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Calculation of Gas Radiation from a Rectangular Parallelepiped Gas Mass Surrounded by Transparent Gas*

1st Report, The Emissivity of a Gas Mass Radiating to Furnace Walls

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On the assumptions that a radiating gas mass having a uniform coefficient of absorption exists in the center of the furnace and that between radiating gas mass and walls exists transparent gas, the emissivity of a gas mass radiating to furnace walls and the frequency distribution of the heat-flow intensity on furnace walls are calculated. When the volume ratio of radiating gas to furnace is about 0.2, the frequency distribution calculated coincides closely with that of actual furnaces, which is independent of the form of furnace or operating conditions.

The average absorptivity of the gas mass for the radiation between any two of the furnace walls is also presented in the present paper. The results will be able to be utilized for calculating the heat absorption in any one of the furnace walls.

1. Introduction

Since 1945, a series of furnace performance tests on three pulverized-coal-fired⁽¹⁾, one natural-gas-fired⁽²⁾ and one spreader-stoker-fired steam boilers⁽³⁾ has been conducted under the auspices of the ASME Special Research Committee. Using these data, the frequency distribution on the intensity of heat-absorption has been analyzed⁽⁴⁾, but the frequency distribution calculated on the assumptions that an uniform radiating gas mass fills the furnace is dissimilar to that of actual furnaces. Then, in this report, we consider a model furnace, in which flame having relatively higher coefficient of absorption and a shape similar to the furnace exists in center and the space between flame and wall is occupied by a transparent gas. In this case heterogeneity of radiating gas is taken into consideration as the volume ratio of radiating gas to furnace. For calculating frequency distribution of heat absorption rates on the wall, in the first place the calculating methods of emissivity of a gas mass radiating to furnace walls are shown.

Recently general equation for radiation in a

furnace consisting of any number of gray source-sink surface and refractory surface has been deduced by Yagi and Kunii⁽⁵⁾ and by H.C. Hottel⁽⁶⁾. A. K. Oppenheim has derived the network method of radiation analysis in which the overall interchange factor of radiation is equivalent to the conductance between any two nodes of the network⁽⁷⁾. By the utilization of these methods complicated calculation of radiation heat transfer is to be made easily. For this calculation, however, transmittance of gas mass for radiation emitting from one surface to the other must be known. These values calculated are also shown in this report.

2. Calculating method of emissivity of gas mass (flame) at furnace walls

A furnace is assumed to be a rectangular parallelepiped and a flame involved to be similar in shape to the furnace and to occupy the same relative position to any furnace wall respectively. It is assumed that flame is uniform both in constitution and in temperature and has a uniform coefficient of absorption over entire region of wave length of radiation. The radiation from flame is assumed to follow Beer's law, i. e.,

$$\epsilon = 1 - e^{-kl} \dots\dots\dots (1)$$

where ϵ = flame (gas) emissivity
 k = coefficient of absorption, m^{-1}

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l =path length, m

e =base of hyperbolic logarithms

Beer's law is derived originally for monochromatic wave. However, in calculating the absorptivity of gas mass for evaluation of heat absorbed in furnace, the total radiation must be considered. Experimental results by Hottel⁽⁸⁾ about emissivity of carbon dioxide, water vapor and luminous flames follow only approximately Beer's law. About emissivity of pulverized-coal flame due to solid particles, Haslam and Hottel have derived exponential law⁽⁹⁾, using experimental results by Wohlenberg and Morrow⁽¹⁰⁾. Here, taking into account that considerable deviation is seen in experimental results of gas radiation according to experimenters, Beer's law was determined to be used for gas radiation ranging over entire wave length.

Next, enclosure is assumed to be constructed by black surfaces. In order to get the emissivity of gas mass at any point on the furnace walls, in the first place the emissivity of the wedge-shaped gas mass is obtained.

(a) Emissivity ϵ_g at the point A of the wedge-shaped gas mass $A'A''B'B''C'C''$ (Fig. 1)

Wedge-shape $A'A''B'B''C'C''$ is divided into two parts, I and II. Considering the element with the elementary surface df cut from the part I, the emissivity at point A is

$$\Delta\epsilon_{gI} = \frac{1}{\pi} (1 - e^{-kh_1}) \cos\varphi \cdot \sin\varphi \cdot d\varphi \cdot d\theta \quad \dots (2)$$

So the emissivity integrated over the part I is

$$\begin{aligned} \epsilon_{gI} &= \frac{d\theta}{\pi} \int_0^{\varphi_0} (1 - e^{-\frac{kh_1}{\cos\varphi}}) \cdot \cos\varphi \cdot \sin\varphi \cdot d\varphi \\ &= \frac{1}{2\pi} \cdot \left[\sin^2\varphi_0 - \Phi(kh_1) + \cos^2\varphi_0 \cdot \Phi\left(\frac{kh_1}{\cos\varphi_0}\right) \right] \end{aligned} \quad \dots (3)$$

where $\Phi(z) \equiv e^{-z} - z \cdot e^{-z} - z^2 Ei(-z)$

$$Ei(z) = \int \frac{e^{-z}}{z} dz$$

The emissivity of the gas mass II is also given

Fig. 1 Emissivity at A of the wedge-shaped gas mass $A'A''B'B''C'C''$

by Eq. (2), if only we put

$$l_1 = \frac{r}{\sin\varphi} - \frac{h_2}{\cos\varphi}$$

The calculation of emissivities by numerical integration is done in the case of $kh_1=0.5, 1, 2, 3$, $kh_2/kh_1=0.0386, 0.0928, 0.1785, 0.355$ (correspond respectively to volume ratio of 0.8, 0.6, 0.4, and 0.2). One example of the results is shown in Fig. 3.

(b) Emissivity ϵ_g at the point A of the gas mass $B'B''C'C''D'D''E'E''$ (Fig. 2)

As shown in Fig. 2 (a, b), according to the relative position of the gas mass and point A, two cases are considered separately. At any rate, the emissivity of the element gas mass with infinitesimal area df is given by Eq. (2), where path length l is given by the next relation.

i) Case (a) in Fig. 2.

$$\text{for } \varphi_1 > \varphi > \varphi_0 \quad l_1 = \frac{h_1 + h_2}{\cos\varphi} - \frac{r_1}{\sin\varphi}$$

$$\text{for } \varphi_2 > \varphi > \varphi_1 \quad l_1 = \frac{h_1}{\cos\varphi}$$

$$\text{for } \varphi_3 > \varphi > \varphi_2 \quad l_1 = \frac{r_2}{\sin\varphi} - \frac{h_2}{\cos\varphi}$$

Fig. 2 Emissivity at A of the gas mass $B'B''C'C''D'D''E'E''$

Fig. 3 An example of emissivity at A for the radiation of the wedge-shaped gas mass. (where $kh_1=2.0$ and $kh_2=0.357$ -correspond to the volume ratio of 0.4)

ii) Case (b) in Fig. 2.

$$\text{for } \varphi_1 > \varphi > \varphi_0 \quad l_1 = \frac{h_1 + h_2}{\cos \varphi} - \frac{r_1}{\sin \varphi}$$

$$\text{for } \varphi_2 > \varphi > \varphi_1 \quad l_1 = \frac{r_2 - r_1}{\sin \varphi}$$

$$\text{for } \varphi_3 > \varphi > \varphi_2 \quad l_1 = \frac{r_2}{\sin \varphi} - \frac{h_2}{\cos \varphi}$$

The emissivity is calculated in the cases of $kh_1 = 0.5, 1, 2, 3$, $h_2/h_1 = 0.0386, 0.0928, 0.1785, 0.355$, $kr_2 = 0.5, 1, 2, 3, 5$, $r_1/r_2 = 0.1, 0.2, 0.6$. One example of the results is shown in Fig. 3.

(c) Emissivity ε_g of the gas mass at any point on furnace walls and average emissivity ε_{gm} over the wall

The emissivity ε_g of the gas at arbitrary point on the wall is given as the sum of the emissivity of wedge-shaped gas masses as shown in Figs. 1 and 2. An example of the distribution of ε_g in the case of the 1-1-2 rectangular parallelepiped furnace walls is shown in Fig. 4. From ε_g -distribution, the average emissivity ε_{gm} of the gas mass having arbitrary volume ratio is evaluated. While, if the

Fig. 4 ε_g -distribution diagram on the 1-1-2 rectangular parallelepiped furnace wall for the radiation of the gas mass (characterizing dimension multiplied by $k = kh = 3\sqrt{1 \times 1 \times 2}$) having volume ratio of 0.2

Fig. 5 Average emissivity on the wall for the radiation of gas mass having each volume ratio to furnace

average emissivity in the furnace which is completely filled with the same radiating gas, is specified as ε_{gm1} , ε_{gm} can be written as follows:

$$\varepsilon_{gm} = \varepsilon_{gm1} \cdot \frac{F_1}{F} \quad (4)$$

where F = wall surface in the case of an arbitrary volume ratio

F_1 = surface of the gas mass

= wall surface in the case of volume ratio of 1.

Using this relation, the accuracy of the calculation of ε_{gm} by ε_g -distribution on the walls will be checked. ε_{gm} calculated from Eq. (4) is more accurate than the one from ε_g -distribution, and maximum deviation is about 3%. ε_{gm} 's from Eq. (4) are shown in Fig. 5.

3. Frequency distribution of heat absorption rates on the walls

In the boiler tests by the ASME committee, temperature gradients through water walls are plotted as isotherms on the unfolded view of the furnace. The temperature gradient that is surface wall temperature minus saturation temperature is proportional to the heat flow through the water wall at that point, therefore to the furnace heat absorption rates. So isotherms are equivalent to equal emissivity lines.

It is generally concluded by these tests that pattern of isotherms depends on path of combustion gas, that is position of burners and type of furnace. Therefore, this pattern differs according to the types of steam boilers and is complicated even in the same boiler. From these facts, therefore, general conclusion would be difficult to be derived by the isotherm pattern itself.

So turning the view point, here we take up the frequency distribution of temperature gradients, that is heat absorption rates on the walls.

As the measuring points of surface temperature in the furnace test above mentioned are uniformly distributed on the walls, these points are regarded as the unbiased sample of the points on the walls. The points are assigned to approximately equal area, so the number of points of equal temperature gradient is proportional to the area where heat is absorbed at the equal ratio equivalent to the temperature gradient.

Mumford and Bice derived slag-ash factor in order to evaluate the effect of wall cleanliness⁽¹¹⁾. This factor is the ratio of heat absorbed by ash coated wall to that

Fig. 6 Frequency distribution of the logarithm of the heat absorption rates as corrected to clean surface basis (S: standard deviation)

absorbed by clean ideal wall. In the furnace tests conducted at Paddy's Run Station and Willow Island Station, ash coverage diagrams on walls are shown. Therefore heat absorption rates corrected to clean wall condition can be obtained using the slang ash factor.

From our previous analysis⁽¹²⁾, it has been concluded that the frequency distribution of the logarithm of the intensity as corrected to clean surface basis is approximately normal and independent of both furnace types and operating conditions. Some example is shown in Fig. 6. There, a model

of heat source, which gives the frequency distribution above mentioned, would become a clue in deriving a general calculation method of heat absorption in furnaces.

In this point of view, first, uniform radiating gas, which fills the furnace, is considered as heat source. Frequency of heat absorption rates calculated on this assumption increases with the increase of heat absorption rate, and the curve of the distribution differs from that of actual furnace (see Figs. 7 (1) and (5)).

Now, a calculating method of absorptivity of flame surrounded by transparent gas at arbitrary point on furnace walls is derived, and frequency distribution of the absorptivity on the walls, that is, of heat absorption rates in this model furnace is obtained. Fig. 7 shows this frequency distribution at the volume ratios of 0.2, 0.4 and 0.6 both for the cubic gas mass of $kh=1$ and for the 1-1-2 rectangular parallelepiped gas mass with $kh = \sqrt[3]{1 \times 1 \times 2}$ as characterizing dimension multiplied by k . It is found that the frequency distribution calculated coincides closely with that of actual furnaces in the case of volume ratio of 0.2. In the rectangular parallelepiped gas mass, not only for the entire walls, but also for the 1-2 side and the 1-1 base surface ϵ_g -distributions are obtained. As for the distribution curve same trend is shown in all cases.

Besides the examples in Fig. 7, for the cubic gas mass with $kh=0.5, 2, 3$ and for the rectangular parallelepiped gas mass with $kh = \sqrt[3]{0.5 \times 0.5 \times 1}$ and $\sqrt[3]{2 \times 2 \times 4}$ similar trends were found.

4. Average absorptivity of flame with arbitrary volume ratio to furnace concerning the solid radiation from one surface to another, ϵ_{gij} ⁽¹³⁾

Now, the absorptivity of gas mass as shown in Fig. 8 is considered for the solid radiation emitting from the arbitrary point p on the base $b'c'g'f'$ to the side $a'b'f'e'$.

In Fig. 8 (B) let Q be the intersection of line $P-a'e'$ and $ae-bf$, and the length of $Q-bf$ is known. In Fig. 8 (C) let $\angle e'f' \cdot P \cdot ab$ be θ_1 and $\angle ab \cdot P \cdot a'b'$ be θ_2 . For the range of θ_1 , the absorptivity at P for solid radiation is obtained as the sum of the emissivity of wedge-shaped gas layers with the thickness $Q-bf$. For the range of θ_2 , layer of gas mass changes in thickness along the line QS , but here the gas mass is assumed to have an average constant thickness as thickness. Then, the absorp-

(1) Cubic gas mass with $kh=1$,
volume ratio 1.0

(5) Rectangular parallelepiped gas
mass with $kh=\sqrt[3]{1 \times 1 \times 2}$ volume ratio 1.0

(2) Cubic gas mass with $kh=1$,
volume ratio 0.6

(6) Rectangular parallelepiped gas
mass with $kh=\sqrt[3]{1 \times 1 \times 2}$, volume ratio 0.6

(3) Cubic gas mass with $kh=1$,
volume ratio 0.4

(7) Rectangular parallelepiped
gas mass with $kh=\sqrt[3]{1 \times 1 \times 2}$,
volume ratio 0.4

(4) Cubic gas mass with $kh=1$,
volume ratio 0.2

(8) Rectangular parallelepiped gas
mass with $kh=\sqrt[3]{1 \times 1 \times 2}$, volume ratio 0.2

Fig. 7 Frequency distribution of the emissivity on the furnace wall for the radiation of gas mass having each volume ratio to furnace (M represents average emissivity)

Fig. 8

Fig. 9 Average absorptivities of flame with arbitrary volume ratio to furnace concerning the solid radiation from one surface to another for the cube

Fig. 10 Average absorptivities of flame with arbitrary volume ratio to furnace concerning the solid radiation from one surface to another for 1-2 rectangular parallelepiped.

tivity at any point P is obtained as the sum of the absorptivity of infinitesimal wedge-shaped gas masses with the average constant thickness. For example, the average value for the solid radiation emitting from the base and reaching to the side $a'b'f'e'$, ϵ_{gs} , is shown in Figs. 9 and 10.

The average emissivity of the gas mass $abcdefgh$ at the side $a'b'f'e'$, ϵ_{gms} , is equivalent to the average absorptivity for the solid radiation from the entire walls to the side $a'b'f'e'$, and the latter consists of absorptivities, $2\epsilon_{gSB}$, $2\epsilon_{gSNS}$ and ϵ_{gSOS} . Here ϵ_{gSB} is the absorptivity for the solid radiation from the one base to the side $a'b'f'e'$, ϵ_{gSNS} is that from the one neighbouring side to the side $a'b'f'e'$ and ϵ_{gSOS} is that from the opposite surface to the side $a'b'f'e'$. This relation is expressed as follows:

$$\epsilon_{gms} = 2\epsilon_{gSB} + 2\epsilon_{gSNS} + \epsilon_{gSOS}$$

Using this relation calculation of the absorptivity at each surface is simplified. The values for the cube and 1-1-2 rectangular parallelepiped are shown in Figs. 9 and 10.

5. Conclusion

In a model furnace of the cube and the 1-1-2 rectangular parallelepiped which is filled with the uniform flame surrounded by transparent gas and having a shape similar to the furnace, the average emissivity of the gas mass radiating on the whole furnace walls and the average absorptivity of the gas mass for the solid radiation between any two of the furnace walls are evaluated. These values could be utilized in calculating the heat absorption on the furnace wall.

Considering the accuracy of the emissivity of the radiating gas by the approximate calculation method expressed by the next relation

$$\epsilon_{gm} = 1 - e^{-0.6kh}$$

for the cubic furnace in the case of volume ratio of 1, the deviation of the value from the rigorously calculated one shown in Fig. 5 is more than 10 per cents in the range beyond $kh=2.0$. The deviation

increases with the decrease of volume ratio of the flame. In the 1-1-2 rectangular parallelepiped gas mass the error becomes larger than in the cube.

The frequency distribution of the emissivity on the walls for the radiation of flame with the volume ratio of about 0.2 to the furnace coincides closely with that of heat absorption ratios in the actual furnace, which is independent of the form of furnaces or operating conditions. So this ideal furnace may be utilized in deriving an improved method of calculation of heat-absorption in boiler furnaces. This will be discussed in the next report.

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- (13) refer to (4).
- (14) ϵ_{gij} has the next relation with gas emissivity averaged over all paths with ends on A_i and A_j , ϵ_{ij} , used in Hottel's general equation of radiant-heat transmission

$$\epsilon_{ij} = \frac{\epsilon_{gij}}{F_{ij}}$$

where F_{ij} : view factor.