this experimental range—the maximum volume fraction of air in the tube is about 85%.

4. From the experiments with obstacles in the flow, it has been shown that the temperature rise of the tube wall may occur at the low water rate in the flow patterns (c) and (d).

Acknowledgment

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References

(1) Martinelli, R.C., and others, Trans. ASME., 66, 2, 139-151, 1944.
(4) Behringer, P., Forschungsheft 365, 4-12, 1934.
(5) Schurig, W., Forschungsheft 365, 13-23, 1934.

Pressure Drop due to Friction for Parallel Flow of Mixtures of Vapour and Water in a Downcomer

By Tetsuji Katsuhara

In this paper, the author experimentally investigated the pressure drop due to friction for the flow of water containing bubbles in a non-heated downcomer.

First, the pressure drop due to friction and dryness fraction for the parallel downward flow of the mixtures of air and water were measured in a vertical 1500 mm long and 19 mm inside diameter glass pipe. Next, the ratios of the coefficient of friction in high pressure regions were extensively computed from the results of the above experiment. Finally, in case we used non-heated downcomer, it was proposed that we might adopt 0.03 as an approximate value of the coefficient of friction.

1. Introduction

In the calculation of a water circulation in a steam boiler, we must find the pressure drops of water and steam in the circuit. Generally speaking, however, many problems have not yet been clear. Consequently, we cannot help contenting with the rough calculations. Especially, when water containing steam bubbles flows in a downcomer, as far as the writer is aware, the pressure drop for the flow of water has not yet been made clear.

It has been already reported that there are many flow modes in an evaporation pipe with the downward flow of water. In a non-heated downcomer, however, even if steam bubbles mix in water, the quantity of bubbles is very little. Therefore, bubbles flow in parallel with water, and the pressure drop of water is mainly due to friction loss.

In the first part of this paper, the pressure drop due to friction and the dryness fraction for the parallel downward flow of the mixtures of air and water are measured in a glass pipe, and in the second, the ratios of the coefficient of friction in high pressure regions are extensively computed from the results of the experiments.

The writer regrets that there are few experimental data for a downcomer to justify the above extension. According to the successful trial for an evaporation pipe by Nelson and Martinelli(9), the results of the extension will be believable in this
case because of the simpler flow.

2. Experimental Apparatus and Measurements

Fig. 1 shows the general arrangement for the experiment.

Water was supplied to the head tank $T_1$ which was 4 meters high from the floor of the laboratory, and kept the constant water level by the overflow method. $V_2$ was the discharge control valve, which was connected with a rubber hose to the upper cock $C_1$. Then, water in the head tank was discharged to the vertical pipe of glass $GP$ through $V_2$ and $C_1$.

![Diagram]

Fig. 1 General arrangement for the experiment

Air was compressed by the compressor $K$, and reserved in the reservoir $AR$. In order to regulate the flow rate of air, the needle valve was adopted at the outlet of the reservoir. The valve was communicated with the gas meter $AG$ by a rubber tube of 3/8″ in diameter. The flow rate of air was measured by the gas meter. From the outlet of the gas meter, air was guided to the cock $C_1$, where air was blown into the water, through a nozzle of a single hole type.

The distance between the upper cock and the glass pipe had been previously so determined that the accelerations of bubbles might be neglected and the flow mode might be stable. It was about fifty times as long as the inner diameter of the hose.

The glass pipe $GP$ was 1,500 mm long, and 19 mm in inside diameter. The pipe was set vertically. In order to measure the static pressure, the first, second and third measuring holes were installed at a distance of 250, 750, 1,250 mm, respectively, from the top end of the pipe.

The mixtures of air and water through the pipe were discharged to the midway tank $T_2$, in which air was separated from water. Then, water alone was discharged to the tank $T_3$, and the flow rate was measured on its way.

$M_1$ and $M_4$ are the mercury manometers, $M_2$ and $M_5$ the carbon tetrachloride manometers. The pressure of air at the outlet of the flow meter was measured by $M_1$. The pressure drops of the mixtures between the first and the second measuring holes and between the second and the third measuring holes were measured by $M_2$ and $M_3$, respectively.

The pressure of the mixtures at the second measuring holes was measured by $M_4$. The pressure losses of the mixtures $\Delta P_{TP1}$ between the first and the second measuring holes, and $\Delta P_{TP2}$ between the second and the third measuring holes are respectively given by

$$\Delta P_{TP1} = (\gamma - \gamma_w)H_0 + (\gamma_{CC4} - \gamma_w)H_1$$

$$\Delta P_{TP2} = (\gamma - \gamma_w)H_0 + (\gamma_{CC4} - \gamma_w)H_2$$

where $\gamma$, $\gamma_w$ and $\gamma_{CC4}$ are the specific weight of the mixtures, water and carbon tetrachloride, $H_0$ the length between the measuring holes, $H_1$ and $H_2$ the indicated heads of the manometer $M_1$ and $M_2$.

The specific weight of the mixtures can be written as follows:

$$\gamma = \psi \gamma_d + (1 - \psi)\gamma_w$$

where $\psi$ is the dryness fraction, $\gamma_d$ the specific weight of air. In this experiment, $\gamma_d$ is much smaller than $\gamma_w$, therefore we can put

$$\gamma \approx (1 - \psi)\gamma_w$$

In order to find the values of $\gamma$, $\psi$ must be measured. With the object of the measurement, the bottom cock $C_5$ was quickly closed, as soon as the upper rubber hose connected with the glass pipe was taken off.

3. Results of Experiments

1) Preliminary Test

In order to find the coefficient of friction $\lambda_w$ for the single flow in the glass pipe, water alone was discharged, and the pressure drops were measured. As a result, $\lambda_w$ was given as follows:

$$\lambda_w = 0.317 R_{Re}^{-0.25}$$

where $R_{Re}$ is the Reynolds number for the single flow of water.

2) $\sqrt{\Delta P_{TP1} - \Delta P_{w1}} - \sqrt{\Delta P_{w2}}$ $\Delta P_{TP2}$

$\Delta P_{TP1}$ and $\Delta P_{TP2}$ are given by the equation (1) and (2). In case that both values were nearly equal, $\Delta P_{TP2}$ was taken as the representative value of the pressure drop for the mixtures. If they were very different, the length of the rubber hose between the upper cock and the glass pipe was extended
till they had become nearly equal.

The pressure drop $\Delta P_w$ for the water flowing alone and $\Delta P_d$ for the air flowing alone can be written as follows:

$$\Delta P_w = \frac{H_0 w_0^2}{d} \frac{\omega_w}{2g} \gamma_w$$

$$\Delta P_d = \frac{H_0 w_0^2}{d} \frac{\omega_d}{2g} \gamma_d$$

where $d$ is the inside diameter of pipe, $\lambda$ the coefficient of friction for the single flow of air, $w_0$ and $w_d$ the mean velocities of the single flow of water and air, respectively.

In fact, generally, the flow of water in a downcomer is turbulent. From this standpoint, the single flows of water were experimented for the only turbulent region in which the Reynolds numbers were $75 \leq R_w \leq 2880$, and the values of $\lambda_w$ known from the results of the preliminary test were adopted for the calculations of $\Delta P_w$.

As it is usual that the flow rates of air in the study are very small, the single flow of air is laminar in most cases. Consequently, the experiments were carried out in laminar regions for the single flow of air, and the values of $\lambda_d$ were adopted $\lambda_d = 64/R_d$, where $R_d$ was the Reynolds number for the single flow of air.

Fig. 2 shows the relations between the values of $\sqrt{\Delta P_{TF}/\Delta P_w}$ and $\sqrt{\Delta P_d/\Delta P_w}$. The full line in the figure represents the mean values for the results of experiment in a horizontal pipe by Lockhart and Martinelli$^4$.

![Fig. 2 $\sqrt{\Delta P_{TF}/\Delta P_w} = \sqrt{\Delta P_d/\Delta P_w}$](image)

From the figure it will be understood that the correlation method by Lockhart and Martinelli will be applicable to the parallel flow of the mixtures. However, as the pressure drops in a downcomer are larger than that in a horizontal pipe, it is no wonder that the curve for a horizontal pipe should not be applicable to the present case.

As the direction of the buoyancy force of bubbles is opposite to the flow direction, the measured values of pressure drops are dispersed. This tendency is strongly revealed, when the sizes of bubbles are large.

$$\varphi = 8.466 \gamma_0^{-1.544} \quad (4)$$

where $m$ equals to $\sqrt{\Delta P_w/\Delta P_d}$.

![Fig. 3 $\varphi = \sqrt{\Delta P_w/\Delta P_d}$](image)

(4) **The Ratios of the Coefficient of Friction**

The coefficient of friction $\lambda$ in two-phase flow is defined by

$$\Delta P_{TF} = \lambda \frac{l}{d} \frac{w^2}{2g} \gamma$$

where $l$ is the length of pipe, $d$, the inner diameter of pipe, $w$, the mean velocity of mixtures based on the principle of continuity, $\gamma$, the mean specific weight expressed by the equation (3) and $\Delta P_{TF}$, the pressure drop due to friction.

As the values of $\sqrt{\Delta P_{TF}/\Delta P_w}$ are represented by $\sqrt{\Delta P_d/\Delta P_w}$ as shown in Fig. 2, we have

$$\sqrt{\Delta P_{TF}/\Delta P_w} = f_1(\sqrt{\Delta P_d/\Delta P_w})$$

Moreover, using the discharges of air and water, we have

$$\sqrt{\Delta P_{TF}/\Delta P_w} = \sqrt{\gamma_0 (G_d + G_w) \gamma_w}$$

Inserting the above equation in the equation (6), we have

$$\lambda = \frac{1}{\lambda_w \left(1 + \frac{G_d}{G_w}\right) \gamma_w} \left[f_1(m)\right]^2$$

As the values of $G_d/G_w$ are negligible comparing with 1.0 for the flow in the present case, we have

$$\lambda = \frac{\lambda_w \gamma_d}{\gamma_w} \left[f_1(m)\right]^2$$

$\varphi$ is the function of $m$, generally, we put

$$\varphi = f_1(m)$$

Considering the equation (3), we have
In the above figure, the parameters of the curves are of Reynolds numbers 5,000, 10,000, 20,000, 50,000 and 100,000 from the upper curve, respectively.

Fig. 6 The ratios of the coefficient of friction for high pressure regions
\[ \lambda_w = [1-f_2(m)]^2 \quad \text{where} \quad \lambda_w = [1-f_2(m)]^2 \quad \text{and} \quad \lambda = \frac{\lambda}{\lambda_w} \]

Accordingly, the ratios of the coefficient of friction \( \lambda/\lambda_w \) can be represented by \( m \).

As the values of \( f_2(m) \) are regularized for \( m \) and the errors of measurements are less than that of \( f_1(m) \), the values of \( f_2(m) \) are calculated from the equation (4), and the values of \( f_1(m) \) are used as the experimental values shown in Fig. 2. Substituting these values of \( f_1(m) \) and \( f_2(m) \) in the equation (7b), the relations between the values of \( \lambda/\lambda_w \) and \( m \) are obtained as shown in Fig. 4.

Eliminating \( m \) from the equations (4) and (7b), \( \lambda/\lambda_w \) is able to be connected with \( \varphi \). By means of using the values on the dotted line in Fig. 4, the relations between the values of \( \lambda/\lambda_w \) and \( \varphi \) are obtained as Fig. 5.

According to Fig. 5 the ratios of the coefficient of friction are in the range from 1.5 to 5.5. So that, it will be able to understand that, even if the quantities of bubbles contain very little in a downcomer, the pressure drop due to friction is larger than what is generally expected. For that reason it will be also noted that bubbles disturb the water circulation and make a cause of unstable circulation.

4. Extensions for High Pressure Regions

As the experiments are carried out under the atmospheric pressure, the results of the experiments cannot be directly applied to a downcomer of high pressure boiler. Consequently, it is necessary to extend the data to high pressure regions.

The writer regrets that the law of similarity for the extension is neither found experimentally nor theoretically. Hence, for the sake of convenience, it is assumed that both \( f_1(m) \) and \( f_2(m) \) are independent of pressure and temperature. The physical constants of vapour and water depend upon those values.

Next, considering that the ratios of \( G \) to \( G_0 \) are negligible comparing with 1.0, the ratios of the coefficient of friction can be calculated by the equation (7a). Substituting the equation (3) in (7a), and representing \( \xi \) by \( f_2(m) \), we have

\[ \frac{\lambda}{\lambda_w} = [1-(1-\xi)^2 f_2(m)] [f_1(m)]^2 \quad \text{where} \quad \xi = \frac{1-\xi}{\xi} \]

in which

\[ m = \sqrt{\frac{\Delta P_s}{\Delta P_x}} \frac{\lambda_w}{\lambda} \frac{\lambda}{\lambda_w} \]

Employing the results of the preliminary test for \( \lambda \) and taking \( \lambda = 64/R_4 \), the following equation is derived.

\[ m = 0.0703 \sqrt{\frac{\lambda}{\lambda_w}} \frac{\lambda_w}{\lambda} \frac{\lambda}{\lambda_w} \]

where \( \nu \) and \( \nu_0 \) are the coefficients of kinematic viscosity of vapour and water, respectively.

In a steam boiler, the pressure in the steam generator is the same as the saturated steam and water, respectively. So \( \nu \) and \( \nu_0 \) take the values for saturated steam, and \( \nu \) and \( \nu_0 \) the values for saturated water. If the values of \( \nu_0/\nu \) and \( R_4 \) are known, the values of \( m \) can be calculated by the equation (10). Then, substituting the values of \( m \) in the equation (9), the values of \( \lambda/\lambda_w \) are obtained. Using the parameter \( R_4 \), Fig. 6 shows the relations between the values of \( \lambda/\lambda_w \) and \( w_0/w_0 \) at the following pressures.

1, 10, 20, 40, 60, 80, 100 and 150 kg/cm²

5. Summary

The results of the investigations may be summarized briefly as follows:

(1) The correlation method of the pressure drop by Lockhart and Martinelli is applicable to the parallel flow of the mixtures of vapour and water in a vertical downcomer as well as that in a horizontal pipe.

(2) In case the flow rate of the mixtures is same, the frictional pressure drop in a downcomer is larger than that in a horizontal pipe.

(3) The ratios of the coefficient of friction are in the range from 1.5 to 5.5, and the values in high pressure regions are computed as shown in Fig. 6.

(4) Generally speaking, even if a very little quantity of bubbles is contained in a downcomer, the water circulation of steam boiler is disturbed.

(5) Even if a steam boiler is provided with a superior separator, it is never expected to contain no bubble in a non-heated downcomer. Therefore, the writer proposes that the coefficient of friction in practice may be approximately set at 0.03.

The foregoing experiments were made in the Steam Power Laboratory of the Kyushu Institute of Technology. The writer cannot conclude the present paper without expressing his genuine gratitude to Prof. Dr. K. Yamagata of Kyushu University, who gave him valuable advice and kind encouragement.

References

(1) Yamagata, K., Preprint of the 7 the ordinary meeting of the Kyushu branch of JSMCE., 131-140, March 1956.

