assumed as $\eta = \alpha + \frac{\gamma}{\xi + \beta}$, where α , β , and γ are constants and η , ξ are the sum and the number of the terms, respectively. On the other side when the stress values converge periodically, the mean values during one period are assumed as their sums. In this way the errors in the stress values caused by calculating infinite terms are considered to be mostly kept under 1 %.

The errors of this method are secondly caused by the fact that all boundary conditions are not satisfied. This is the reason why this method is approximate, but this makes numerical calculations very easy. As the examples show, the simultaneous equations of eight or ten unknowns will give satisfactorily good results as far as the stress values in the interior of the plate are concerned. Besides, if the relations like Eqs. (14), (16), (17) or (18) are prepared, the trouble to solve the simultaneous equations is saved.

5. Conclusions

In the forgoing discussion the outline of this method is explained, and its accuracy is investigated by numerical examples.

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620. 194. 8

Effects of Cycle Frequency on the Corrosion Fatigue Strength*

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The effects of cycle frequency on the fatigue strength of carbon steels under simultaneous corrosion in tap water and saline are studied. The stress σ_c given under corrosion fatigue increases to $\sigma_c k$ owing to the notch effect of corrosion pits. At a certain repetition number of N , $\sigma_c k$ reaches the fatigue strength σ of the same repetition number in air and fractures by corrosion fatigue occur. And the condition of fracture is given by Eq. (5), under consideration that the corrosion effect k increases with time and σ is a function of N alone. Various experimental results are well explained by this criterion. This conception is further extended to the corrosion fatigue combining the static pre-corrosion and also changing cycle frequencies during runs. For the corrosion fatigue under varying stress amplitudes the cumulative damage criterion was found to be applicable. And so the strengths of corrosion fatigue under wide stress conditions may be known.

1. Introduction

It is a special feature of corrosion fatigue that a little variation of test conditions makes rich test results obtained in the past inapplicable. Especially, the effects of cycle frequency may be important. The systematic researches by McAdam⁽¹⁾, where the effect of cyclic stresses in corrosive

agents on fatigue strength in air was studied under various cycle frequencies, are not to be applied to so-called corrosion fatigue. Furthermore, not only corrosion fatigue under constant stress repetitions but corrosion fatigue where cycle frequency or stress amplitude is varied in the course of running and static corrosion in rest may give rise to discussion.

In the present paper, these problems are studied with some corrosion fatigue tests.

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Table 1 Chemical composition and mechanical properties of the material used

Mark	Chemical composition %					Heat treatment	Mechanical properties				
	C	Mn	Si	P	S		Yield point σ_s kg/mm ²	Ultimate strength σ_B kg/mm ²	True tensile strength σ_T kg/mm ²	Elongation ϕ %	Reduction of area ψ %
A	0.44	0.08	0.21	0.033	0.026	Cooled in air from 860°C	36.2	66.6	104.0	25.9	43.5
B	0.17	0.42	0.144	0.20	0.053	Cooled in air from 900°C	32.9	45.8	82.5	40.0	59.4
C	0.44	0.49	0.27	0.016	0.021	Cooled in furnace from 800°C	41.7	64.6	81.0	28.7	48.2

2. Fatigue tests

Chemical composition and mechanical properties of materials used are shown in Table 1. The test piece shown in Fig. 1 (a) is made from the material A and is subjected to alternating plane bending where cycle frequency f is varied in 245, 725, and 2550 cpm under corrosion, while f is 2550 cpm in the air. The testing machine⁽²⁾ is found by inspection to show no effect of frequency on the magnitude of stresses. The test piece shown in Fig. 1 (b) is made from the material B, and is used for the Ono's rotating bending fatigue testing machine under $f=1020$ and 2640 r.p.m. The material C is steel wire and is used for the wire rotating

bending fatigue testing machine reported previously⁽³⁾. The size of specimens is 3 mm in diameter and 450 mm in length and the rotating velocity is 600 and 2300 r.p.m.

Tap water and solution of 1 % NaCl are dropped on the middle part of specimens as corrosive agents.

Each test result is plotted on the S-N diagram and S-t diagram (t in hr) in Fig. 2~4. Fig. 5, where the results of the material A are compared, for example, with the results of Ni-Si steel by McAdam⁽⁴⁾, shows good coincidence. When the cycle frequency is lower, the number of repetition, until the test piece breaks, is fewer and the corresponding time is longer, as already known. According to Speller⁽⁵⁾, the ratios of repetition number or time till break between different frequencies remain constant when the intensity of corrosive action is reduced by inhibitors. And this suggestion is used to explain the mechanism of corrosion fatigue⁽⁶⁾. In this case, however, the test time seems rather shorter if we consider the corrosion fatigue. In the results of the authors, too, the curves in the S-N or S-t diagram are nearly paralleled with each frequency and the repetition number ratio or time ratio are roughly constant for various stress magnitudes when the test time is short. However, when the time till break is sufficiently long under consider-

(a) The material A, for plane bending
(b) The material B, for rotating bending
Fig. 1 Shape of test pieces

Fig. 2 S-N and S-t diagrams of the material A under plane bending

Fig. 3 S-N and S-t diagrams of the material B under rotating bending

Fig. 4 S-N and S-t diagrams of the wiry material C under rotating bending

Fig. 5 Relation between cycle frequency and the corrosion fatigue strength at $N=10^6$ and 5×10^6 (The material A, plane bending)

ably low stress, S - t curves for tap water converge in one curve and the time ratio is reduced to unity. Moreover, the ratio of repetition number or time required for breaking under different cycle frequencies becomes unequal under corrosion of tap water and saline for low stresses.

3. Consideration on corrosion effect

It was shown by McAdam⁽¹⁾ that the mean damage effect of corrosion fatigue R per unit time was shown in $R=CS^n$ when the specimen was subjected to the stress of S . And n denoted a constant whose value was 3~4 for steel under corrosion in tap water and C was a function of cycle frequency f . Since $C=af^{0.8}$ is given from his test results, the damage during t hr becomes

$$\sum R = CS^n t = af^{0.8} S^n t = a' N S^n / f^{0.2}$$

When $R=1$, the specimen is to break, and so the S - N curve may be obtained as

$$N = \frac{f^{0.2}}{a'} \frac{1}{S^n} \quad \dots \dots \dots (1)$$

The equation shows the linear relation between $\log S$ and $\log N$ and the constancy of frequency effect under various magnitudes of stress. Fig. 2~4 do not agree with the equation and the reason for this may be found in his test method differing from the so-called corrosion fatigue.

It is understood that the energy of repeated stresses makes corrosion damage more severe than static corrosion from the fact⁽⁷⁾ that repeated stresses accelerate the diffusion or precipitation at the surface of steel. The reduction of fatigue strength by corrosion is considered due to the notch effect

of corrosion pits. The notch effect k named the corrosion effect, which depends on the corrosive agent and the notch sensitivity of the material, may be a function of time t and cycle frequency f .

When the test of corrosion fatigue is made under a stress of σ_c , the stress actually working on the test piece is $\sigma_c k$, where k is increasing from unity with passage of time. Although the working stress varies with time, the influence of understressing in earlier stage may be little. And the test piece is considered to break at the repetition number N , when $\sigma_c k$ reaches the magnitude σ of the fatigue strength in air for the corresponding repetition number.

The S-N curve in Fig. 6 explains the condition schematically. The test piece breaks at N which is the number of repetition indicated by a point where the S-N curve in air is intersected by the curve showing $\sigma_c k$. And the point A or B is on the S-N curve of corrosion fatigue for each cycle frequency. In this case the S-N curve in air is, as is well known, not influenced by cycle frequency. The condition of fracture under corrosion fatigue is expressed by

$$\sigma_c k(f, t) = \sigma_{(N)} \quad (2)$$

the fatigue strength σ in air being a function of N alone and k being a function of f and t .

Fig. 6 Scheme explaining the effect of cycle frequency on S-N diagram of corrosion fatigue

Fig. 7 Scheme explaining the effect of cycle frequency on S-t diagram of corrosion fatigue

If k is not affected by f , $\sigma_c k$ is higher when f is lower and so the time till the same repetition is longer. Consequently, the S-N curve of corrosion fatigue in lower f is located below the higher one. In the scheme of S-t diagram, Fig. 7, $\sigma_c k$ is expressed by one curve, while the S-t curve in air shifts rightwards when the frequency is lower. The cross points of both curves give the fracture times, and the S-t curve of corrosion fatigue passes A or B in the same manner. Thus, it is understood why number of repetition until the test piece breaks is larger and the corresponding time is shorter when cycle frequency is lower.

The corrosion effect k is given as follows. If the corrosion fatigue test is made by a stress of σ_c and the test piece breaks at N , the ratio of σ/σ_c is equal to the corrosion effect k at the time t required for the same repetition, where σ is a fatigue strength in air at the repetition of N . Fig. 8~10 show k vs. t obtained from test results, and suggest the experimental formula as

$$k = 1 + a \log (bc^f t + d) \quad (3)$$

In the figure, k in saline is smaller when f is lower, and this seems reasonable since repeated stresses accelerate the corrosion. The term of c^f is

Fig. 8 The corrosion effect k of the material A

Fig. 9 The corrosion effect k of the material B

Table 2 Experimental formula of the corrosion effect k

Corrosion fatigue in tap water	The material A, plane bending	$k=1+0.30 \log (0.60 t+1)$
	The material B, rotating bending	$k=1+0.24 \log (0.60 t+1)$
	The material C, rotating bending	$k=1+0.78 \log (0.05 t+1)$
Corrosion fatigue in saline	The material A, plane bending	$k=1+2.2 \log (0.0135 \times 1.00052^f t+1)$
	The material C, rotating bending	$k=1+2.2 \log (0.0178 \times 1.00043^f t+1)$

to be included in k , though c is nearly equal to unity. In mild corrosive agents like tap water, k is almost unrelated to f , namely $c=1$, for every material and every testing condition, and is reduced to

$$k=1+a \log (bt+d) \quad (4)$$

This is the same equation as Tamman got for the thickness of oxide film of metals, the values of the constants a and b being not yet known⁽⁸⁾; d denotes a constant decided by the starting conditions and is equal to unity for the present case.

These constants are estimated from Fig. 8~10 as is shown in Table 2. However, discussion of the values of these constants is not the main subject, but these equations are expected to be substantial for the condition of fracture under corrosion fatigue.

The curves drawn in Fig. 2~4 are the S - N and S - t curves obtained from the condition

$$\sigma_c \{1+a \log (bc^f t+d)\} = \sigma \quad (5)$$

by using the values in Table 2. These closely coincide with the test results. When working stress is low enough to intersect the curves of $\sigma_c k$ with the horizontal line of S - t curve in air as shown in lower portion of Fig. 7, the value of σ in Eq. (5) is constant as it is equal to the endurance limit. Consequently, the time till break remains constant under various cycle frequencies and the S - t curves are united in one curve when k is a function of t alone such as in tap water.

Fig. 10 The corrosion effect k of the material C

4. Influence of static corrosion and of varying cycle frequency

In the experimental formula mentioned above, d took the value of unity because corrosion was never given and k should be unity when $t=0$. If the test piece is subjected to static corrosion previous to running, the value of d must vary. When the test piece breaks at repetition of N' under cyclic stress σ' in air after suffering static corrosion during t' hr, σ/σ' indicates the effect of pre-corrosion, where σ is the fatigue strength in air at N' without any corrosion.

Fig. 11 is obtained from the fatigue tests in air under $\sigma'=25 \text{ kg/mm}^2$ following the static pre-corrosion of t' hr in tap water for the material B. And σ/σ' is given from the figure as

$$\sigma/\sigma' = 1+a \log (b't'+1) = 1+0.24 \log (0.04 t'+1) \quad (6)$$

showing similar values with the test results of the same kind reported by McAdam⁽¹⁾.

When corrosion fatigue tests are made under σ_c' after static corrosion, the total effect of corruptions may be given by

$$\sigma/\sigma_c' = 1+a \log (bt+b't'+1) = 1+0.24 \log (0.6 t+0.04 t'+1) \quad (7)$$

The same material is subjected to corrosion fatigue.

Fig. 11 The effect of static pre-corrosion on the fatigue strength in air and the corrosion fatigue strength, the material B, rotating bending, corrosion in tap water
In fatigue tests in air, $\sigma'=25 \text{ kg/mm}^2$
In fatigue tests in tap water, $\sigma_c'=20 \text{ kg/mm}^2$, $f=2630 \text{ r.p.m.}$

Table 3 Results of corrosion fatigue tests where the cycle frequency is varied in the course of running, the material B, rotating bending, corrosion in tap water

Test stress kg/mm ²	First stage tests			Second stage tests						
	f_1 r.p.m.	t_1 hr	N_1	f_2 rpm	Experimental results			Calculated results		
					t_2 hr	N_2	(t_1+t_2) hr	t_2 hr	N_2	(t_1+t_2) hr
18	1020	24.5	1.5×10^6	2640	32.2	5.10×10^6	56.7	26.5	4.25×10^6	51.0
21	2640	5.43	8.6×10^5	1020	13.5	8.28×10^5	18.93	11.4	6.98×10^5	16.83
23	2640	3.5	5.6×10^5	1020	4.73	2.90×10^5	8.23	5.0	3.06×10^5	8.5

of $\sigma_c' = 20$ kg/mm² after static corrosion both in tap water, and the effect of pre-corrosion σ_c/σ_c' is estimated by using the strength σ_c at the same number of repetition under corrosion fatigue without static corrosion. The test results show close coincidence to the calculated values given by a curve in Fig. 11.

When a test piece is subjected to the corrosion fatigue during t_2 hr or repetitions of N_2 under cycle frequency of f_2 following the corrosion fatigue during t_1 hr or N_1 under frequency of f_1 , the corrosion effect may be given by

$$k = 1 + a \log (bt_1 + bt_2 + 1) \quad (8)$$

and $\sigma_c k$ increases as is shown by the thick curve in Fig. 12 schematically. As the fatigue strength in air depends on repetition number only, the condition of fracture under corrosion fatigue becomes

$$\sigma_c \{1 + a \log (bt_1 + bt_2 + 1)\} = \sigma_{(N_1+N_2)} \quad (9)$$

Table 3 shows the results on the material B tested in tap water, which seems to coincide with the results calculated by Eq. (9) sufficiently for the nature of this sort of tests. When working stress is considerably low, the condition of fracture is appointed only by the sum of time under each frequency since $\sigma_{(N_1+N_2)}$ is invariably the endurance limit.

Altogether, the most general condition of fracture under corrosion fatigue is brought to

Fig. 12 Scheme of S-N diagram of corrosion fatigue where cycle frequency is varied in the course of running

$$\sigma_c [1 + a \log \{b(c^f t_1 + c^f t_2 + \dots) + b' t' + 1\}] = \sigma_{(N_1+N_2, \dots)} \quad (10)$$

where static corrosion is given in rest and also the cycle frequency is varied in the course of running under corrosion in tap water and in saline.

5. Conclusion

Fatigue tests of three materials were carried out under alternating plane bending and rotating bending in various cycle frequencies with corrosion in tap water and in saline. When the cycle frequency is lower, the number of repetition, until the test piece breaks, is fewer while the corresponding time is longer. The stress σ_c given under corrosion fatigue increases to $\sigma_c k$ owing to the notch effect of corrosion pits. At a certain repetition number of N , $\sigma_c k$ reaches the fatigue strength σ of the same repetition number in air and fractures by corrosion fatigue occur. And the condition of fracture is given by Eq. (5), under consideration that the corrosion effect k becomes higher with time and σ is a function of N alone.

When tap water is used as a corrosive agent, the value of c in Eq. (5) is unity and the corrosion effect k is a function of time but not of cycle frequency. Consequently, the time till break becomes constant under various frequencies when the working stress is sufficiently low, and the S - t curves are united in one curve at their lower portions. When an experimental formula of k is made from an S - N curve in air and an S - N curve of corrosion fatigue under an optional frequency, S - N curves of corrosion fatigue may be known under given cycle frequencies.

In the intense corrosive agents such as saline, the corrosion effect k is affected by the cycle frequency to some degree.

When static corrosion is given in rest and the cycle frequency is varied in the course of running, Eq. (10) may be used to estimate the life in corrosion fatigue.

For the corrosion fatigue fracture under varying stress amplitudes the cumulative damage cri-

terion, that is $\sum n_i/N_i=1$, is found to be applicable, and the paper is prepared to be issued shortly⁽⁹⁾.

And so the strengths of corrosion fatigue under wide range of stress conditions can be known.

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534. 121. 1

Vibration of Rectangular Plates with Circular Holes*

By Shin TAKAHASHI**

The frequencies in vibration of rectangular plates with circular holes are studied. The method is due to Ritz and the displacement is expressed in terms of the products of bar-solutions which satisfy the boundary conditions of plate. The kinetic and potential energies are given in the cases of clamped-clamped, clamped-free and clamped-supported ends. Numerical calculations are carried out in the case of the plate of which all edges are fixed and with a hole in centre. Experimental data are also given in the above mentioned case in which the ratio of edge lengths is 1/2 and the agreement with calculations is good.

D. Young and G.B. Warburton⁽¹⁾ have solved the problem of vibration of plates using the bar-solutions. The author has studied the vibrations of rectangular bars with many holes, and moreover he has studied, as sequel to that paper, the vibration of rectangular plates with circular holes. The method followed is, according to that of Ritz, to apply bar-solutions, which satisfy the boundary conditions of the edges (without considering the conditions round the holes) and gives the frequencies of any mode.

Nomenclatures

a, b : lengths of both sides of rectangular plate
 l, m, n : la, mb, ra are x - and y -coordinates and

$$\begin{aligned} S(nl, pl) &= (1 + \lambda_n \lambda_p) \cos(g_n - g_p)l + (-\lambda_n + \lambda_p) \sin(g_n - g_p)l \\ C(nl, pl) &= (1 - \lambda_n \lambda_p) \cos(g_n + g_p)l + (-\lambda_n - \lambda_p) \sin(g_n + g_p)l \\ S'(nl, pl) &= (1 - \lambda_n \lambda_p) \cosh(g_n - g_p)l + (\lambda_n - \lambda_p) \sinh(g_n - g_p)l \\ C'(nl, pl) &= (1 + \lambda_n \lambda_p) \cosh(g_n + g_p)l + (\lambda_n + \lambda_p) \sinh(g_n + g_p)l \\ K(nl, pl) &= \sin g_n l \sinh g_p l + \lambda_n \cos g_n l \sinh g_p l + \lambda_p \sin g_n l \cosh g_p l + \lambda_n \lambda_p \cos g_n l \cosh g_p l \\ G(nl, pl) &= \cos g_n l \cosh g_p l - \lambda_n \sin g_n l \cosh g_p l + \lambda_p \cos g_n l \sinh g_p l - \lambda_n \lambda_p \sin g_n l \sinh g_p l \end{aligned}$$

radius of hole
 p : circular frequency
 q : a/b
 h : thickness of plate
 T, V : maximum kinetic and potential energies
 w : $w \sin pt$ is displacement of plate
 $X_n(x), X_s(y)$: bar solutions
 ν : Poisson's ratio
 D : flexural rigidity of plate
 g_n, λ_n : constants determined from the boundary conditions
 $Q(x)$: $J_1(x)/x$, where J_1 is the Bessel function of the first kind
 $(2R), (2\mathcal{I})$: twice of the real and imaginary parts of the complex followed, respectively

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