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Measurement of Plant Dynamics*

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There have been developed many methods to identify a process. Among them, statistical methods using the correlation function are most popular and practical methods. But these methods have been developed independently.

In this paper, it is shown that they can be dealt with systematically when input is white noise. And from the practical point of view, the specified signal named pseudo-random signal is introduced in the analysis.

Some consideration about the characteristics of this signal are presented and it is shown that it is applicable for measuring a zeromemory nonlinearity with ease. Measurement data of a distillation column are also presented. From consideration about these data, the obtained results are found to present the average dynamics of the plant.

1. Introduction

To synthesize a control system, dynamic characteristics of a plant must be determined first, then controllers will be designed to satisfy the given specification. In recently developed adaptive or optimum control systems, identification plays an important role, and the on-line measurement of the dynamics of a plant is required, because the plant has usually time-varying parameters.

Realizing the importance of process identification, many researches concerning these techniques have been developed. Among them, the statistical method developed by N. Wiener and others was found to be one of the useful techniques after the appearance of digital computers. They also presented the philosophy⁽¹⁾ of identification methods to express unknown characteristics of a plant (black box) by several terms of the orthogonal functions (white box) with coefficients determined by products of outputs of both boxes. The method using such a known system is called "model method", and this model method is very popular in the adaptive and optimum control systems.

If "model method" is restricted to systems which are physically realizable, general identification methods developed until now are classified into either the methods with model or the ones without model (see Table 1).

Plants have essentially nonlinear characteristics, but usual dynamics of plants can be approximated as linear with sufficient accuracy. When a plant is linear, its characteristics are expressed in one of the following forms.

- a) weighting function or impulse response
- b) frequency characteristics
- c) transfer function
- d) differential equations or transition equations

To measure dynamics, output signals expressing the plant characteristics must be observed. To obtain these output signals, either i) specified signal or ii) naturally generated signal is applied to the plant as an input test signal. In Table 1, several typical identification methods are classified from the above-mentioned point of view. But the latest methods using pattern recognitions and Lyapunov function are omitted.

2. Basic principle of correlation methods

Numerical treatment of correlation techniques first appeared in the research of heredity, and recently it is used in various fields. In the research of a radar for example, it is used in a form of the matched filter. In the automatic control engineering, it gives useful methods to determine dynamic characteristics and others.

Let the weighting function of the plant, its input and the output, be $g(t)$, $x(t)$ and $y(t)$ respectively, then Eq. (1) exists.

$$y(t) = \int_0^\infty g(\tau)x(t-\tau)d\tau \dots\dots\dots (1)$$

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By multiplying $x(t-\alpha)$ to Eq. (1) and averaging in the time domain, Eq. (2) is derived

$$\phi_{xy}(\alpha) = \int_0^\infty g(\tau) \phi_{xx}(\alpha-\tau) d\tau \quad \dots\dots\dots (2)$$

where $\phi_{xy}(\alpha)$ and $\phi_{xx}(\alpha)$ are the cross-correlation function between $x(t)$ and $y(t)$, and the auto-correlation function of the input respectively. They are defined as follows.

$$\left. \begin{aligned} \phi_{xy}(\alpha) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) y(t+\alpha) dt \\ \phi_{xx}(\alpha) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) x(t+\alpha) dt \end{aligned} \right\} \dots\dots\dots (3)$$

These correlation functions are considered to exist in this paper. Comparing Eq. (1) with Eq. (2), $\phi_{xy}(\alpha)$ can be considered as the output of the system to the input $\phi_{xx}(\alpha)$. If $\phi_{xx}(\alpha)$ is a delta function, $\phi_{xy}(\alpha)$ gives the weighting function directly. This principle gives a powerful identification method, for the test signal needs not have large amplitude even if the output signal is contaminated by noise. This gives possibility to measure the dynamics during the operation of the plant.

In this section let the characteristic property P (later let us simply call this property) be considered when the white noise input is applied to the plant. This property is measured by the circuit shown in Fig. 1.

Let the weighting function of the white box and its output be $h(t)$ and $s(t)$ respectively, then

$$s(t) = \int_0^\infty h(\tau) x(t-\tau) d\tau \quad \dots\dots\dots (1)'$$

From Eqs. (1) and (1)', the property of the plant defined by $P = \phi_{sy}(0)$ is expressed as

$$P = \int_0^\infty h(\sigma) \phi_{xy}(\sigma) d\sigma \quad \dots\dots\dots (4)$$

where the input $x(t)$ is white noise, i.e., auto-correlation function $\phi_{xx}(\alpha) = \delta(\alpha)$. By substituting this relation into Eq. (4), Eq. (5) will be derived.

$$P = \int_0^\infty h(\sigma) g(\sigma) d\sigma \quad \dots\dots\dots (5)$$

The properties of the plant will be given in various forms according to a white box, and Table 2 shows several properties and corresponding weighting functions of white boxes. When $h(t) = t^n e^{-st}$, then $P = G^n(s)$. When a family of orthonormal

Table 2

	$h(\sigma)$	P						
1	$\delta(\sigma-\alpha)$	$g(\alpha)$						
2	σ^n	$G^{(n)}(0)$						
3	<table><tr><td rowspan="2">$e^{-j\omega\sigma}$</td><td>$\cos \omega\sigma$</td></tr><tr><td>$\sin \omega\sigma$</td></tr></table>	$e^{-j\omega\sigma}$	$\cos \omega\sigma$	$\sin \omega\sigma$	<table><tr><td rowspan="2">$G(j\omega)$</td><td>$\Re[G(j\omega)]$</td></tr><tr><td>$-\Im[G(j\omega)]$</td></tr></table>	$G(j\omega)$	$\Re[G(j\omega)]$	$-\Im[G(j\omega)]$
$e^{-j\omega\sigma}$	$\cos \omega\sigma$							
	$\sin \omega\sigma$							
$G(j\omega)$	$\Re[G(j\omega)]$							
	$-\Im[G(j\omega)]$							
4	$\sigma^n e^{-s\sigma}$	$G^n(s)$						
5	$\{\phi_i\}$	a_i						

$$\delta_{ij} = \int_0^\infty \phi_i(\alpha) \phi_j(\alpha) d\alpha, \quad a_i = \int_0^\infty \phi_i(\alpha) g(\alpha) d\alpha$$

Table 1 Several typical identification methods

	Weighting function	Frequency characteristics	Transfer function	Differential function	Nonlinear
Methods using model		Ueno ⁽⁵⁾ (Maclaurin series) Sagara ⁽⁶⁾ (Taylor series) • Kitamori ⁽⁷⁾ (steepest descend)	• Kalman ⁽²⁾ (pulse transfer function) Kitamori ⁽³⁾ (Lagurre func.) Braun et al. ⁽⁴⁾ (orthogonal func.)		Singleton ⁽⁸⁾ , Bose ⁽⁹⁾ (Lagurre-Hermite function) Ikehara ⁽¹⁰⁾ (Volterra series) Gyftopoulos ⁽¹¹⁾ (ternary M-sequence • 2nd order Volterra series) • Kumar ⁽¹²⁾ (differential eq.)
Methods without model	• Goodman ⁽¹³⁾ Anderson ⁽¹⁴⁾ (binary random signal) Balcomb ⁽¹⁵⁾ Furuta • Izawa ⁽¹⁶⁾ , Sato ⁽¹⁷⁾ (pseudo-random binary signal)	Frequency Resp. (sin wave) Hougen ⁽¹⁸⁾ (Pulse Test Method) • Kaya ⁽¹⁹⁾		• Lee ⁽²⁰⁾	

• Denotes the specified input signal

functions $\{\phi_i(t)\}$ is chosen as $\{h_i(t)\}$, the i -th member of the property is given by

$$p_i = a_i = \int_0^\infty \phi_i(\tau) g(\tau) d\tau \quad (6)$$

where

$$\int_0^\infty \phi_i(\tau) \phi_j(\tau) d\tau = \delta_{ij} \quad (7)$$

In Eq. (7), δ_{ij} is Kronecker's delta. Though properties of the plant can be measured by the system shown in Fig. 1 using white noise, it is wellknown that it is impossible to obtain white noise from available physical systems. When the input is not white, i.e., $\phi_{xy}(\alpha)$ can not be regarded as negligible for $\alpha < 0$, the measurement system⁽²¹⁾ shown in Fig. 2 can be used. Both p_1 and p_2 in Fig. 2 can be expressed as follows.

$$\left. \begin{aligned} p_1 &= \int_0^\infty h_1(\sigma) \phi_{xy}(\sigma) d\sigma \\ p_2 &= \int_0^\infty h_2(\sigma) \phi_{yx}(\sigma) d\sigma \end{aligned} \right\} \quad (8)$$

By using the relation $\phi_{xy}(\sigma) = \phi_{yx}(-\sigma)$, and by considering that both $h_1(\sigma)$ and $h_2(\sigma)$ vanish for $\sigma < 0$, the property defined as the sum of p_1 and p_2 gives Eq. (9)

$$P = p_1 + p_2 = \int_{-\infty}^\infty \{h_1(\sigma) + h_2(-\sigma)\} \phi_{xy}(\sigma) d\sigma \quad (9)$$

This relation assures that the analysis of the dynamics of the plant can be achieved with any kind of inputs. When

$$h_1(t) = h_2(t) = \cos \omega t \quad (10)$$

Fig. 1 Measuring system of plant properties

Fig. 2 Improved measuring system of Fig. 1

P is defined as

$$P = \Re[\Phi_{xy}(j\omega)] \quad (11)$$

and when

$$h_1(t) = e^{-j\omega t}, \quad h_2(t) = e^{j\omega t} \quad (12)$$

then

$$P = \Phi_{xy}(j\omega) \quad (13)$$

where $\Phi_{xy}(j\omega)$ is the Fourier transform of $\phi_{xy}(\alpha)$. Frequency characteristics of the plant $G(j\omega)$ can be measured using this P and $\Phi_{xx}(j\omega)$;

$$G(j\omega) = \frac{\Phi_{xy}(j\omega)}{\Phi_{xx}(j\omega)} = \frac{P}{\Phi_{xx}(j\omega)} \quad (14)$$

where $\Phi_{xx}(j\omega)$ is Fourier transform of $\phi_{xx}(\alpha)$.

3. Pseudo-random signal

In the previous chapter, the correlation method with property has been analysed, and been found to be useful for identification of the plant. In this analysis it has been shown that it is desirable to use white noise or white noise-like signal as the input test signal. But it is impossible to obtain white noise from available physical systems as mentioned in Chapter 2. Even if it is available, the correlation calculation defined in Eq. (3) by averaging from the minus infinite to the plus infinite can not be achieved. To cover these shortcomings, the "window" concept is introduced in the power spectrum analysis, but it will still give errors.

If both $x(t)$ and $y(t)$ have the same period, the cross-correlation function defined by Eq. (3) will be rewritten as

$$\left. \begin{aligned} \phi_{xy}(\alpha) &= \frac{1}{ND} \int_{t-ND}^t x(\tau-\alpha) y(\tau) d\tau \\ \phi_{xx}(\alpha) &= \frac{1}{ND} \int_{t-ND}^t x(\tau-\alpha) x(\tau) d\tau \end{aligned} \right\} \quad (15)$$

These equations can be easily calculated.

When $x(t)$ and $y(t)$ are non-periodic stationary signals and only the measured data are available within the period of $(-\frac{T_l}{2}, \frac{T_l}{2})$, their cross-correlation function and auto-correlation function are approximately calculated by Eq. (3)'

$$\left. \begin{aligned} \phi_{xy}(\alpha) &= \frac{1}{T_l - |\alpha|} \int_{-(T_l - |\alpha|)/2}^{(T_l - |\alpha|)/2} x\left(t - \frac{\alpha}{2}\right) \\ &\quad \times y\left(t + \frac{\alpha}{2}\right) dt \\ \phi_{xx}(\alpha) &= \frac{1}{T_l - |\alpha|} \int_{-(T_l - |\alpha|)/2}^{(T_l - |\alpha|)/2} x\left(t - \frac{\alpha}{2}\right) \\ &\quad \times x\left(t + \frac{\alpha}{2}\right) dt \end{aligned} \right\} \quad (3)'$$

These equations contain such errors due to averaging over finite period depending upon α , because the averaging period changes with α .

But when a periodic signal is used as input test signal and the plant is static, i.e.,

$$x(t-N\Delta) = x(t) = x(t+N\Delta)$$

$y(t)$ is also periodic, so $\phi_{xy}(\alpha)$ can be exactly calculated by Eq. (15).

If the signal is periodic and its auto-correlation function has a form like white noise⁽²²⁾ as shown in Fig. 3 (c), this signal will be very preferable for the test signal. This kind of signals are called pseudo-random signals⁽²³⁾. M-sequence⁽²⁴⁾, L-sequence⁽²⁵⁾ and TP sequence⁽²³⁾, etc. have this characteristic. Fig. 3 (a) gives one of pseudo-random signals (M-sequence $N=31$) and hereafter let the pseudo-random signal with the value of 1 and -1 be $I(t)$. The auto-correlation function of this signal $\phi_{II}(\alpha)$, which is given in Eq. (16), is shown in Fig. 3 (b)

$$\phi_{II}(\alpha) = \begin{cases} 1 & ; \alpha = kN\Delta \\ -\frac{1}{N} & ; kN\Delta + \Delta \leq \alpha \leq kN\Delta + (N-1)\Delta \end{cases} \quad (16)$$

where k is integer.

When the actual input signal $x(t)$ is chosen to be

$$x(t) = AI(t) + B \quad (17)$$

where A and B are constant

The auto-correlation function of $x(t)$ is

$$\phi_{xx}(\alpha) = A^2 \phi_{II}(\alpha) + 2AB\bar{I}(\alpha) + B^2 \quad (18)$$

If A and B satisfy Eq. (19), the auto-correlation function $\phi_{xx}(\alpha)$ gives the form shown in Fig. 3 (c)

Fig. 3 Pseudo-random binary signal and its auto-correlation function

$$B = A \left\{ \frac{-1 \pm \sqrt{1+N}}{N} \right\} \quad (19)$$

Thus $\phi_{xx}(\alpha)$ has no more d.c. component between $kN\Delta + \Delta$ and $kN\Delta + (N-1)\Delta$. If Eq. (19) is not satisfied, a new variable $x^0(t)$ is introduced ($NB + A \neq 0$)

$$x^0(t) = I(t) + C \quad (20)$$

The cross-correlation function $\phi_{x^0x}(\alpha)$ between $x^0(t)$ and $x(t)$ has the same form as auto-correlation function $\phi_{xx}(\alpha)$ shown in Fig. 3 (c) under the condition:

$$C = \frac{A-B}{NB+A} \quad (21)$$

then

$$\phi_{x^0x}(\alpha) = \begin{cases} \frac{N+1}{N} A & ; \alpha = kN\Delta \\ 0 & ; kN\Delta + \Delta \leq \alpha \leq kN\Delta + (N-1)\Delta \end{cases} \quad (22)$$

Following the derivation of Eq. (2) from Eq. (1), Eq. (23) can be obtained.

$$\phi_{x^0y}(\alpha) = \int_0^\infty g(\tau) \phi_{x^0x}(\alpha - \tau) d\tau \quad (23)$$

The measuring system of dynamics using this pseudo-random signals is shown in Fig. 4 instead of Fig. 1. By choosing parameters of this pseudo-random signal, it is wellknown that the following equation exists.

$$\phi_{x^0y}(\alpha) = \phi_{x^0x}(0) \Delta \cdot g(\alpha) \triangleq \Phi_{x^0x}(0) g(\alpha) \quad (24)$$

By substituting Eq. (24) into Eq. (5), the properties measured from the system shown in Fig. 4 are approximately given as

$$P = \int_0^\infty \Phi_{x^0x}(0) h(\sigma) g(\sigma) d\sigma \quad (25)$$

where the weighting function of white box $h(\sigma)$ will vanish for $\sigma \geq N\Delta$;

$$h(\sigma) = 0, \quad \sigma \geq N\Delta \quad (26)$$

If the value of $g(\sigma)$ is negligible for $\sigma_m < \sigma$, where $\sigma_m < N\Delta$ Eq. (27) can be used instead of Eq. (25).

$$P = \int_0^{\sigma_m} \Phi_{x^0x}(0) h(\sigma) g(\sigma) d\sigma \quad (27)$$

4. Simultaneous measurement of multiple properties

In the previous chapter, the measurement of one property is discussed. But in multi-variable plants, [shown in Fig. 5 (a)] simultaneous meas-

Fig. 4 Measuring system using pseudo-random binary signal

urement of several properties is frequently required.

When this multi-variable system is linear, one of the outputs of this system can be considered as the linear combination of inputs as shown in Fig. 5 (b) and the authors⁽²⁶⁾ proposed the method of simultaneous measurement of weighting functions of this multi-variable system.

In this paper, this method is applied to identify a system containing nonlinear characteristics with a nonmemory and a linear system such as Fig. 6. Let the nonlinear characteristics be

$$f(x) = ax^3 + bx^2 + cx \quad (28)$$

and the transfer function of the linear part $G(s)$ be

$$G(0) = 1 \quad (29)$$

and let us suppose to determine coefficients of Eq. (26); a , b and c . Input test signal $x(t)$ is made as Eq. (30) from M-sequence signal $I(t)$ which is a kind of pseudo-random signal and has the binary characteristics with the positive value 1 and the negative value -1 .

$$x(t) = A_1 I(t) + A_2 I(t - M\Delta) \quad (30)$$

Output of the nonlinear part is given by

$$\begin{aligned} f(x) = & \{a(A_1^3 + 3A_1A_2^2) + cA_1\}I(t) \\ & + \{a(A_2^3 + 3A_2A_1^2) + cA_2\}I(t - M\Delta) \\ & + 2bA_1A_2I(t - M\Delta) + b(A_1^2 + A_2^2) \dots (31) \end{aligned}$$

Fig. 5 Multi-variable system

Fig. 6 One model of nonlinear plant

For the linear part of the system, the following Eq. (32) exists.

$$\phi_{x^0f}(\alpha) = \int_0^\infty g(\tau) \phi_{x^0f}(\alpha - \tau) d\tau \quad (32)$$

Let $b=0$, first. And let the variable $x^0(t)$ be

$$x^0(t) = I(t) + 1 \quad (33)$$

then $\phi_{x^0f}(\alpha)$ is given by

$$\phi_{x^0f}(\alpha) = \begin{cases} E_1 \frac{N+1}{N} : \alpha - kN\Delta = 0 \\ E_2 \frac{N+1}{N} : \alpha - kN\Delta = M\Delta \\ 0 : \Delta < \alpha - kN\Delta < (M-1)\Delta \\ \text{or} \\ (M+1)\Delta < \alpha - kN\Delta < (N-1)\Delta \end{cases} \quad (34)$$

where k is an integer including zero, and

$$\begin{aligned} E_1 &= a(A_1^3 + 3A_1A_2^2) + cA_1 \\ E_2 &= a(A_2^3 + 3A_2A_1^2) + cA_2 \end{aligned} \quad (35)$$

From Eqs. (32) and (34), Eq. (36) is derived

$$\left. \begin{aligned} \int_{-\Delta}^{(M-1)\Delta} \phi_{x^0f}(\alpha) d\alpha &= E_1 \frac{N+1}{N} \\ \int_{(M-1)\Delta}^{(N-1)\Delta} \phi_{x^0f}(\alpha) d\alpha &= E_2 \frac{N+1}{N} \end{aligned} \right\} \quad (36)$$

Substituting Eq. (36) into Eq. (35), simultaneous equations about unknown parameters a and c are obtained, solvable for a and c . The properties given in the left hand side of Eq. (36) can be measured by the previously mentioned method.

When b of Eq. (28) is not zero secondary, this parameter can be measured as follows. Let M-sequence with binary state 1 or 0 be $\langle m \rangle$, and the M and L units delays denote $D^M \langle m \rangle$ and $D^L \langle m \rangle$ respectively. With the aid of favorable characteristics of M-sequence, the modulo 2 addition $\oplus$ of $\langle m \rangle$ and $D^M \langle m \rangle$ gives certain-units-delayed M-sequence $D^L \langle m \rangle$, such as

$$\langle m \rangle \oplus D^M \langle m \rangle = D^L \langle m \rangle \quad (37)$$

where L can be determined by this M and the characteristic equation to generate M-sequence $\langle m \rangle$. The M-sequence signal $I(t)$ is given by Eq. (38) using $\langle m \rangle$.

$$I(t) = 2m(t) - 1 \quad (38)$$

Considering Eqs. (37) and (38) (see Appendix)

$$I(t) I(t - M\Delta) = -I(t - L\Delta) \quad (39)$$

Substitute Eq. (39) into Eq. (31)

$$\begin{aligned} f(x) = & E_1 I(t) + E_2 I(t - M\Delta) \\ & - E_3 I(t - L\Delta) + E_4 \end{aligned} \quad (40)$$

where E_1 and E_2 are given in Eq. (35) and E_3 and E_4 are given in Eq. (41)

$$\begin{aligned} E_3 &= 2bA_1A_2 \\ E_4 &= b(A_1^2 + A_2^2) \end{aligned} \quad (41)$$

If $x^0(t)$ is introduced as Eq. (33), the following equation holds like Eq. (34).

$$\phi_{x^0 f}(\alpha) = \begin{cases} (E_1 + E_4) \frac{N+1}{N} : \alpha - kN\Delta = 0 \\ (E_2 + E_4) \frac{N+1}{N} : \alpha - kN\Delta = M\Delta \\ (-E_3 + E_4) \frac{N+1}{N} : \alpha - kN\Delta = L\Delta \\ E_4 \left(\frac{N+1}{N} \right) : \text{elsepoints far more than} \\ \Delta \text{ from the above points} \end{cases} \dots\dots\dots(42)$$

Proper choice of M makes $g(\alpha)$ vanish for $\min\{L\Delta, (M-L)\Delta, (N-M)\Delta\} \leq \alpha, (M > L)$. Similarly as Eq. (36), Eq. (43) is obtained

$$\left. \begin{aligned} \int_{-\Delta}^{(L-1)\Delta} \phi_{x^0 y}(\alpha) d\alpha - E_4 \frac{N+1}{N} L\Delta &= E_1 \frac{N+1}{N} \Delta \\ \int_{(L-1)\Delta}^{(M-1)\Delta} \phi_{x^0 y}(\alpha) d\alpha - E_4 \frac{N+1}{N} (M-L)\Delta &= -E_3 \frac{N+1}{N} \Delta \\ \int_{(M-1)\Delta}^{(N-1)\Delta} \phi_{x^0 y}(\alpha) d\alpha - E_4 \frac{N+1}{N} (N-M)\Delta &= E_2 \frac{N+1}{N} \Delta \end{aligned} \right\} \dots\dots\dots(43)$$

From simultaneous Eqs. (35), (41), and (43), values of a , b , and c can be determined. The case, when M -sequence with a period 31Δ generated by

$$\langle m \rangle = D^6 \langle m \rangle \oplus D^2 \langle m \rangle$$

and $M=19$ is used as the input signal, is shown in Fig. 7. This equation tells that $L=11$.

5. Examples of plant dynamics measurement

Correlation methods are generally known to be also applicable for measuring the dynamics of nonlinear systems when input is white, Gaussian and ergodic noise, however dynamics of an ordinary plant containing the nonlinear characteristics are usually expressed by its equivalent linear.

In this chapter, the measured results of dynamics of a distillation column by using L -sequence input signal, are presented*. Considering the plant as linear, the properties are determined by the cross-correlation function between input and output.

In this case, input to the plant is the

reflux flow-rate, which has an amplitude of L' l/hr with a mean of $L l/hr$, and output $y(t)$ is the temperature of certain tray. The cross-correlation function between $x(t)$ and $y(t)$ is calculated using sampled data in the following way.

$$\phi_{xy}(\alpha) = \frac{1}{N\Delta} \sum_{i=1}^{N\Delta/\Delta} x(is-\alpha)y(is)s \dots\dots\dots(44)$$

where $N\Delta$ is the period of the input. The minimum pulse duration Δ is chosen very small in comparison with the time constant of the plant, so the weighting function of the plant is given by

Fig. 7 The cross-correlation function between input and outputs of nonlinear plant

Fig. 8 Distillation column and reflux flow rate and $g(\alpha)$ calculated from correlation function

Fig. 9 $g(t)$ of 9th tray

* These data were obtained by H. Kato and others at HITACHI Laboratory of HITACHI Works, and authors appreciate their kind presentation of data.

$$g(\alpha) = [\phi_{xy}(\alpha) - \bar{\phi}_{xy}] / \bar{\Phi}_{xx}(0) \dots \dots \dots (45)$$

where

$$\bar{\Phi}_{xx}(0) = \frac{N+1}{N} \left(\frac{L'}{2} \right)^2 \Delta$$

and ϕ_{xy} is a mean value of $\phi_{xy}(\alpha)$ for α near but less than $N\Delta$.

In Fig. 8, the dynamics of each liquid and vapor temperature on the 9th tray as output are presented, where $N\Delta = 79$ min and $L' = 16$ l/hr.

In Fig. 9, the dynamics of the same tray measured by using different inputs are presented.

In Figs. 10 and 11, some examples of the dynamics of the liquid and vapor temperatures on the 16th and the 19th tray as output are presented. The feed tray of this distillation column is the 15th tray.

Comparing Figs. 9, 10 and 11, the small amplitude oscillation contained in $g(\alpha)$ becomes more observable for the lower tray (as tray number increases.) These phenomena would be explained due to the nonlinear characteristics as mentioned later.

Fig. 10 $g(t)$ of 16th tray

Fig. 11 $g(t)$ of 19th tray

From the step response of 9th tray of this distillation column, both time constants for the positive step and negative step response are observed larger than the period of here-used input test signal $N\Delta$, so the dynamics calculated by correlation function can be considered as only showing the dynamics of the transportation lag due to input flow change.

From the step response of the 16th tray, the gain for the negative step is more than three times of that for the positive step, and this is in turn to add the oscillatory portion to the weighting function. For the case of the 19th tray having somewhat faster response, the same phenomena were observed. And the time constant here obtained gives about the mean value of the time constants of the negative and positive step response. From Figs. 8, 10 and 11, the dynamics of the liquid temperature and vapor temperature on the 9th tray have different tendencies concerning oscillatory portions, but the dynamics of the liquid temperature and vapor temperature on the 16th

and the 19th tray have the same tendency concerning oscillatory portions. This phenomenon also confirms that the curve of the 9th tray in Fig. 8 shows only the dynamics due to the transportation.

To theoretically investigate this oscillatory phenomenon due to the nonlinear characteristics, the plant is assumed to be described by the following equation,

$$\frac{d}{dt} Y = A(x)Y + B(x)x \dots \dots \dots (46)$$

where Y is state variables of the plant, $x(t)$ is input, A is matrix $l \times l$, $B'(x) = (B_1, \dots, B_l)$. Input signal is binary with positive 1 and negative -1 values and the change of the value occurs only at the discrete time and it holds its value for Δ , i.e., let the discrete time be t_k , $\Delta = t_{k+1} - t_k$. The solution of Eq. (46) is given by

$$Y(t) = \Phi_k(t - t_k) Y(t_k) + \int_{t_k}^t \Phi_k(t - \tau) B(x) \cdot x(\tau) d\tau \dots (47)$$

where

$$t - t_k \leq \Delta$$

and $\Phi_k(t)$ is the solution of Eq. (48)

$$\frac{d}{dt} \Phi_k(t) = A[x(t_k)] \Phi_k(t) \dots \dots \dots (48)$$

where $\Phi_k(0) = I$

After denoting $Y(t_k) = Y_k$, $x(t_k) = x_k$, $\Phi_k(\Delta) = \Phi_k$ Eq. (47) can be rewritten as Eq. (47)'

$$Y_{k+1} = \Phi_k Y_k + G_k x_k \dots \dots \dots (47)'$$

where

$$G_k = \int_{t_k}^{t_{k+1}} \Phi_k(t_{k+1} - \tau) B(x_k) d\tau \quad (49)$$

Eq. (47)' is reduced to

$$Y_{k+1} = \sum_{i=0}^{\infty} \Phi_k \Phi_{k-1} \dots \Phi_{k-i+1} G_{k-i} x_{k-i} \quad (47)''$$

If the present input does not affect the output at more than N future time, Eq. (47) can be rewritten as

$$Y_{k+1} = \sum_{i=0}^{N-1} \Phi_k \dots \Phi_{k-i+1} G_{k-i} x_{k-i} \quad (47)'''$$

The cross-correlation function between input and output $\phi_{xy}(j\Delta)$ is approximately expressed to be Eq. (50) with sufficient accuracy

$$\phi_{xy}(j\Delta) = \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{k=1}^{L-1} x(k-j\Delta) Y(k\Delta) \quad (50)$$

From Eqs. (47)''' and (50)

$$\begin{aligned} \phi_{xy}(j\Delta) = \lim_{L \rightarrow \infty} \frac{1}{L} \left\{ \sum_{k=0}^{L-1} \Phi_k \dots \Phi_{k-j+1} G_{k-j+1} 1 \right. \\ \left. + \sum_{\substack{i=0 \\ i \neq j}}^{N-1} \sum_{k=0}^{L-1} \Phi_k \dots G_{k-i} x_{k-i} x_{k-j} \right\} \quad (51) \end{aligned}$$

The first term of Eq. (51) expresses the case for $i=j$, and the second term does for $i \neq j$. For the linear system, $\Phi_0 = \Phi_1 = \Phi_2 = \dots$, $G_0 = G_1 = \dots$, and the second term can be expressed as

$$\begin{aligned} \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{k=0}^{L-1} \Phi_k \dots G_{k-i} x_{k-i} x_{k-j} = \Phi_0 \dots \\ G_0 \lim_{L \rightarrow \infty} \frac{1}{L} \sum_{k=0}^{L-1} x_{k-i} x_{k-j} \quad (52) \end{aligned}$$

for a certain i . In linear system, the second term of Eq. (51) vanishes if $\phi_{xx}(i-j\Delta) = 0$ for $i \neq j$. But in nonlinear system this does not vanish by the following reason. $x(t)$ is binary signal and Φ_k and G_k are functions of $x(t)$. Let us express Φ_k and G_k at $x=1$ as ${}^{+1}\Phi$ and ${}^{+1}G$, and similarly ${}^{-1}\Phi$ and ${}^{-1}G$ for $x=-1$. To simplify the second term of

Eq. (51), let us define the function of $x_k, x_{k-1}, \dots, \dots, x_{k-i}$ and x_{k-j} as

$$Q_i = Q_i(x_k, x_{k-1}, \dots, x_{k-i}; x_{k-j}) \quad (53)$$

That function denotes the number of coincidence in one period that the successive input signals and x_{k-j} have specified values shown in the parenthesis of Eq. (53). Using this Q_i , Eq. (51) is reduced to

$$\begin{aligned} \phi_{xy}(j\Delta) = \lim_{L \rightarrow \infty} \frac{1}{L} \{ [{}^{-1}\Phi \dots {}^{-1}G \Phi_j(-1, \dots, -1; -1) \\ + {}^{-1}\Phi {}^{+1}\Phi \dots {}^{-1}G \Phi_j(-1, +1, \dots, -1; -1) + \dots] \\ + [{}^{-1}G\{Q_0(-1; -1) - Q_0(-1; +1)\} \\ + {}^{+1}G\{Q_0(1; 1) - Q_0(1; -1)\} + \dots] \quad (54) \end{aligned}$$

If the second term and the following successive terms of Eq. (54) can be considered negligible, only the first term exists. When $x(t)$ is pseudo-random signal, N is substituted for L of Eq. (54), with Q_i obtained beforehand. One example of Q_i for M-sequence with $N=7$ is presented in Table 3 ($i \leq 2$). The Q_i ($i \geq 2$) can be obtained by using the property of M-sequence. From Table 3, each Q_i can be regarded as equal for $0 \leq i \leq n-2$ and $N=2^n-1$. For the input signal to the distillation column, Q_i can not be regarded as equal for larger i , and oscillatory portion was considered to be introduced.

6. Conclusion

The identification methods using correlation function are achieved in various forms, and this paper shows a systematic consideration of these methods. When the white box is linear, the here introduced property expresses the dynamics of the plant and this property can be easily obtained using pseudo-random input signal. Some consideration about the case when the model is linear for the nonlinear plant is presented. In this case, the obtained dynamics can be considered to show

Table 3 Obtained from M-sequence 00101110010 generated by $(D^3 \oplus D^2 \oplus I)$

Q	$j-i$	0	1	2	3	4	5	6	Q	$j-i$	0	1	2	3	4	5	6
Q_0	(1:1)	4	2	2	2	2	2	2	Q_2	(111:1)	1	0	1	0	0	1	1
Q_0	(0:1)	0	2	2	2	2	2	2	Q_2	(011:1)	1	1	0	1	0	0	1
Q_0	(1:0)	0	2	2	2	2	2	2	Q_2	(001:1)	1	1	1	0	1	0	0
Q_0	(0:0)	3	1	1	1	1	1	1	Q_2	(100:1)	0	1	1	1	0	1	0
Q_1	(11:1)	2	1	1	1	1	1	2	Q_2	(010:1)	0	0	1	1	1	0	1
Q_1	(01:1)	2	1	1	1	1	1	0	Q_2	(001:1)	1	0	0	1	1	1	0
Q_1	(10:1)	0	1	1	1	1	1	2	Q_2	(110:1)	0	1	0	0	1	1	1
Q_1	(00:1)	0	1	1	1	1	1	0	Q_2	(000:1)	0	0	0	0	0	0	0
Q_1	(11:0)	0	1	1	1	1	1	0	Q_2	(111:0)	0	1	0	1	1	0	0
Q_1	(01:0)	0	1	1	1	1	1	2	Q_2	(011:0)	0	0	1	0	1	1	0
Q_1	(10:0)	2	1	1	1	1	1	0	Q_2	(001:0)	0	0	0	1	0	1	1
Q_1	(00:0)	1	0	0	0	0	0	1	Q_2	(100:0)	1	0	0	0	1	0	1
									Q_2	(010:0)	1	1	0	0	0	1	0
									Q_2	(101:0)	0	1	1	0	0	0	1
									Q_2	(110:0)	1	0	1	1	0	0	0
									Q_2	(000:0)	0	0	0	0	0	0	0

certain averaged dynamics.

Appendix Pseudo-random signal

The signal, whose auto-correlation has the form shown in Fig. 3 (b) or (c), is called pseudo-random signal, and it belongs to one of the following signals.

- a) Maximum length null sequence
- b) Legendre sequence
- c) Twin Prime sequence
- d) Tompkin sequence⁽²⁸⁾, etc.

In this appendix, considerations about a) and b) are presented.

M-sequence is the binary sequence found by Huffman first. And it is generated from the feedback circuit consisting of shift register and modulo 2 adder. Fig. 12 shows its circuit where the clock pulse period is Δ , and the generated signal has the period $N\Delta$ which depends upon the feedback logic. If $N=2^n-1$, the period is maximum and the signal of this case is called M-sequence. This satisfies Eq. (a).

$$(D^n \oplus C_1 D^{n-1} \oplus \dots \oplus C_{n-1} D) \langle m \rangle = \langle m \rangle \dots (a)$$

where $D^k \langle m \rangle$ denotes k unit delayed sequence of a sequence $\langle m \rangle$, $\oplus$ denotes the modulo 2 addition. From Eq. (a), the arbitrary delayed signal $D^l \langle m \rangle$ can be obtained by modulo 2 addition of certain signals: $D^n \langle m \rangle, \dots, D \langle m \rangle$.

L-sequence is produced from the number theory and satisfies Eq. (b)

$$I(k\Delta) = \left(\frac{k}{N} \right) \dots \dots \dots (b)$$

N is the prime number and $N=4k-1$ and the right hand of Eq. (b) is Legendre symbol, which satisfies

$$k \equiv r^2 \pmod{N}, \text{ then } \left(\frac{k}{N} \right) = 1$$

Fig. 12

$$k \equiv r^2 \pmod{N}, \text{ then } \left(\frac{k}{N} \right) = -1$$

$$k \equiv 0 \pmod{N}, \text{ then } \left(\frac{k}{N} \right) = +1 \text{ or } -1$$

This signal is also generated from the feedback shift register, but the feedback logic becomes non-linear.

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