532. 57. 082 : 627. 13

Study on Gibson Method of Flow Measuring*

(Influence of Coefficient of Pipe Friction Losses)

By Takao NISHIKAWA** and Minoru SUIA***

Recently, the Gibson method has been used for flow measuring at hydraulic power plants, but the obtained results have never been compared with volume or weight measurements. Therefore the accuracy of the method is uncertain.

Authors made comparative experiments between the weight measurements and the Gibson method by means of a test apparatus of 3 in. diameter pipe line. As a result of the tests, the relative errors of the Gibson method were found to be positive for rapid gate closure, and negative for slow gate closure. The variations of discharge did not affect the discrepancies. The authors proposed such a new treatment of fluid friction coefficient in the Gibson method analysis that the relative errors might become smaller particularly for slow gate closure.

1. Introduction

At the present time the Gibson method is commonly used at hydraulic power plants for flow measuring, but the results obtained by this method have not been compared with those of quantity or weight measurements because of the difficulties involved in the field test. Therefore, the accuracy of the Gibson method needs to be further checked.

Authors have carried out model experiments with a pipe line of three in. in diameter to check the Gibson method in comparison with weight measurements. In this report the results of a series of experiments with a model setup are discussed particularly referring to the friction coefficients of the pipe line.

2. Experimental apparatus^{(1) (2)}

The arrangement of the experimental apparatus is shown in Fig. 1. The zinc-plated pipe line of 75.27 meters in length and 80.7 millimeters in inner diameter leads straight down from the reservoir at a gradient of about 5 degrees. At the entrance to the pipe line a bell mouth is installed to avoid disturbance in flow. The same caution is paid to the joints of pipes, where the inner surface of the pipes makes a smooth continuity. Thus the change of sectional area is avoided throughout the pipe line. The entrance

bell mouth is located more than 1.5 meters or about $20d$ (d is the pipe diameter) off the bottom and the side wall of the reservoir. Thus the side wall and the bottom have conceivably no influences upon the flow through the pipe line. The static head at the outlet of the pipe line is about 9.6 meters. The pipe line constant, l/F , is 1.467×10^4 l/m, with allowances for the entrance bell mouth. In the foregoing, l is the length of the pipe line, and F is the pipe sectional area. The reservoir's dimensions are 10 meters by 10 meters and 4.75 meters in depth. The reservoir is fed by a centrifugal pump with a capacity of 30 l/sec, and the water level is kept unchanged by means of an overflow of 5 meters in length across the vessel. The needle valve at the outlet of the pipe line is operated hydraulically. Initial flow can be varied with the initial opening of the needle valve. The discharge out of the pipe line is measured by weighing. The pressure pulsation picked up by a pressure transducer of strain gauge type is recorded on the electromagnetic oscillograph.

Fig. 1 The arrangement of the experimental apparatus

* Received 14th September, 1965.

** Assistant, Tokyo Institute of Technology.

*** Assistant Professor, Faculty of Engineering, Ibaraki University, Hitachi.

3. Experimental results

As initial quantity flow, 3, 5, 9, 12 l/sec were chosen. The closing time of the valve was varied from 1 to 10 sec. The error of the Gibson method, ε , was obtained for each test.

$$\varepsilon = \frac{Q_g - Q}{Q} \dots \dots \dots (1)$$

where Q_g is the flow rate estimated by the Gibson method and Q is the flow rate by the weighing method. The results of the experiments are shown in Fig. 2. The abscissa is taken as T/μ , where T is the closing time, μ is $2l/\alpha$, i. e. the time during which a pressure wave proceeds forth and back through the pipe line, l is the total length of the pipe line, and α is the travelling velocity of pressure wave. For this particular setup, α is 1 285 m/sec and μ is 0.117 sec. As shown in the figure, while the error turns out to be positive for small T/μ , it decreases with larger T/μ , and when T/μ becomes much larger, the error tends to be negative. The errors spread from +4 % to -4 %, which are considerably large.

4. Treatment of pipe friction coefficient

The causes why the errors vary with T/μ have not been ascertained yet, but it is conceivable that one of the causes lies in the treatment of the friction coefficient. Although in the Gibson method the friction coefficient is usually taken as constant, the velocity through the pipe line changes after the valve starts moving and consequently, the

friction coefficient must vary with flow. If it should be analysed strictly, the friction coefficients corresponding to successive transient flows would have to be used instead of those in the steady state. Since this kind of coefficients was not known, however, we tentatively tried to find out empirically an appropriate formula for it. The loss head was found proportional to $v^{1.87}$ and the coefficient remained constant regardless of the velocity within the range of the experiments.

In accordance with the above-mentioned results, the recovery of the head was taken into account to correct the pressure response curves in the Gibson method. By making use of the new formula for head loss, the values of ε in Fig. 2 were corrected under a treatment to be described later.

For the first step of our experiments, the friction coefficients, λ , as defined in the following expression were determined.

$$h = \lambda \frac{l}{d} \frac{v^2}{2g} \dots \dots \dots (2)$$

where

- h : friction loss head m
- l : length of conduit m
- d : inner diameter of conduit m
- v : flow velocity m/sec
- g : gravitational constant m/sec²

The λ vs. R_e diagram of the conduit is Fig. 3,

where R_e is the Reynolds number $R_e = \frac{vd}{\nu}$, and ν

is kinematic viscosity. In accordance with the change of R_e from 2.7×10^4 to 2.1×10^5 , λ takes the value from 0.025 to 0.018 as shown in Fig. 3. This range of R_e corresponds to the quantity flow 1.75 through 10.5 l/sec, or to the velocity 0.35 through 2.1 m/sec. In the Gibson method, the velocity changes from its maximum value to naught. When the formula of $h \propto v^2$ is employed in making allowance for the friction loss head, λ values for the corresponding R_e should be used at the successive transitional stages. In Fig. 3, Blasius' formula for smooth pipe, $\lambda = 0.3164 R_e^{-1/4}$, and the results of Nikuradse's experiments on rough pipes are compared with our test results. A glance at Fig. 3 indicates that the relative roughness of the conduit is nearly 1.5×10^{-3} .

Let h_i be the loss head consisting of the velocity head and the friction loss head of the conduit. Fig. 4 represents the relation between h_i and v in steady flow. Fig. 4 clarifies the existence of the

Fig. 2 ε vs. T/μ diagram

Fig. 3 λ vs. R_e diagram

formula of $h_i \propto v^{1.87}$ except in the range of small R_e , where the flow assumes a laminar state. In this state, the power order of velocity of 1.87 will take another value. In the Gibson method, however, the Reynolds number involved ranges mostly from 3.5×10^5 to 2.5×10^7 , and the duration of the laminar flow has only minor effect on the analysis. According to the Gibson method commonly used, the formula of $h_i \propto v^2$ with a constant λ has been employed. We made trial calculations of the friction correction based on the formula of $h_i \propto v^{1.87}$ and compared the results with those obtained by the common method.

5. Proposal of a new method of correction owing to pipe friction

Let us assume a pressure vs. time diagram as shown in Fig. 5, where

h_i : head rise at time t while valve is closing m

H_0 : total head when valve is closed m

T : closing time of valve sec

h_{i0} : sum of velocity head and friction loss head at time $t=0$ m

h_i : sum of velocity head and friction loss head at time t m

h_{i0} and h_i in the Gibson method are as follows, respectively,

$$\left. \begin{aligned} h_{i0} &= \left(1 + \lambda_0 \frac{l}{d}\right) \frac{v_0^2}{2g} \\ h_i &= \left(1 + \lambda \frac{l}{d}\right) \frac{v^2}{2g} \end{aligned} \right\} \dots\dots\dots (3)$$

where v_0 is the flow velocity in m/sec in the initial steady state, v is the flow velocity in a transitional state at time t , λ_0 is the friction coefficient of pipe at $v=v_0$, and λ is the one at v . From Eq.

Fig. 4 h_i vs. v diagram

(3) we get

$$h_i = h_{i0} \frac{1 + \lambda \frac{l}{d}}{1 + \lambda_0 \frac{l}{d}} \frac{v^2}{v_0^2} = h_{i0} \frac{\frac{d}{l} + \lambda}{\frac{d}{l} + \lambda_0} \frac{v^2}{v_0^2} \dots\dots\dots (4)$$

In the Gibson method, the next assumption is employed.

$$\lambda = \lambda_0 \dots\dots\dots (5)$$

Thus,

$$h_i = h_{i0} \frac{v^2}{v_0^2} \dots\dots\dots (6)$$

The error due to assumption (5) may be affected by d/l of each pipe line. As already mentioned, we have obtained the formula of $h_i \propto v^{1.87}$ for the conduit. Hence it results in

$$\left. \begin{aligned} h_{i0} &= K_0 v_0'^{1.87} \\ h_i &= K v_0'^{1.87} \end{aligned} \right\} \dots\dots\dots (7)$$

instead of Eq. (3), where K_0 and K are constants, and $K_0 = K$ in this case. While the quantities derived from the formula of $h_i \propto v^{1.87}$ are designated with primes, those derived from the formula $h_i \propto v^2$ have no prime. From Eq. (7) we have

$$h_i = h_{i0} \left(\frac{v'}{v_0'} \right)^{1.87} \dots\dots\dots (8)$$

On the other hand, in the Gibson method

$$\int_0^T h_i dt = \frac{l}{g} (v_0 - v) = \frac{A_i}{\gamma S} \dots\dots\dots (9)$$

where A_i is the area of the pressure vs. time diagram from $t=0$ to t , γ is the pressure coefficient in cm/m, and S is the time coefficient, cm/sec. At $t=T$, $v=0$, hence

$$v_0 = \frac{(g/l) A_T}{\gamma S} \dots\dots\dots (10)$$

From Eqs. (9), (10)

$$v = \frac{A_T - A_i}{\gamma S (l/g)} \dots\dots\dots (11)$$

From Eqs. (10), (11)

$$\left. \begin{aligned} v &= C(A_T - A_i) \\ v_0 &= C A_T \end{aligned} \right\} \dots\dots\dots (12)$$

where $C = \frac{g/l}{\gamma S}$. A_T is the enclosed area of the diagram from $t=0$ to T . Eqs. (9)~(12) are supposed to be derived from the formula that the losses are proportional to the second power of the

Fig. 5 Pressure vs. time diagram

velocity. The equations corresponding to Eq. (12) are

$$\left. \begin{aligned} v' &= C(A_T' - A_t') \\ v_0' &= CA_T' \end{aligned} \right\} \dots\dots\dots (13)$$

Let Δh_t be the difference between h_t and h_t' at time t . Substituting Eqs. (6) and (8) in the following

$$\Delta h_t = h_t - h_t' = h_{t0} \left(\frac{v}{v_0} \right)^2 - h_{t0} \left(\frac{v'}{v_0'} \right)^{1.87} \dots\dots\dots (14)$$

From Eqs. (13), (14)

$$\frac{v}{v_0} = 1 - \frac{A_t}{A_T}, \quad \frac{v'}{v_0'} = 1 - \frac{A_t'}{A_T'}$$

Thus Eq. (14) becomes

$$\Delta h_t = h_{t0} \left(1 - \frac{A_t}{A_T} \right)^2 - h_{t0} \left(1 - \frac{A_t'}{A_T'} \right)^{1.87} \dots\dots\dots (15)$$

Integrating Eq. (15) from $t=0$ to T , we get the area enclosed by OABCO in Fig. 5. Let the relative error, ε_t , be the ratio of the quantity flow corresponding to the abovementioned area to the one corresponding to A_T' ;

$$\varepsilon_t = \frac{Fg/l \int_0^T \Delta h_t dt}{\gamma S F C A_T'} = \frac{1}{A_T'} \int_0^T \Delta h_t dt \dots\dots\dots (16)$$

where F is the cross-sectional area of the conduit, m^2 . ε_t is a negative value. The calculation of the equation can be solved only by the trial and error method. Now, simplifying the problem, relative errors were estimated with an assumed pressure wave pattern. As for the pressure vs. time diagram, we made use of the following assumptions:

- (1) The pressure rises linearly from the time when the valve has started moving until the initial reflected wave comes back.
- (2) The pressure is constant from the time when the initial reflected wave comes back until the valve is closed.
- (3) After the valve is closed, the pressure is subjected to a damped oscillation.

The above-mentioned behavior is the one theoretically derived on the condition that the valve is operated at a constant speed, the losses due to velocity head and friction head being neglected. Therefore we get $Q = Q_0(1 - t/T)$, where Q_0 is the

initial quantity flow, and Q represents quantity flow at time t . Let h_m be the mean pressure head rise, m , during $t=0$ and T , then

$$A_T' = \gamma S h_m T \dots\dots\dots (17)$$

And also $h_t' \propto v^{1.87}$, hence

$$h_t' = K Q^{1.87} = K Q_0^{1.87} \left(1 - \frac{t}{T} \right)^{1.87} \dots\dots\dots (18)$$

$h_{t0} = h_{t0}$ at $t=0$, therefore $K = h_{t0}/Q_0^{1.87}$.

From Eq. (18)

$$h_t' = h_{t0} \left(1 - \frac{t}{T} \right)^{1.87} \dots\dots\dots (19)$$

The foregoing equation is the one representing the recover head, when friction losses are proportional to 1.87 th power of velocity.

In the case of $h_t \propto v^2$, similarly

$$h_t = h_{t0} \left(1 - \frac{t}{T} \right)^2 \dots\dots\dots (20)$$

Eqs. (19), (20) and (15) are substituted in Eq. (16), and the relative error denoted as ϵ instead of ε_t becomes

$$\epsilon = \frac{h_{t0}}{A_T'} \int_0^T \left\{ \left(1 - \frac{t}{T} \right)^2 - \left(1 - \frac{t}{T} \right)^{1.87} \right\} dt$$

With A_T' obtained by Eq. (17)

$$\epsilon = \frac{13}{861} \frac{h_{t0}}{h_m} = 1.51 \times \frac{h_{t0}}{h_m} \% \dots\dots\dots (21)$$

This formula gives the correction to the errors, which are calculated with the formula of $h_t \propto v^2$ associated with Eqs. (1), (2) and (3). Since ϵ in Eq. (21) becomes negative, the amount of correction to our proposed treatment, δ , is given as $\delta = \epsilon + \epsilon$, where ϵ is as in Eq. (1).

The final result (21) is applicable only to our particular pipe line. In actual hydroelectric power stations, the order of power of velocity is larger than 1.87, and hence the amount of correction in actual pipe lines is smaller than in our model. Fig. 6 is obtained through the foregoing procedure of correction to ϵ in Fig. 2. In view of Fig. 6, which shows that the errors, δ , are nearly equal to zero for large T/μ regardless of the quantity flow, the effect of the correction to the errors becomes unnoticeable towards small T/μ . The errors remaining at small T/μ are conceivably due to other causes than friction losses.

In the above discussion, the unsteadiness of flow has not been taken into account. Even so, it was found the friction coefficient of the conduit had a considerable effect upon the results in the Gibson method.

6. Summary

(1) The errors of the common Gibson method applied to 3 in. pipe line are illustrated in Fig. 2.

(2) It is found that there is room for

Fig. 6 Corrected δ vs. T/μ diagram

argument about reducing Eq. (5) with assumption of $\lambda = \lambda_0$, associated with the formula of $h_i \propto v^2$.

(3) The general formula of the correction to the treatment according to item (2) is Eq. (16).

(4) Eq. (21) is the correction to Eq. (2) for our setup, provided the valve is operated at a constant speed.

(5) Fig. 6 is obtained by making corrections to Fig. 2, assuming the valve is shut off at a constant speed.

(6) The correction for the pipe line of hy-

draulic power plants is smaller than that for our model setup.

In closing authors would like to express their greatest gratitude to Professor J. Itaya, Tokyo Institute of Technology, for guiding their project.

References

- (1) J. Itaya et al.: *Trans. Japan Soc. Mech. Engrs.*, Vol. 26, No. 162 (1960), p. 187.
- (2) T. Nishikawa and M. Suita: *Trans. Japan Soc. Mech. Engrs.*, Vol. 30, No. 210 (1964), p. 197.

536. 24: 532. 526

Studies on the Convective Heat Transfer from Rotating Disk*

(1st Report, The Effect of Dissipative Energy on the Laminar Heat Transfer from a Disk Rotating in Uniform Forced Stream)

By IKUO MABUCHI**, TOSHIO TANAKA***, and YUKIHIRO SAKAKIBARA***

This paper deals with a theoretical investigation on the effect of the dissipative energy on the laminar heat transfer from a disk rotating in uniform forced stream. Under the conditions that the fluid is incompressible and its physical properties are constant, the energy equation is solved exactly for the cases where the variation of the surface temperature is expressed by the general quadratic distribution along the disk radius and the disk surface is thermally insulated.

1. Introduction

The laminar flow and heat transfer about a rotating disk were analysed at first† by von Kármán⁽¹⁾ and Millsaps and Pohlhausen⁽²⁾, respectively. Recently, the theoretical extensions to make clear the effects of compressibility⁽³⁾, Prandtl number⁽⁴⁾, variation of surface temperature^{(5) (6)}, etc. have been reported, but these studies are made for a disk rotating in a quiescent fluid.

The laminar flow field about a disk rotating in uniform stream was analysed at first†† by Han-nah⁽⁷⁾. Recently, Schlichting and Truckenbrodt⁽⁹⁾ obtained an approximate solution by applying the momentum-integral method. On the other hand, Yamaga⁽¹⁰⁾ extended the integral method to the heat transfer problem. Recently, the exact nu-

merical solution for heat transfer has been obtained by Tien and Tsuji⁽¹¹⁾. However, the effect of the dissipative energy is not considered in their analytical treatments for heat transfer.

The purpose of this report is to make clear the effect of the dissipative energy on laminar heat transfer from a disk rotating in uniform stream. In this analysis, usual boundary-layer approximation is applied for the thermal field near a disk and the variation of the surface temperature includes the following cases, i.e. (1) isothermal case (2) quadratic distribution (3) thermally insulated case.

2. Nomenclature

a, b : constant of velocity field defined by Eq. (5)

c : specific heat of fluid, and specific heat at constant pressure should be used for the gas.

E_{cr} : local Eckert number of rotating disk with axial forced flow defined by Eq. (37)

F, G, H : dimensionless function of velocity field

* Received 28th October, 1965.

** Professor, Faculty of Engineering, Gifu University, Kagamigahara.

*** Assistant, Faculty of Engineering, Gifu University.

† According to Dorfman⁽¹²⁾, a primary analysis for heat transfer was performed by Kibel (1947).

†† Recently, the exact solution was independently rediscovered by Tifford and Chu⁽⁸⁾.