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On the Distribution of Surface Roughness when Two Surfaces Are Pressed Together*

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In the study of contact of two surfaces, surface roughness is especially important. So first it is necessary to study how an initial roughness changes under the given conditions. And distribution constants of the profile in terms of an estimation of roughness are useful. We press two surfaces each other under the given conditions, study the process of change of roughness and obtain a transformation law from distribution of the pressed surface profile to that of the initial profile. Using the results, we consider about the contacting ratio, the gap between two contacting surfaces and the coefficients of changing roughness determined by combination of two materials of the surfaces, etc.

1. Introduction

Of many problems on contact it is the most important basic one to study the process of change of roughness under various conditions. Now as a method of analysing roughness statistics of distribution of the profile may be used, because surface profile is arranged at random. Then we proceed to study about the change of roughness when the rough surface is pressed on the mirror surface, next about the rough surface pressed on another rough surface. Using the results, we further consider about contacting ratio, the gap between two contacting surfaces and the coefficient of changing roughness determined by combination of two materials of the surfaces, etc.

2. Pressing the rough surface on the mirror surface

(1) Procedure of experiment

Test pieces are made of spring steel H and mild steel S . Their contacting surfaces are ground

to various roughnesses. Mirror surfaces are made of blockgauge B_0 , spring steel H_0 , and mild steel S_0 . They are combined as (HB_0) , (HH_0) , (HS_0) , (SB_0) , (SH_0) , (SS_0) and set in the jig (Fig. 1). Each pair of them is pressed under several pressures ($P_1=3.0$, $P_2=7.0$, $P_3=14.0$, $P_4=24.0$, $P_5=35.0$, $P_6=55.0$ kg/mm²). Profiles of the surfaces are recorded after each pressing. Then mean value $\bar{x}$, standard deviation s , median x_{med} of distribution of profile are calculated.

(2) Results of experiment

Fig. 2 indicates change of roughness, Fig. 3 indicates the relation of $\log \bar{x}$ and $\log s$ on the graph. Then we obtain the following linear relation.

$$\sigma = \alpha m^{1-n} \dots\dots\dots (1)$$

where

m : mean value, σ : standard deviation

α : coefficient decided by initial roughness

n : index number decided by combination of the materials

Under the pressure beyond broken lines on the figure, linearity does not exist, so this pressure may be called the critical pressure. The type of change of roughness depends on the index number n within the critical pressure, so n is called the coefficient

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Fig. 1 Jig for pressing

Fig. 2 Change of surface profile

Fig. 3 Relation between mean value $\bar{x}$ and standard deviation s

of changing roughness. Besides this several relations among various distribution constants are found.

(3) Change of distribution of profile

We can easily know the shape of the distribution

under any given pressure if transformation formula of variable from the final distribution to the initial one is obtained. Now putting the initial distribution function $F_1(x)$, final one $F_2(x)$, in order to get the relation $F_1(y) = F_2(x)$, we must set

$$y = x^n \dots \dots \dots (2)$$

If the original point is taken at the highest point of profile, Fig. 4 indicates the calculated value of density function $f_1(y)y'$ according to transformation formula(2). This nearly conforms to the results of measurements.

Now if in place of this calculation we use probability paper which has x^n abscissa $F_2(x)$ is drawn as a straight line. Then we may draw the straight lines with gradient $1/\alpha$ through calculated median. Here α is the standard deviation of x^n . Density of profile near the original point, $x=0$ distributes uniformly with value $\pm \alpha \theta / 2f_\theta$ (θ : probability at $x=0$). Fig. 5 indicates these distribution functions on the probability paper. We find θ is not related to roughness but to pressure.

(4) Assumption of contacting ratio

In the experiments ($H B_0$), ($S B_0$) blockgauge

B_0 is almost rigid, so probability at $x=0$ is about equal to contacting ratio θ . Fig. 6 (a) indicates the relation between pressure P and contacting

Fig. 4 Density of the pressed surface profile

Fig. 5 Distribution function of the pressed surface profile

ratio θ . So we obtain the following relation

$$a = A \cdot \theta = A \cdot P / P_f \quad \dots \dots \dots (3)$$

where a : real contacting area, A : apparent contacting area, P_f : flowing pressure (generally about equal to their Brinell hardness H : 320 kg/mm², S : 180 kg/mm²). On the other hand in the experiments

$$\theta = \int_0^\delta f_p(x) dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\delta x^{n-1} \exp \left\{ -\frac{(x^n - x_{med}^n)^2}{2\alpha^2} \right\} dx \quad \dots \dots \dots (4)$$

where $f_p(x)$: density function under pressure P .

Fig. 6 (b) indicates θ at $x=\delta$. This is almost equal to Fig. 6 (a). Further changes of caving in and yielding of the peaks are calculated and measured. From the results the relation between maximum height H_{max} and pressure P is obtained as Fig. 7.

$$H_{max} = h_2 - h_1 \quad \dots \dots \dots (5)$$

where

$$F_p(h_1) = 0.5\%, \quad F_p(h_2) = 99.5\%, \quad F_p(x) = \int_{-\infty}^x f_p(x) dx$$

3. The assumption of the numbers of contacting points

We consider the numbers and the size of contacting points by analysing the distribution of surface profiles.

(1) The theory of the assumption

Generally speaking quantiles Z_p (p : probability) from universe $f(x)$ are asymptotically distributed according to the following normal distribution

$$N \left\{ \zeta_p, 1/f(\zeta_p) \sqrt{\frac{pq}{N}} \right\} \quad \dots \dots \dots (6)$$

$$\sigma = \frac{1}{f(\zeta_p)} \sqrt{\frac{pq}{N}} \quad \dots \dots \dots (7)$$

where

q : $q=1-p$, ζ_p : quantile

N : sampling size

σ : standard deviation

($H H_0$), ($H S_0$), ($S H_0$), ($S S_0$) the peaks of rough surface cave in the mirror surface and yield themselves, too. Contacting ratio θ is equal to the probability at $x=\delta$ (δ : depth of the cave) because the peaks of profile from $x=0$ to $x=\delta$ are in strict contact. Then θ is written as follows:

Now if a rigid mirror finite surface is put on a rough infinite surface under mean pressure P the peaks of the surface profile yield due to the pressure and settle at the position Z_0 , which is the sample value of ζ_p . Here θ is contacting ratio.

N corresponds to the width of the mirror surface. Actually the distribution of surface profiles changes from $F(x)$ to $F_p(x)$, so $f(\zeta_p)$ changes to $f_p(\zeta_p)$. Then σ of (7) is changed as follows:

$$\sigma_p = \frac{1}{\phi_p(t_a)} \sqrt{\frac{\theta(1-\theta)}{N}} \quad \dots \dots \dots (8)$$

where

ϕ_p : density of normal distribution of x^n

t_a : standardized median of distribution of surface profile

$$f(\zeta_p) \doteq \phi_p(t_a)$$

Fig. 8 indicates the calculations of (8)

Fig. 6 Contacting ratio

Fig. 7 Change of maximum height of roughness

Fig. 8 Standard deviation of fluctuation of the indenter

(2) Contents and results of the experiments

As in Fig. 9 a blockgauge is put on a rough surface and moved in the shown direction. Fig. 10 shows the instrument and Fig. 11 indicates the surface profile and fluctuation of the blockgauge.

Then σ_p of (8) is calculated by the fluctuation of the blockgauge. Also $\phi_p(t_d)$ and θ are calculated in terms of the size of test pieces, combined materials, initial roughness and mean pressure. So sampling size N is obtained. As a result the number and the sizes of contacting points depend on the finished method. The size is not related to the apparent area but to combined materials and mean pressure. On the other hand the number is not related to combined materials but to the mean pressure and apparent area. For example in the combination of mild steel and blockgauge the number is 18, the size is $9 \times 9 \mu$ in the average under mean pressure 1 kg/mm^2 , apparent area is 0.5 mm^2 .

4. Pressing the rough surface on the other rough surface

(1) Biting between the peaks and valley of two surfaces

Let us get the depth of biting between the peaks and the valley of two surfaces, which is not seen in the case of pressing on the mirror surface. See Fig. 12 (a), both profiles, which distribute normally as $N(m_1, \sigma_1^2)$, $N(m_2, \sigma_2^2)$, contact each other at their highest points; (b), the upper profile moves horizontally; (c), it comes down by δ and settles. Here δ is the minimum gap of the two profiles, so biting depth is distributed according to the sampling distribution of the minimum value of the gap. The gap is distributed as the normal distribution $N(m_1 - m_2, \sigma_1^2 + \sigma_2^2)$. Generally density function of the minimum value in universe $F(x)$

Fig. 9 Method of experiment

Fig. 10 The apparatus for the experiment

Fig. 11 Surface profile and fluctuation of the indenter

is written as follows :

$$g(x)dx = N\{1-F(x)\}^{N-1}f(x)dx \dots\dots (9)$$

provided that $f(x)=F'(x)$, N is the sampling size, which is written as follows :

$$N = \left(\frac{1}{4} \sim \frac{1}{8}\right) \frac{L}{\sigma} \div \left(1 \sim \frac{1}{2}\right) \frac{L}{H_{\max}} \dots\dots (10)$$

L : length of contacting part

(2) Change of distribution of profile by pressing

In Fig. 13 two surfaces are biting with biting depth δ under pressure P .

Let us put

$f_1(x_1)$: density function of pressed surface

$f_2(x_2)$: that of pressing surface

$F_1'(x_1)=f_1(x_1)$, $F_2'(x_2)=f_2(x_2)$

where x_1, x_2 are taken from their highest points of the profiles. If the origin O' is transferred into O , $F_2(x_2)$ is written as follows :

$$\int_{x_2}^{\infty} f_2(x_2)dx_2 = \int_{-\infty}^{x_1} f_2(\delta-x_1)dx_1$$

Fig. 12 Biting of two profiles

Fig. 13 Corresponding pressure

Pressure at $x_1 \geq \delta$ is P . So change of roughness at $x_1 \geq \delta$ is equal to change under pressure P in the case of pressing on the mirror surface. Also pressure at a given ordinate x_{1i} is as follows :

$$P \left\{ \int_{-\infty}^{x_{1i}} f_2(\delta-x_1)dx_1 \right\} = P_i \dots\dots (11)$$

So change of roughness at $x = x_{1i} \pm \Delta x_1/2$ is equal to change under the corresponding pressure P_i in the case of pressing on the mirror surface. These partial changes are integrated in the whole range as Fig. 14. Fig. 15 indicates the comparison between the distribution pressed on the mirror surface and on the rough one.

(3) Contacting ratio and the gap between two surfaces

In Fig. 16 contacting ratio θ_i from $x_1=0$ to $x_1=x_{1i}$ is written as

$$\theta_i = P_i/P_f = P \int_{-\infty}^{x_{1i}} f_2(\delta-x_1)dx_1 / P_f$$

because pressure at x_{1i} is equal to P_i . So in the whole range

$$\theta = P \int_{-\infty}^{\delta} f_2(\delta-x_1)dx_1 / P_f = P/P_f \dots\dots (12)$$

This is quite the same as the formula (3).

Also the gap between $f_{1p}(x_1)$ and $f_{2p}(x_2)$, which are the density function, changes under pressure P , distributes according to the distribution of difference of x_1 and x_2 . Namely it is written as follows :

$$\left. \begin{aligned} E[g_p] &= \bar{x}_{1p} + \bar{x}_{2p} - \delta_p \\ D^2[g_p] &= \sigma_{1p}^2 + \sigma_{2p}^2 + \rho \end{aligned} \right\} \dots\dots (13)$$

where

$E[g_p]$: mean value of the gap

$D^2[g_p]$: variance of the gap

δ_p : biting depth under pressure P

ρ : correlation coefficient between x_{1p} and x_{2p} , this is almost 0

Fig. 17 indicates an example.

Fig. 14 Change of distribution of pressed surface profile

Fig. 15 Distribution function of pressed surface profile

Fig. 16 Contacting ratio

Fig. 17 x and σ of the distribution of the gap between surface I and II

5. The coefficient of changing roughness

The coefficient of changing roughness is very difficult to get as shown in 2, (2). So instead of getting n directly we try to measure relative hardness to two metals. The thin plates of hard steel (K), mild steel (N), brass (O), and copper

Fig. 18 Instrument for the experiment

(D) are used as test pieces. And, for example, the coefficient of (K) for (N) is written as $n(K_A N_B)$.

(1) The relative hardness of two metals

Two test pieces, whose contacting surfaces are mirrorpolished, are set as Fig. 18, and pressed. Then both deformations are measured by the roughness meter. The profiles are shown in Fig. 19. Fig. 20 shows the relation between mean pressure and mean depth of their deformation. Their gradient coefficient $k(A, B)$ is called the relative hardness of metal A for metal B . The minimum pressure by which A begins to be deformed only depends on its own hardness—their Brinell hardness ($K: 250 \text{ kg/mm}^2$, $N: 75 \text{ kg/mm}^2$, $O: 60 \text{ kg/mm}^2$, $D: 40 \text{ kg/mm}^2$). In order to get the

relation between $\kappa=1-n$ and k , the minimum pressure P and k are plotted on log-log paper and the fitting curves are given as Fig. 21. Parallel lines $D_1 D_2, O_1 O_2, N_1 N_2 \dots$ are called 'the relative lines' and shown as

$$k(AB) = y_1 H_B^\beta = y H_A^{-\alpha} H_B^\beta \dots (14)$$

Here y, α, β are constants, y is called the material line; H_A and H_B are each minimum pressure of A and B . The relative line is horizontal beyond $k(AG)$ line. This means that B acts as rigid against A . For example, mild steel acts as rigid against copper. So $k(AG)$ may be called 'the passive-rigid line' and shown as

$$k(AG) = g_1 H_{AG}^{-\gamma_1} = y_1 H_A^{-\alpha} H_{AG}^\beta \dots (15)$$

$$H_{AG} = (g_1/y_1)^{\frac{1}{\gamma_1+\beta}} \cdot H_A^{\frac{\alpha}{\gamma_1+\beta}} = \lambda_1 H_A^{\gamma_2} \dots (16)$$

where g_1, γ_1 are constants.

Fig. 19 Profiles of the deformed surface of the test pieces

$$\lambda_1 = (g_1/y_1)^{\frac{1}{\gamma_1+\beta}}, \quad \gamma_2 = \frac{\alpha}{\gamma_1+\beta_1}$$

H_{AG} is the minimum pressure of B which acts as rigid for A . Next, for example, beginning point of the relative line of brass, O_1 is obtained according to the broken lines. The line $k(GB)$, which is formed by these beginning points, may be called 'active-rigid line'. This means that A acts as rigid for B . So

$$k(GB) = g_2 H_{GB}^{-\gamma_2} = y H_A^{-\alpha} H_{GB}^{-\alpha} \dots (17)$$

$$H_{GB} = (g_2/y)^{\frac{1}{\beta+\gamma_2}} H_A^{\frac{1}{\beta+\gamma_2}} = \lambda_2 H_A^{\gamma_4} \dots (18)$$

where g, γ_4 are constants.

$$\lambda_2 = (g_2/y)^{\frac{1}{\beta+\gamma_2}}, \quad \gamma_4 = \frac{\alpha}{\beta+\gamma_2}$$

H_{GB} is the minimum pressure of A which acts as rigid for B .

(2) Relation between $k(AB)$ and $\kappa(AB)$

$k(AB)$ and $\kappa(AB)$ are plotted on log-log paper as Fig. 22. The relation is written as

$$\kappa(AB) = y_0 k(AB)^{\nu - \log \frac{g_2}{y_0} / \log k(AG)} = y_0 k^{\mu(A)}(AB) \dots (19)$$

Here y_0, g_0, ν are constants.

(3) Assumption of the coefficient of changing roughness

In order to get the coefficient n , we measure Brinell hardness of the materials A, B , calculated H_{AG} from formula (16), substitute it into (15) and calculate $k(AG)$. Also we calculate $k(AB)$ from

Fig. 20 Mean depth of the deformed surface of the test pieces