Free Lateral Vibrations of Rectangular Bars with Many Circular Holes (Continued)*

By Shin TAKAHASHI**

Free lateral vibrations of rectangular bars with many circular holes are studied, in which the vibrations are produced in the direction perpendicular to the axes of holes. The boundary conditions considered are as follows: one end built-in and the other free; built-in ends; free ends; hinged ends and one end built-in and the other hinged. The method used is to determine the coefficients of normal functions according to Ritz, and it is possible to obtain the frequencies of any mode. Numerical calculations are carried out for the bars with one end built-in and the other free with respect to the first and second modes, and a part of the calculated results is compared with experiments.

The free lateral vibration of rectangular bar with many circular holes, occurring in the direction parallel to the axes of holes has been discussed in the preceding paper⁽¹⁾, and in this paper the case is discussed by Ritz method, in which the bar vibrates perpendicularly to the axes. The axes of holes coincide with one of the symmetric axes of rectangular section and the boundary conditions are as follows: both ends built-in; free or simply supported or one end built-in and the other free or simply supported. The results of the preceding paper are possible to be applied to the bar with symmetric semicircular notches, while the results in this paper are applied only to the holes. When the axes of holes coincide with both axes of symmetry in rectangular section, we are able to calculate the frequencies using the results of the preceding and this papers.

Notations

- x, y : Cartesian coordinates, basing on the centre line of bar and one of the principal axes (direction of deflection) respectively
 t : time
 p : circular frequency
 X : normal function in lateral vibration of uniform bar
 l, b, h : length, breadth and depth of the bar respectively

k_n, λ_n, α_n : constants determined by the boundary conditions

l_1, l_2 : x -coordinates of centres of holes

r_1, r_2 : radii of holes

I : moment of inertia, I_0 , that in the holeless cross section

A : sectional area, A_0 , that in the holeless cross section

E : Young's modulus

γ : specific weight

V, T : strain and kinetic energies, respectively

A, B, C : constants

μ : $\mu = p^2 A_0 \gamma l^4 / EI_0 g$

ζ : $\zeta_1^2 = r_1^2 - (x - l_1)^2$

1. Ritz method

Let the deflection curve of the bar be

$$y = \sum_n (A_n \cos p_n t + B_n \sin p_n t) X_n$$

where

$X_n = C_n [(\sin k_n x \pm \sinh k_n x) + \lambda_n (\cos k_n x \pm \cosh k_n x)]$
 and for the case of both ends built-in; one end built-in and the other free or simply supported, we use the negative signs in the double signs in the above equation and for the case of both ends free, the positive signs. $k_n l$'s are the n -th roots of the following equations and the values of λ_n are

$\cos k_n l \cosh k_n l = -1$ for one end built-in and the other free

$$\lambda_n = \frac{\cos k_n l \mp 1}{\sin k_n l} \quad (-) n : \text{odd}, (+) n : \text{even}$$

$\cos k_n l \cosh k_n l = 1$ for both ends built-in or free

$$\lambda_n = \frac{\cos k_n l \mp 1}{\sin k_n l} \quad (+) n : \text{odd}, (-) n : \text{even}$$

* Received 24th April, 1957.

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$\tan k_n l = \tanh k_n l$ for one end built-in and the other simply supported.

$$\lambda_n = -\tan k_n l$$

The dimensions of bar are shown in Fig. 1, and the moment of inertia and the sectional area in the section at the portion of circle of radius r_1 are

$$I = I_0 - \frac{8b\zeta_1^3}{12}, \quad A = A_0 - 2b\zeta_1, \quad \zeta_1 = \sqrt{r_1^2 - (x - l_1)^2}$$

Fig. 1 Direction of vibration

The maximum strain and kinetic energies are

$$\frac{2V}{E} = I_0 \int_0^l (X'')^2 dx - \sum_{l_1} \frac{2b}{3} \int_{l_1-r_1}^{l_1+r_1} (X'')^2 \zeta_1^3 dx$$

$$\frac{2T}{E} = \frac{p^2 A_0 \gamma}{Eg} \int_0^l X^2 dx - \sum_{l_1} 2 \frac{p^2 b \gamma}{Eg} \int_{l_1-r}^{l_1+r} X^2 \zeta_1 dx$$

The first terms, respectively, represent the energies of uniform sectional bar and the second do those to subtract from the energies above mentioned on account of the holes. By $\sum_{l_1}$ we mean the summation as many as the number of holes. After integration

$$\frac{2V}{E} = \frac{I_0}{l^3} \left[\sum_n C_n^2 \lambda_n^2 \alpha_n^4 - \left(\frac{2l}{h} \right)^3 \sum_{l_1} (B_{11} C_1^2 \alpha_1^4 + B_{22} C_2^2 \alpha_2^4 + 2B_{12} C_1 C_2 \alpha_1^2 \alpha_2^2 + \dots) \right]$$

$$\frac{2T}{E} = \frac{p^2 A_0 \gamma l}{Eg} \left[\sum_n C_n^2 \lambda_n^2 - \frac{2l}{h} \sum_{l_1} (A_{11} C_1^2 + A_{22} C_2^2 + 2A_{12} C_1 C_2 + \dots) \right]$$

A and B are the non-dimensional quantities and they are

$$A_{nn} = \frac{\pi \lambda_n^2}{2} \left(\frac{r}{l} \right)^2 + \frac{\pi}{4\alpha_n} \left(\frac{r}{l} \right) [(\phi_n^2 - \psi_n^2) J_1(2k_n r) + (\Phi_n^2 + \Psi_n^2) I_1(2k_n r)]$$

$$\mp \frac{\pi}{\alpha} \left(\frac{r}{l} \right) [(\phi_n \Phi_n + \psi_n \Psi_n) \xi_1(k_n r, k_n r) + (\phi_n \Phi_n - \psi_n \Psi_n) \gamma_1(k_n r, k_n r)]$$

$$A_{np} = \frac{\pi}{2} \left(\frac{r}{l} \right) \left[(\phi_n \phi_p + \psi_n \psi_p) \frac{J_1(k_n r - k_p r)}{\alpha_n - \alpha_p} + (\phi_n \phi_p - \psi_n \psi_p) \frac{J_1(k_n r + k_p r)}{\alpha_n + \alpha_p} \right]$$

$$+ \frac{\pi}{2} \left(\frac{r}{l} \right) \left[(\Phi_n \Phi_p - \Psi_n \Psi_p) \frac{I_1(k_n r - k_p r)}{\alpha_n - \alpha_p} + (\Phi_n \Phi_p + \Psi_n \Psi_p) \frac{I_1(k_n r + k_p r)}{\alpha_n + \alpha_p} \right]$$

$$\mp \frac{\pi}{\alpha_n^2 + \alpha_p^2} \left(\frac{r}{l} \right) [(\alpha_n \phi_n \Phi_p + \alpha_p \psi_n \Psi_p) \xi_1(k_n r, k_p r) + (\alpha_p \phi_n \Phi_p - \alpha_n \psi_n \Psi_p) \gamma_1(k_n r, k_p r)]$$

$$\mp \frac{\pi}{\alpha_n^2 + \alpha_p^2} \left(\frac{r}{l} \right) [(\alpha_p \phi_p \Phi_n + \alpha_n \psi_p \Psi_n) \xi_1(k_p r, k_n r) + (\alpha_n \phi_p \Phi_n - \alpha_p \psi_p \Psi_n) \gamma_1(k_p r, k_n r)]$$

$$B_{nn} = \frac{3\pi \lambda_n^2}{8} \left(\frac{r}{l} \right)^4 + \frac{3\pi}{8\alpha_n^2} \left(\frac{r}{l} \right)^2 [(\phi_n^2 - \psi_n^2) J_2(2k_n r) + (\Phi_n^2 + \Psi_n^2) I_2(2k_n r)]$$

$$\pm \frac{3\pi}{\alpha_n^2} \left(\frac{r}{l} \right)^2 [\psi_n \Psi_n \xi_2(k_n r, k_n r) + \phi_n \Phi_n \gamma_2(k_n r, k_n r)]$$

$$B_{np} = \frac{3\pi}{2(\alpha_n - \alpha_p)^2} \left(\frac{r}{l} \right)^2 [(\phi_n \phi_p + \psi_n \psi_p) J_2(k_n r - k_p r) + (\Phi_n \Phi_p - \Psi_n \Psi_p) I_2(k_n r - k_p r)]$$

$$+ \frac{3\pi}{2(\alpha_n + \alpha_p)^2} \left(\frac{r}{l} \right)^2 [(\phi_n \phi_p - \psi_n \psi_p) J_2(k_n r + k_p r) + (\Phi_n \Phi_p + \Psi_n \Psi_p) I_2(k_n r + k_p r)]$$

$$\pm \frac{3\pi}{(\alpha_n^2 + \alpha_p^2)^2} \left(\frac{r}{l} \right)^2 [(\alpha_n^2 - \alpha_p^2) \phi_n \Phi_p + 2\alpha_n \alpha_p \psi_n \Psi_p] \xi_2(k_n r, k_p r)$$

$$+ \{ -(\alpha_n^2 - \alpha_p^2) \psi_n \Psi_p + 2\alpha_n \alpha_p \phi_n \Phi_p \} \gamma_2(k_n r, k_p r)]$$

$$\pm \frac{3\pi}{(\alpha_n^2 + \alpha_p^2)^2} \left(\frac{r}{l} \right)^2 [(\alpha_p^2 - \alpha_n^2) \phi_p \Phi_n + 2\alpha_n \alpha_p \psi_p \Psi_n] \xi_2(k_p r, k_n r)$$

$$+ \{ -(\alpha_p^2 - \alpha_n^2) \psi_p \Psi_n + 2\alpha_n \alpha_p \phi_p \Phi_n \} \gamma_2(k_p r, k_n r)]$$

where

$$\phi_n(l_1/l) = \sin k_n l_1 + \lambda_n \cos k_n l_1, \quad \psi_n(l_1/l) = \cos k_n l_1 - \lambda_n \sin k_n l_1,$$

$$\Phi_n(l_1/l) = \sinh k_n l_1 + \lambda_n \cosh k_n l_1, \quad \Psi_n(l_1/l) = \cosh k_n l_1 + \lambda_n \sinh k_n l_1,$$

and the real and imaginary parts of $J_1(\mu r + i\nu r)$, $J_2(\mu r + i\nu r)$ are $\xi_1(\mu r, \nu r)$, $\gamma_1(\mu r, \nu r)$ and $\xi_2(\mu r, \nu r)$, $\gamma_2(\mu r, \nu r)$ respectively, and these are expressed in series form as follows:

$$\xi_1(\mu r, \nu r) = J_1(\mu r) [I_0(\nu r) + I_2(\nu r)] - J_3(\mu r) [I_2(\nu r) + I_4(\nu r)]$$

$$\begin{aligned}
& +J_5(\mu r)[I_4(\nu r)+I_6(\nu r)]-J_7(\mu r)[I_6(\nu r)+I_8(\nu r)]+ \cdots \\
\eta_1(\mu r, \nu r) &= J_0(\mu r)I_1(\nu r)-J_2(\mu r)[I_1(\nu r)+I_3(\nu r)] \\
& +J_4(\mu r)[I_3(\nu r)+I_5(\nu r)]-J_6(\mu r)[I_5(\nu r)+I_7(\nu r)]+ \cdots \\
\xi_2(\mu r, \nu r) &= -J_0(\mu r)I_2(\nu r)+J_2(\mu r)[I_0(\nu r)+I_4(\nu r)] \\
& -J_4(\mu r)[I_2(\nu r)+I_6(\nu r)]+J_6(\mu r)[I_4(\nu r)+I_8(\nu r)]- \cdots \\
\eta_2(\mu r, \nu r) &= J_1(\mu r)[I_1(\nu r)+I_3(\nu r)]-J_3(\mu r)[I_1(\nu r)+I_5(\nu r)] \\
& +J_5(\mu r)[I_3(\nu r)+I_7(\nu r)]-J_7(\mu r)[I_5(\nu r)+I_9(\nu r)]+ \cdots
\end{aligned}$$

These series converge rapidly. In numerical example, these values are obtained directly with expanding $J_1(\mu r+i\nu r)$ etc. J and I are the Bessel and modified Bessel functions of the first kind.

And

$$\begin{aligned}
\phi_n(1-l_1/l) &= (-1)^n \phi_n(l_1/l), & \psi_n(1-l_1/l) &= (-1)^{n-1} \psi_n(l_1/l), \\
\Phi_n(1-l_1/l) &= (-1)^{n-1} \Phi_n(l_1/l), & \Psi_n(1-l_1/l) &= (-1)^n \Psi_n(l_1/l)
\end{aligned}$$

especially

$$l_1/l = 0.5$$

$$n: \text{ odd ; } \phi_n=0, \psi_n=1/\cos k_n l_1, \Phi_n=-1/\sinh k_n l_1, \Psi_n=0$$

$$n: \text{ even ; } \phi_n=1/\sin k_n l_1, \psi_n=0, \Phi_n=0, \Psi_n=1/\cosh k_n l_1$$

In above equations of A and B , we take the lower signs in the double signs for the case of both ends free and the upper signs for the other cases.

In the case of both ends simply supported

$$\alpha_n \equiv k_n l = n\pi, \quad \lambda_n = \frac{1}{2}$$

$$A_{nn} = \frac{\pi}{4} \left(\frac{r}{l} \right)^2 - \frac{\pi}{4\alpha_n} \left(\frac{r}{l} \right) \cos 2k_n l_1 \cdot J_1(2k_n r)$$

$$A_{np} = \frac{\pi}{2(\alpha_n - \alpha_p)} \left(\frac{r}{l} \right) \cos(k_n l_1 - k_p l_1) J_1(k_n r - k_p r) - \frac{\pi}{2(\alpha_n + \alpha_p)} \left(\frac{r}{l} \right) \cos(k_n l_1 + k_p l_1) J_1(k_n r + k_p r)$$

$$B_{nn} = \frac{3\pi}{16} \left(\frac{r}{l} \right)^4 - \frac{3\pi}{8\alpha_n^2} \left(\frac{r}{l} \right)^2 \cos 2k_n l_1 \cdot J_2(2k_n r)$$

$$B_{np} = \frac{3\pi}{2(\alpha_n - \alpha_p)^2} \left(\frac{r}{l} \right)^2 \cos(k_n l_1 - k_p l_1) \cdot J_2(k_n r - k_p r) - \frac{3\pi}{2(\alpha_n + \alpha_p)^2} \left(\frac{r}{l} \right)^2 \cos(k_n l_1 + k_p l_1) \cdot J_2(k_n r + k_p r)$$

Putting

$$\begin{aligned}
\mu &= p^2 A_0 \gamma l / EI_0 g \\
\lambda_n^2 - \frac{2l}{h} A_{nn} &= a_{nn}, \quad \lambda_n^2 \alpha^4 - \left(\frac{2l}{h} \right)^3 B_{nn} \alpha_n^4 = b_{nn} \\
\frac{2l}{h} A_{np} &= a_{np}, \quad \left(\frac{2l}{h} \right)^3 B_{np} \alpha_n^2 \alpha_p^2 = b_{np}
\end{aligned}$$

then the frequency is obtained using Ritz method from the following determinant (where $a_{np} = a_{pn}$)

$$\begin{vmatrix}
(a_{11}\mu - b_{11}) & (a_{12}\mu - b_{12}) & \cdots \\
(a_{21}\mu - b_{21}) & (a_{22}\mu - b_{22}) & \cdots \\
\cdots & \cdots & \cdots
\end{vmatrix} = 0$$

2. Numerical example and comparison with the experimental results

Numerical calculations and comparison with experimental results are carried out in the case of one end built-in and the other free. The numerical values of above mentioned quantities are following :

$$\begin{aligned}
k_1 l &\equiv \alpha_1 = 1.875 & 104 & 068 & 712 & 0 \\
k_2 l &\equiv \alpha_2 = 4.694 & 091 & 132 & 974 & 2 \\
\lambda_1 &= -1.362 & 220 & 557 & 485 & \\
\lambda_2 &= -0.981 & 867 & 539 & 175 &
\end{aligned}$$

l_1/l	$\phi_1(l_1/l)$	$\psi_1(l_1/l)$	$\Phi_1(l_1/l)$	$\Psi_1(l_1/l)$
0.1	-1.151 93	1.236 41	-1.197 63	0.760 70
0.2	-0.901 25	1.429 47	-1.075 27	0.548 23
0.3	-0.618 98	1.572 42	-0.990 82	0.355 09
0.4	-0.315 01	1.660 24	-0.941 32	0.174 47
0.5	0	1.689 87	-0.925 01	0
l_1/l	$\phi_2(l_1/l)$	$\psi_2(l_1/l)$	$\Phi_2(l_1/l)$	$\Psi_2(l_1/l)$
0.1	-0.423 31	1.335 99	-0.605 20	0.634 20
0.2	0.226 83	1.382 97	-0.364 36	0.410 73
0.3	0.827 90	1.130 78	-0.205 29	0.279 43
0.4	1.249 86	0.633 96	-0.092 29	0.210 84
0.5	1.401 45	0	0	0.189 57

The calculations are carried out for the first and second modes to the second approximation for $h/l = 1/10, 1/20$ and $1/30$, the results are expressed with non-dimensional quantity μ , and they are given in Figs. 2 to 9. For comparison, μ in the case of vibration in the axial direction of the hole are shown in Figs. 3 and 5. With experiment, the first mode in $h/l = 1/30$ are obtained. The test pieces are the square bar, 300mm $\times$ 10mm $\times$ 10mm, of mild steel with one hole, which has been made larger successively. The value of $A_0 \gamma l^4 / EI_0 g$ is determined such as the empirical data of uniform bar coincide with the calculated values, and after a hole was

bored p^2 is measured and the values of μ 's are obtained by multiplying p^2 by $A_0 \gamma l^4 / EI_0 g$ as mentioned above. The experimental results coincide approximately with the calculated ones.

The author wishes to express his gratitude to Mr. Fukashi Nakamura and Mr. Seiji Tsuchida for their efforts in carrying out the experiments.

Appendix

In calculating the energies T and V , the fol-

$$I[\sin \mu x \sin \nu x] = \frac{\pi r}{2(\mu - \nu)} \cos(\mu l_1 - \nu l_1) J_1(\mu r - \nu r) - \frac{\pi r}{2(\mu + \nu)} \cos(\mu l_1 + \nu l_1) J_1(\mu r + \nu r)$$

$$I[\cos \mu x \cos \nu x] = \frac{\pi r}{2(\mu - \nu)} \cos(\mu l_1 - \nu l_1) J_1(\mu r - \nu r) + \frac{\pi r}{2(\mu + \nu)} \cos(\mu l_1 + \nu l_1) J_1(\mu r + \nu r)$$

lowing integrations are necessary

$$I[\psi(x)] \equiv \int_{l_1-r}^{l_1+r} \psi(x) [r^2 - (l_1 - x)^2]^{\frac{1}{2}} dx$$

$$I'[\psi(x)] \equiv \int_{l_1-r}^{l_1+r} \psi(x) [r^2 - (l_1 - x)^2]^{\frac{3}{2}} dx$$

Using the Bessel's second integral (2)

$$J_n(x) = \frac{1}{\sqrt{\pi} \Gamma\left(n + \frac{1}{2}\right)} \left(\frac{x}{2}\right)^n \int_0^\pi e^{\pm i x \cos \phi} (\sin \phi)^{2n} d\phi$$

Fig. 2 First mode

Fig. 4 Second mode

Fig. 3 First mode

Fig. 5 Second mode

Fig. 6 First mode

Fig. 8 First mode

Fig. 7 Second mode

Fig. 9 Second mode

$$\begin{aligned}
 I[\sin \mu x \cos \nu x] &= \frac{\pi r}{2(\mu - \nu)} \sin(\mu l_1 - \nu l_1) J_1(\mu r - \nu r) + \frac{\pi r}{2(\mu + \nu)} \sin(\mu l_1 + \nu l_1) J_1(\mu r + \nu r) \\
 I'[\sin \mu x \sin \nu x] &= \frac{3\pi r^2}{2(\mu - \nu)^2} \cos(\mu l_1 - \nu l_1) J_2(\mu r - \nu r) - \frac{3\pi r^2}{2(\mu + \nu)^2} \cos(\mu l_1 + \nu l_1) J_2(\mu r + \nu r) \\
 I'[\cos \mu x \cos \nu x] &= \frac{3\pi r^2}{2(\mu - \nu)^2} \cos(\mu l_1 - \nu l_1) J_2(\mu r - \nu r) + \frac{3\pi r^2}{2(\mu + \nu)^2} \cos(\mu l_1 + \nu l_1) J_2(\mu r + \nu r) \\
 I'[\sin \mu x \sin \nu x] &= \frac{3\pi r^2}{2(\mu - \nu)^2} \sin(\mu l_1 - \nu l_1) J_2(\mu r - \nu r) + \frac{3\pi r^2}{2(\mu + \nu)^2} \sin(\mu l_1 + \nu l_1) J_2(\mu r + \nu r)
 \end{aligned}$$

In the case $\mu = \nu$, the limits must be used and when these integrations contain $\sinh$ and $\cosh$, μ and ν are imaginary and the modified Bessel functions of the first kind or ξ , η must be used. For examples,

$$\begin{aligned}
 I[\sin \mu x \sinh \nu x] &= \frac{\pi r}{\mu^2 + \nu^2} [\xi_1(\mu r, \nu r) (\mu \sin \mu l_1 \sinh \nu l_1 + \nu \cos \mu l_1 \cosh \nu l_1) \\
 &\quad + \eta_1(\mu r, \nu r) (-\mu \cos \mu l_1 \cosh \nu l_1 + \nu \sin \mu l_1 \sinh \nu l_1)] \\
 I[\cos \mu x \cosh \nu x] &= \frac{\pi r}{\mu^2 + \nu^2} [\xi_2(\mu r, \nu r) (\mu \cos \mu l_1 \cosh \nu l_1 - \nu \sin \mu l_1 \sinh \nu l_1)
 \end{aligned}$$

$$\begin{aligned}
& +\eta_1(\mu r, \nu r)(\mu \sin \mu l_1 \sinh \nu l_1 + \nu \cos \mu l_1 \cosh \nu l_1)] \\
I[\sin \mu x \cosh \nu x] &= \frac{\pi r}{\mu^2 + \nu^2} [\xi_1(\mu r, \nu r)(\mu \sin \mu l_1 \cosh \nu l_1 + \nu \cos \mu l_1 \sinh \nu l_1) \\
& +\eta_1(\mu r, \nu r)(-\mu \cos \mu l_1 \sinh \nu l_1 + \nu \sin \mu l_1 \cosh \nu l_1)] \\
I[\cos \mu x \sinh \nu x] &= \frac{\pi r}{\mu^2 + \nu^2} [\xi_1(\mu r, \nu r)(\mu \cos \mu l_1 \sinh \nu l_1 - \nu \sin \mu l_1 \cosh \nu l_1) \\
& +\eta_1(\mu r, \nu r)(\mu \sin \mu l_1 \cosh \nu l_1 + \nu \cos \mu l_1 \sinh \nu l_1)] \\
I'[\sin \mu x \sinh \nu x] &= \frac{3\pi r^2}{(\mu^2 + \nu^2)^2} [\xi_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \sin \mu l_1 \sinh \nu l_1 + 2\mu\nu \cos \mu l_1 \cosh \nu l_1\} \\
& +\eta_2(\mu r, \nu r)\{-(\mu^2 - \nu^2) \cos \mu l_1 \cosh \nu l_1 + 2\mu\nu \sin \mu l_1 \sinh \nu l_1\}] \\
I'[\cos \mu x \cosh \nu x] &= \frac{3\pi r^2}{(\mu^2 + \nu^2)^2} [\xi_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \cos \mu l_1 \cosh \nu l_1 - 2\mu\nu \sin \mu l_1 \sinh \nu l_1\} \\
& +\eta_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \sin \mu l_1 \sinh \nu l_1 + 2\mu\nu \cos \mu l_1 \cosh \nu l_1\}] \\
I'[\sin \mu x \cosh \nu x] &= \frac{3\pi r^2}{(\mu^2 + \nu^2)^2} [\xi_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \sin \mu l_1 \cosh \nu l_1 + 2\mu\nu \cos \mu l_1 \sinh \nu l_1\} \\
& +\eta_2(\mu r, \nu r)\{-(\mu^2 - \nu^2) \cos \mu l_1 \sinh \nu l_1 + 2\mu\nu \sin \mu l_1 \cosh \nu l_1\}] \\
I'[\cos \mu x \sinh \nu x] &= \frac{3\pi r^2}{(\mu^2 + \nu^2)^2} [\xi_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \cos \mu l_1 \sinh \nu l_1 - 2\mu\nu \sin \mu l_1 \cosh \nu l_1\} \\
& +\eta_2(\mu r, \nu r)\{(\mu^2 - \nu^2) \sin \mu l_1 \cosh \nu l_1 + 2\mu\nu \cos \mu l_1 \sinh \nu l_1\}]
\end{aligned}$$

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Vibration of Corrugated Diaphragm*

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Formerly, one of us (T. A.) studied the elastostatic properties of the corrugated diaphragm, assuming it as an orthotropic elastic plate by smearing out the effect of the corrugations over a wave length, and found that the calculated results agreed well with those of the experiments.

In this paper, analyzing the vibration properties of the corrugated diaphragm under the same assumptions as the static case, we obtained the approximate formulas for the natural frequencies of symmetric and asymmetric modes including the effect of concentrated mass. Furthermore, by the energy method, we studied the effects of initial tension and concentrated mass on the natural frequencies. In order to check this analysis, some experiments were performed. The experimental results showed good coincidence with the correspondig formulas, when the anisotropy parameter α is not so large.

1. Introduction

Corrugated diaphragms find a wide use in pressure-measuring instruments, speaker corn dampers, etc. It is owing to the linearity of the flexural properties under a range of load wider than that

of the flat plate in the same dimmensions as the former. This relation may be illustrated as in Fig. 1, which is quoted from the author's paper.⁽¹⁾

The individual influences on the flexural sensitivities of a corrugated diaphragm, of section figure, dimmensions, elastic properties of the material and its inelastic phenomena (for example hysteresis, creep, residual strain), have been investigated in detail by the experimental methods. However, the experimental stress analyses and the vibration

* Received 28th, March 1957.

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