathbf{x}$ have same direction. Then

$$L_2 dt^2 + 2 M_2 dt dr + N_2 dr^2 = 0 \quad (4.5)$$

This is the differential equation of asymptotic curve.⁽³⁾ So, when $\mathbf{g}_2$ has the direction of the asymptotic direction of the surface which contacts with I along a line, ρ_2 is zero. Therefore, among four parts decided by asymptotic directions on the osculating plane, the range not being $\mathbf{g}_1$ is admitted to exist for $\mathbf{g}_2$.

5. Partner's profile

Partner's profile, II, is calculated as follows.

In the first place, I is expressed by polar tangential co-ordinates⁽⁴⁾ whose origin is the project of the intersection of I-axis and common normal between 2 axes to the rectangular plane to $\mathbf{g}_1$, and starting line is the project of I-axis.

That is

$$r_1 = f_1(\theta_1) \quad (5.1)$$

then

$$\left. \begin{aligned} \cos \theta_1 &= \boldsymbol{\omega}_1 \cdot \mathbf{n} / \sqrt{(\boldsymbol{\omega}_1 \times \mathbf{g}_1)^2} \\ &= l_3' / \sqrt{(1 - l_1'^2)} \\ &= g_1(t) \end{aligned} \right\} \quad (5.2)$$

$$\gamma_1 = F_1(t) \quad (5.3)$$

For II, the profile is expressed by polar tangential co-ordinates whose origin is the project of the intersection of II-axis and common normal between 2 axes and starting line is the project of II-axis to the rectangular plane to $\mathbf{g}_2$.

$$\left. \begin{aligned} \cos \theta_2 &= \boldsymbol{\omega}_2 \cdot \mathbf{n} / \sqrt{(\boldsymbol{\omega}_2 \times \mathbf{g}_2)^2} \\ &= l_3'' / \sqrt{(1 - l_2''^2)} \\ &= g_2(t) \end{aligned} \right\} \quad (5.4)$$

$$\left. \begin{aligned} r_2 &= \mathbf{n} \cdot \mathbf{n} - r_1 \\ &= em_3 - r_1 \\ &= F_2(t) \end{aligned} \right\} \quad (5.5)$$

where $\mathbf{n}$ is the common normal of 2 axes.

(5.4) (5.5) express the profile of II in polar tangential co-ordinates by parameter t .

And above constants are,

$$\begin{cases} \omega_1 \cdot \omega / \omega_1 \omega = \cos \beta_1 \\ \omega_2 \cdot \omega / \omega_2 \omega = \cos \beta_2 \\ l_3' = l_3 \cos \beta_1 + n_3 \sin \beta_1 \\ l_3'' = l_3 \cos \beta_2 + n_3 \sin \beta_2 \\ l_1' = l_1 \cos \beta_1 + n_1 \sin \beta_1 \\ l_2'' = l_2 \cos \beta_2 + n_2 \sin \beta_2 \end{cases}$$

6. Conclusion

Cylindrical surfaces, which mesh with each other at one point at a constant angular velocity ratio, are investigated. In the first place, the possibility of this case is investigated, and it is clarified that this is possible when one of the pair is convex and its partner with line contact exists. Then curvatures of these surfaces are made

clear. And interference of these gears is studied basing on this result. When one of the pair is given, the partner's generator direction is limited by asymptotic direction of surface, which contacts with the given cylindrical surface along the line. Lastly, when one of the pair and the partner's generator direction are given, the theory by which the partner's profile is calculated, is clarified. These results are the foundation of cutting gears by milling or shaping.

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Study on the Testing Method of Cemented Carbide Tool*

(On the critical Value of Flank Wear)

by Keiji OKUSHIMA** Shūkichi NAGATOMI*** and Kitao ŌKUSA****

O.W. Boston and others have introduced a mean value of flank wear of 0.03'' as a reasonable indication for the tool life. However, our experiments have revealed that the critical value of flank wear is not always constant, but it should be controlled by the kind of carbide grade, material cut, cutting condition and tool shape. Arranging the experimental data, we have proposed a summary table of the critical value of flank wear.

1. Introduction

We classified the tool failures of carbide tools into nineteen fundamental types in the previous report.⁽¹⁾ Among them, the flank wear, crater and end wear are the most common types which occur in ordinary machining practice. However, the mean value of flank wear (V_B) or the maximum value of flank wear at the nose (V_B') is a most convenient yardstick, with which whether or not the regrinding is necessary.

O.W. Boston⁽²⁾ and the others have proposed the value 0.03'' (0.75 mm) of flank wear as the

Carbide grade TX 3
 Material cut Ni-Cr-Mo steel
 Cutting speed 150 m/min
 Depth of cut 1 mm
 Feed 0.1 mm/rev
 Tool shape 5°-85°-0°-45°-90°-0°-0.2 R

Fig. 1

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** Professor of Mechanical Engineering, Kyoto University, Yoshida, Sakyo-ku, Kyoto.

*** Head of Alloy Department of Tungaloy Manufacturing Company Tsukagoshi, Kawasaki, Kanagawa-ken.

**** Engineer of Alloy Department of Tungaloy Manufacturing Company.